

SOR Example Lecture 6

$$\begin{bmatrix} -2 & 1 & 0 & -1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

Answer:

$$x_1 = x_2 = x_3 = 1$$

Let $\omega = 1.0$ $n=1$ Guess $x_1^{(1)} = x_2^{(1)} = x_3^{(1)} = 0$

$$R_i^{(k)} = \frac{1}{a_{ii}} \left[-\sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i}^n a_{ij} x_j^{(k)} + b_i \right]$$

$$x_i^{(k+1)} = x_i^{(k)} + \omega R_i^{(k)}$$

For $k=1$

$$R_1^{(1)} = \frac{1}{-2} \left[(-2)(0) - 1(0) - 0(0) - 1 \right] = 1/2$$

$\swarrow x_1^{(1)}$ $\swarrow x_2^{(1)}$ $\swarrow x_3^{(1)}$

$$x_1^{(2)} = 0 + (1.0)(1/2) = 1/2$$

$$R_2^{(1)} = \frac{1}{-2} \left[1(1/2) - 2(0) - 1(0) - 0 \right] = 1/4$$

$$x_2^{(2)} = 0 + (1.0)(1/4) = 1/4$$

$$R_3^{(1)} = \frac{1}{-2} \left[-0(1/2) - 1(1/4) - 2(0) - 1 \right] = 5/8$$

$$x_3^{(2)} = 0 + (1.0)(5/8) = 5/8$$

$\omega = 1.0$

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$
0	0	0	0
1	.5	.25	.625
2	.625	.625	.8125
3	.8125	.8125	.90625
4	.90625	.90625	.953125
10	.9985...	.9985...	.99926

For $K=5$ iterations

ω	$x_1^{(5)}$	$x_2^{(5)}$	$x_3^{(5)}$
.5	.651	.545	.704
1.0	.953	.953	.976
1.5	.987	.962	1.00
1.75	.953	.906	1.16
1.99	.995	.990	1.94