Maxwell's Equations - Point Form (Differential Form) Time Domain
^^^ Used for Discrete (numerical) solution

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}\text{source}$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \bullet \mathbf{D} = q_v \text{ source}$$

Constituitive Relationships

Assuming "Simple Media"

- Uniform in space (at least the small discrete space where you are working)
- Single Frequency or Frequency Independent Properties
- NOT time varying

$\mathbf{D} = \varepsilon \mathbf{E}$

$\mathbf{B} = \mu \mathbf{H}$

Units

$\mathbf{E}$ = V/m = N/C — Electric Field
$\mathbf{D}$ = C/m^2 — Electric Flux Density
$\mathbf{H}$ = A/m — Magnetic Field
$\mathbf{B}$ = Wb/m^2 = Tesla = N/(A*m) — Magnetic Flux Density
q_v = C/m^3 — Charge Density (volume)
$\mathbf{J}$ = A/m — Current Density

$$\nabla = \partial \mathbf{x} / \partial x + \partial \mathbf{y} / \partial y + \partial \mathbf{z} / \partial z$$

Maxwell's Equations: Static Form

Substitute $\partial / \partial t = 0$ in above equations.

$$\nabla \times \mathbf{E} = 0 \qquad (1)$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \bullet \mathbf{H} = 0$$

$$\nabla \bullet \mathbf{E} = q_v / \varepsilon \qquad (2)$$

Define Potential = Φ (volts)

$$\mathbf{E} = -\nabla \Phi$$

Substitute into (1) and (2) above.

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$\nabla \times (-\nabla\Phi) = 0$ which satisfies the vector identity: Del x DelA = 0
$\nabla * (-\nabla \Phi) = q_v/\varepsilon$
Which gives:

Poisson's Equation:
$\nabla^2 \Phi = - q_v / \varepsilon$

Laplace's Equation (Poisson's for a source-free region)
$\nabla^2 \Phi = 0$

Maxwell's Equations (Dynamic Forms)

$q_v = 0$ (no free charges floating around)

a) TRANSIENT:
NO further assumptions. Solve the above Maxwell's Equations in the time domain.

b) STEADY STATE:

Assume sinusoidal fields: $\mathbf{E}(t) = \mathbf{E}o\, e^{j\omega t}$

$\frac{\partial \mathbf{E}(t)}{\partial t} = j\omega \mathbf{E}o\, e^{j\omega t} = j\omega\, \mathbf{E}(t)$

Assume Simple Media: $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$

$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ (1)
$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{Jsource}$ (2)
$\nabla \bullet \mathbf{H} = 0$
$\nabla \bullet \mathbf{E} = 0$

Solve for **H** in (1): $\mathbf{H} = \frac{-1}{j\omega\mu} \nabla \times \mathbf{E}$
Plug into (2): $\frac{-1}{j\omega\mu} \nabla \times \nabla \times \mathbf{E} = j\omega\varepsilon\mathbf{E} + \mathbf{J}\text{source}$
Use vector identity: $\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \bullet \mathbf{E}) - \nabla^2 \mathbf{E}$
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ ^^=0

Gives Vector Wave Equation

$(\nabla^2 - \omega^2 \mu\varepsilon)\, \mathbf{E} = j\omega\mu\, \mathbf{J}\text{source}$

wavenumber: $k = \omega \sqrt{(\mu\varepsilon)} = 2\pi / \lambda_\varepsilon$

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wavelength: $\lambda_\varepsilon = c_\varepsilon / f = 2\pi c_\varepsilon / \omega$
speed of propagation : $c_\varepsilon = c_o / \sqrt{(\varepsilon_r)} = 1/\sqrt{(\mu_o \varepsilon_o \varepsilon_r)} = 1/\sqrt{(\mu\varepsilon)}$

This gives the Inhomogenous vector wave equation:

$(\nabla^2 - k^2)\,\mathbf{E} = j\omega\mu\,\mathbf{J}\text{source}$

For a source-free region, we have the Homogeneous vector wave equation

$(\nabla^2 - k^2)\,\mathbf{E} = 0$

For the homogeneous case, the vector equation can be divided into independent x,y,z components, so becomes a scalar wave equation

$(\nabla^2 - k^2)\,E_x = 0$
$(\nabla^2 - k^2)\,E_y = 0$
$(\nabla^2 - k^2)\,E_z = 0$

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