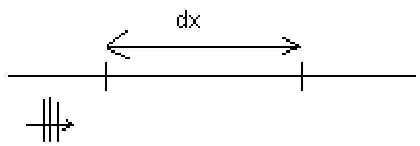
# ECE 5340 / 6340
# FDTD: Stability and Dispersion (Constraints on Cell Size and Time Step)

For FDTD Stability we have to choose the time step (dt) small enough. If the simulation becomes unstable, it will blow up. For stability, dt is related to dx.




How far will wave travel in 1 time step?

$$dx = (dt)\,(c)$$

c is the speed of light. Usually we use the speed of light in vacuum, because that is the fastest it will go. For a simulation that does not have any air cells, we can let c be the fastest velocity of propagation in the model. This is typically in the cell with the lowest conductivity and permittivity.

It is OK to sample more often in time, but not less.

$$dx \geq (dt)(c) \quad ; \quad dt \leq dx / c$$

In 2D: $dt \leq 1/\left(c_{max}\, \mathrm{sqrt}\left(1/dx^2 + 1/dy^2\right)\right)$

For $dx = dy$ → $dt \leq dx/\left(\mathrm{sqrt}(2)\, c_{max}\right)$

In 3D: $dt \leq 1/\left(c_{max}\, \mathrm{sqrt}\left(1/dx^2 + 1/dy^2 + 1/dz^2\right)\right)$

For $dx = dy = dz$ → $dt \leq dx/\left(\mathrm{sqrt}(3)\, c_{max}\right)$

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Numerical Dispersion: (dx)

Dispersion:

- wave propagation velocity in numerical simulation may vary with frequency, direction of propagation, and dx.
- happens naturally in non-air


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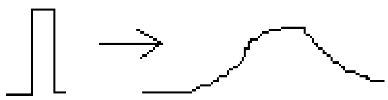
- happens numerically (error) in differential forms because of error in numerical differentiation

Example: Pulse dispersion



See Dispersion curves in Taflove FDTD handout.

dx ≈ $\lambda$ / 10 gives less than 10% error.
This is similar to what you observed in the numerical differentiation homework.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