ECE 5340/6340 HW 6: Finite Element Method

Assignment Objectives:

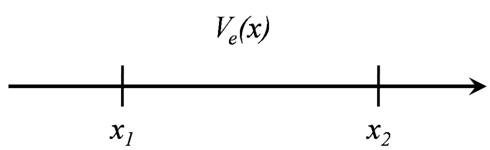
- Understand the theory behind the derivation of the finite element method (FEM).

ASSIGNMENT

This assignment is broken down into two sections. The first section will guide you in the basic MOM theory by taking you step-by-step through the setup. In the second section, you will write a Matlab code that numerically carries out your MOM algorithm.

1. THEORY

A: Begin with a single voltage element in one dimension:



Assume that $V_e(x) = \alpha_1(x)V_1 + \alpha_2(x)V_2$ and derive the elemental shape functions using a linear fit.

$$\alpha_1(x) =$$
$$\alpha_2(x) =$$

Verify the shape function requirements:

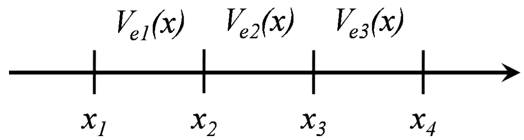
$$\alpha_1(x_1) = \alpha_2(x_2) = 1$$
$$\alpha_1(x_2) = \alpha_2(x_1) = 0$$
$$\alpha_1(x) + \alpha_2(x) = 1 \ .$$

B: Find the elemental coefficient matrix $\mathbf{C}^e$, where each element is defined by the integral

$$C_{ij}^e = \int_{x_1}^{x_2} \frac{\partial \alpha_i}{\partial x} \frac{\partial \alpha_j}{\partial x}\, dx$$

C: Write the matrix equation for the total energy W_e in the element. Be sure to specify each element.

D: Now consider the folowing system consisting of three elements and four points:



Solve for the elemental coefficient matrix of each element.

$$\mathbf{C}^1 =$$
$$\mathbf{C}^2 =$$
$$\mathbf{C}^3 =$$

E: Now use the elemental matrices to assemble the global coefficient matrix.

$$\mathbf{C} =$$

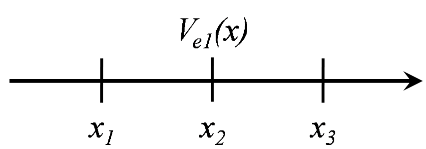
Write the matrix equation for the total energy W of the global system.

F: Minimize the total energy of the system by taking the vector-derivative of W and setting it to zero. HINT: If $\mathbf{A}$ is a symmetric matrix ($\mathbf{A}^T = \mathbf{A}$), then

$$\frac{\partial}{\partial \mathbf{x}} \left(\mathbf{x}^T \mathbf{A} \mathbf{x} \right) = 2\mathbf{A}\mathbf{x}$$

G: Given that $V_1 = 1.0$ V and $V_4 = 0$ V, write the matrix equation you would use to solve for V_2 and V_3 and then solve for them. Given the assumed shape functions, plot $V(x)$ from x_1 to x_4.

H: Go back to step (A) and describe how the size of each of your calculations would have changed if you were using second order linear elements. That is, $V_e = a + bx + cx^2$.



I: For first and second order elements, give the array sizes which would be required by the global coefficient matrix if there were 100 points in the system.

J: Summarize. Would it be more efficient to use first or second order elements for this problem? Would this generally be the case?

K: List the advantages and disadvantages of FEM as you observe them.

2. GRADING BREAKDOWN

- Correct answers, plus introduction and summary sections in your report.
- Total: 50 pts.