# ECE 5340/6340 HW 2: Gaussian Elimination

## Assignment Objectives:

- Understand and program Gaussian elimination.
- Examine the effects of pivoting, scaling, and ill-conditioning.
- Compare results against "canned" matrix solvers in Matlab.
- Learn matrix manipulation with Matlab.

### ASSIGNMENT

Perform the following exercises:

**A:** Attempt to solve the following systems of equations *by hand* using Gaussian elimination WITHOUT pivoting or scaling. Rewrite each system as a matrix and then show all intermediate matrices from your work, specifying each step. Use only 2 decimal places of precision by *truncating* all digits beyond 0.01.

1. Well conditioned system which does not require scaling or pivoting.

$$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 6 \\ 3x_1 &+ 0.5x_2 &+ 5x_3 &= 17 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   Solution: $x_1 = x_2 = x_3 = 2$.

2. Well conditioned system which requires pivoting.

$$\begin{aligned} -x_1 &+ 5x_2 &+ x_3 &= 12 \\ -x_2 &+ 5x_3 &= 13 \\ 5x_1 &- x_2 &= 3 \end{aligned}$$

   Solution: $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.

3. Well-conditioned system which requires scaling. This is a poorly scaled version of Exercise 1. Do not divide by $10^{15}$. Just attempt to solve as is.

$$\begin{aligned} 10^{15}x_1 &+ 10^{15}x_2 &+ 10^{15}x_3 &= 6 \times 10^{15} \\ 3x_1 &+ 0.5x_2 &+ 5x_3 &= 17 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   Solution: $x_1 = x_2 = x_3 = 2$.

4. Ill-conditioned system.

$$\begin{aligned} x_1 &+ x_2 &+ x_3 &= 6 \\ 2x_1 &+ 2.000000001x_2 &+ 1.999999998x_3 &= 12 \\ 2x_1 &+ x_2 &+ 4x_3 &= 14 \end{aligned}$$

   No unique solution.

5. Calculate the determinant of the ill-conditioned matrix in Exercise 4.

**B:** **ECE 6340 students only:** Write a program to solve a matrix equation using Gaussian Elimination. Use your program to solve the systems of equations in Exercises 1 - 4 above. Explain any differences between answers obtained by your program and your initial calculations.

**C:** Use Matlab to solve the systems of equations in Exercises 1 - 4 above. Calculate the condition number and determinant of your matrices. Matlab functions include trap, quad, quad8, and cond (read their help files to see what they are doing).

**D:** Summarize your results.

- Explain when and why pivoting and scaling may be important.
- Explain the effects of ill-conditioning. Can you improve conditioning by going to double precision? If you had infinite precision, could you solve any ill-conditioned system?
- Does Matlab appear to be pivoting? Scaling? Make additional tests if needed to determine this.
- How well does Matlab handle ill-conditioned matrices and what does it do?

## Grading Breakdown

- Hand calculations from part A: 30 pts.
- Program(s) for solving matrix equations (and proof that they work): 50 pts.
- Introduction, summary, conclusions, overall report: 20 pts.
- Total: 100 pts.