How to Become a Field and Charge Separatist
(and use only two field variables instead of four)

Mostly following the notation of Balanis (except for not using script fonts for time domain variables nor **M** for magnetic current density), Maxwell's macroscopic equations are

$$\nabla \cdot \mathbf{D} = q_{ev}$$
$$\nabla \cdot \mathbf{B} = q_{mv}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
$$\nabla \times \mathbf{E} = -\mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}$$

The definition of the flux densities, **D** and **B**, include the usual dipole moment densities, **P** and **M**, and can also include higher order moment densities. See for example de Lange and Raab.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} - \frac{1}{2}\nabla \cdot \mathbf{Q} + \ldots$$
$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} - \frac{1}{2}\mu_0 \nabla \cdot \mathbf{N} + \ldots$$

where the second rank tensors, **Q** and **N**, are the electric and magnetic quadrupole moment densities. If we consider **E** and **H** to be pure fields, then the flux densities, **D** and **B**, are a mixture of field and moment density distribution terms. Substituting these relations into Maxwell's equations we have

$$\nabla \cdot \left(\varepsilon_0 \mathbf{E} + \mathbf{P} - \frac{1}{2}\nabla \cdot \mathbf{Q} + \ldots \right) = q_{ev}$$
$$\nabla \cdot \left(\mu_0 \mathbf{H} + \mu_0 \mathbf{M} - \frac{1}{2}\mu_0 \nabla \cdot \mathbf{N} + \ldots \right) = q_{mv}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\left(\varepsilon_0 \mathbf{E} + \mathbf{P} - \frac{1}{2}\nabla \cdot \mathbf{Q} + \ldots \right)$$
$$\nabla \times \mathbf{E} = -\mathbf{J}_m - \frac{\partial}{\partial t}\left(\mu_0 \mathbf{H} + \mu_0 \mathbf{M} - \frac{1}{2}\mu_0 \nabla \cdot \mathbf{N} + \ldots \right)$$

Now we can separate the field dynamics to the left hand side and the charge dynamics to the right hand side.

$$
\begin{aligned}
\varepsilon_0 \nabla \cdot \mathbf{E} &= \tilde{q}_{ev} \\
\mu_0 \nabla \cdot \mathbf{H} &= \tilde{q}_{mv} \\
\nabla \times \mathbf{H} - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} &= \tilde{\mathbf{J}} \\
-\nabla \times \mathbf{E} - \mu_0 \frac{\partial \mathbf{H}}{\partial t} &= \tilde{\mathbf{J}}_m
\end{aligned}
$$

where we have defined generalized charges and currents that arise when materials possess non-uniform polarization densities.

$$
\begin{aligned}
\tilde{q}_{ev} &= q_{ev} - \nabla \cdot \mathbf{P} + \frac{1}{2} \nabla \cdot \nabla \cdot \mathbf{Q} + \ldots \\
\tilde{q}_{mv} &= q_{mv} - \mu_0 \nabla \cdot \mathbf{M} + \frac{1}{2} \mu_0 \nabla \cdot \nabla \cdot \mathbf{N} + \ldots \\
\tilde{\mathbf{J}} &= \mathbf{J} + \frac{\partial \mathbf{P}}{\partial t} - \frac{1}{2} \nabla \cdot \frac{\partial \mathbf{Q}}{\partial t} + \ldots \\
\tilde{\mathbf{J}}_m &= \mathbf{J}_m + \mu_0 \frac{\partial \mathbf{M}}{\partial t} - \frac{1}{2} \mu_0 \nabla \cdot \frac{\partial \mathbf{N}}{\partial t} + \ldots
\end{aligned}
$$

For example, the generalized electric charge includes electric charge and everything that looks like electric charge with respect to its relationship to electric field. One can show that both the standard and generalized charge-current pairs obey continuity equations.

$$
\begin{aligned}
\nabla \cdot \tilde{\mathbf{J}} + \frac{\partial \tilde{q}_{ev}}{\partial t} &= 0 \\
\nabla \cdot \tilde{\mathbf{J}}_m + \frac{\partial \tilde{q}_{mv}}{\partial t} &= 0
\end{aligned}
$$

For completeness we display the time harmonic forms.

$$\begin{aligned}
\varepsilon_0 \nabla \cdot \mathbf{E} &= \tilde{q}_{ev} \\
\mu_0 \nabla \cdot \mathbf{H} &= \tilde{q}_{mv} \\
\nabla \times \mathbf{H} - j\omega\varepsilon_0 \mathbf{E} &= \tilde{\mathbf{J}} \\
-\nabla \times \mathbf{E} - j\omega\mu_0 \mathbf{H} &= \tilde{\mathbf{J}}_m
\end{aligned}$$

$$\begin{aligned}
\tilde{q}_{ev} &= q_{ev} - \nabla \cdot \mathbf{P} + \frac{1}{2} \nabla \cdot \nabla \cdot \mathbf{Q} + \ldots \\
\tilde{q}_{mv} &= q_{mv} - \mu_0 \nabla \cdot \mathbf{M} + \frac{1}{2} \mu_0 \nabla \cdot \nabla \cdot \mathbf{N} + \ldots \\
\tilde{\mathbf{J}} &= \mathbf{J} + j\omega \mathbf{P} - \frac{1}{2} j\omega \nabla \cdot \mathbf{Q} + \ldots \\
\tilde{\mathbf{J}}_m &= \mathbf{J}_m + j\omega\mu_0 \mathbf{M} - \frac{1}{2} j\omega\mu_0 \nabla \cdot \mathbf{N} + \ldots
\end{aligned}$$

$$\begin{aligned}
\nabla \cdot \tilde{\mathbf{J}} + j\omega \tilde{q}_{ev} &= 0 \\
\nabla \cdot \tilde{\mathbf{J}}_m + j\omega \tilde{q}_{mv} &= 0
\end{aligned}$$