Key

UNIVERSITY OF UTAH
ELECTRICAL ENGINEERING DEPARTMENT

EE 553 FINITE DIFFERENCE TIME DOMAIN (FDTD)

HOMEWORK #3

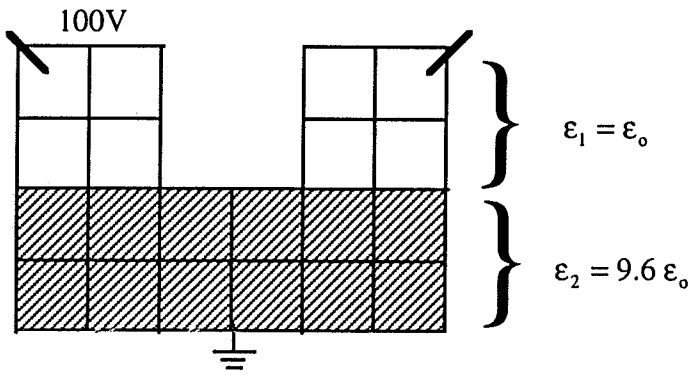
~~April 12, 1996~~
Due: April ~~19~~ 18, 1996

1. Do problem 3.7 in your text (answer given in text).

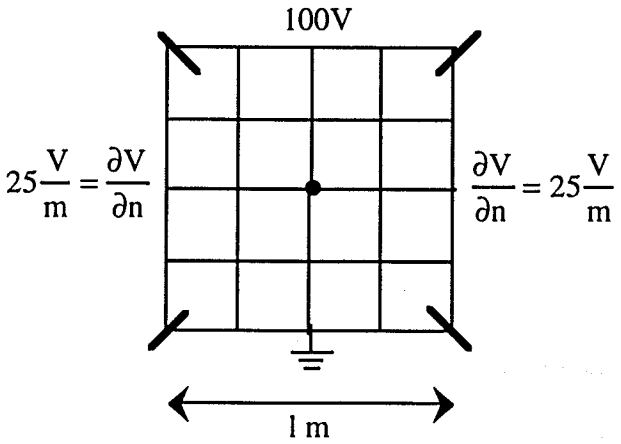
   a. Write the matrix without using symmetry, and solve.

   b. Rewrite the matrix using symmetry and solve again.

2. Repeat for the case below (with symmetry)

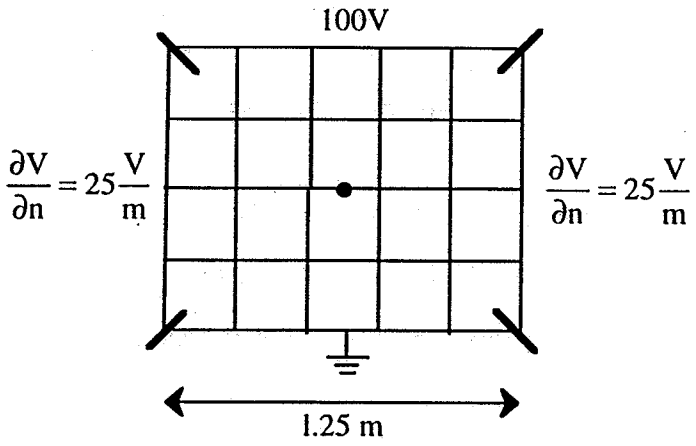


3. Find the potential at the center.



Let h = 0.25m

4. Find the potential at the center



Hand in: All calculations used to compute matrices, including how you numbered the nodes.

Hand in: Matrix equations to be solved (print out matrix $\bar{A}$ and vector $\bar{b}$).

Hand in: Solution, and tell what method you used to solve $\bar{A}\,\bar{x} = \bar{b}$ (i.e. Mathab, Linpack, your gaussian elimination code).

Your grade:

Problem 1a _____ 20
Problem 1b _____ 20
Problem 2 _____ 20
Problem 3 _____ 20
Problem 4 _____ 20

Total _____ 100

How long did you spend on this homework? _____

How much did you learn?

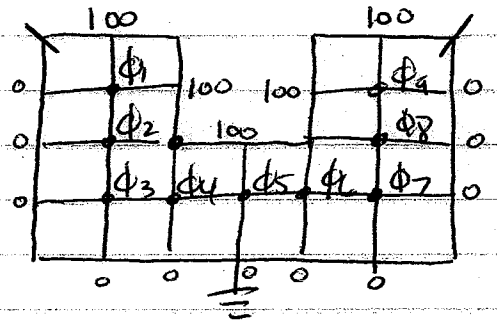
a lot some a little

How difficult was this assignment?

too hard just right too easy

HW #3 Key

#1a



$q_v = 0$

$$\begin{bmatrix} -4 & 1 & & & & & & & \\ 1 & -4 & 1 & & & & & & \\ & 1 & -4 & 1 & & & & & \\ & & 1 & -4 & 1 & & & & \\ & & & 1 & -4 & 1 & & & \\ & & & & 1 & -4 & 1 & & \\ & & & & & 1 & -4 & 1 & \\ & & & & & & 1 & -4 & 1 \\ & & & & & & & 1 & -4 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \\ \phi_9 \end{bmatrix} = \begin{bmatrix} -200 \\ -100 \\ 0 \\ -100 \\ -100 \\ -100 \\ 0 \\ -100 \\ -200 \end{bmatrix}$$

Row 1: $0+100+100+\phi_2-4\phi_1=0$ $\quad Ax=b$

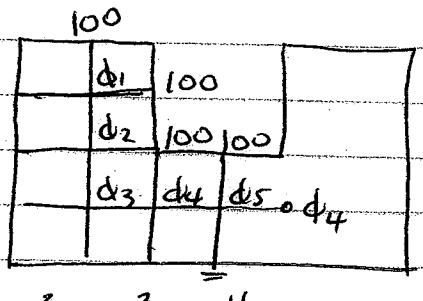
2: $100+\phi_1+\phi_3+0-4\phi_2=0$

Solve: using Matlab

$\phi_1 = \phi_9$	=	61.4641
$\phi_2 = \phi_8$	=	45.8564
$\phi_3 = \phi_7$	=	21.9613
$\phi_4 = \phi_6$	=	41.9890
ϕ_5	=	45.9945

$X = A \backslash b$

#16



$$\begin{bmatrix} -4 & 1 & & & \\ 1 & -4 & 1 & & \\ & 1 & -4 & 1 & \\ & & 1 & -4 & 1 \\ & & & 2 & -4 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} -200 \\ -100 \\ 0 \\ -100 \\ -100 \end{bmatrix}$$

Rows 1-4 are same as before

Row 5: $100 + \phi_4 + \phi_4 + 0 - 4\phi_5 = 0$

$\phi_1 = 61.4641$

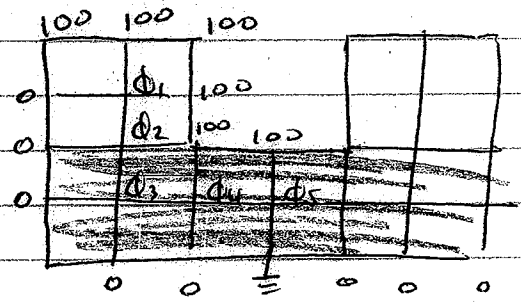
$\phi_2 = 45.8564$

$\phi_3 = 21.9613$

$\phi_4 = 41.9890$

$\phi_5 = 45.9945$

#2



Formula for interface

$$\varepsilon_{r_1}\phi_{ij+1} + \varepsilon_{r_2}\phi_{ij-1} + \left(\frac{\varepsilon_{r_1}+\varepsilon_{r_2}}{2}\right)\left(\phi_{i-1j} + \phi_{i+1j}\right) - 4\left(\frac{\varepsilon_{r_1}+\varepsilon_{r_2}}{2}\right)\phi_{ij} = 0$$

Rows 1, 3, 4, 5 remain the same:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -4 & 1 & & & \\ 1 & -4(5.3) & 9.6 & & \\ & 1 & -4 & 1 & \\ & & 1 & -4 & 1 \\ & & & 2 & -4 \end{bmatrix} \end{array} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} -200 \\ -530 \\ 0 \\ -100 \\ -100 \end{bmatrix}$$

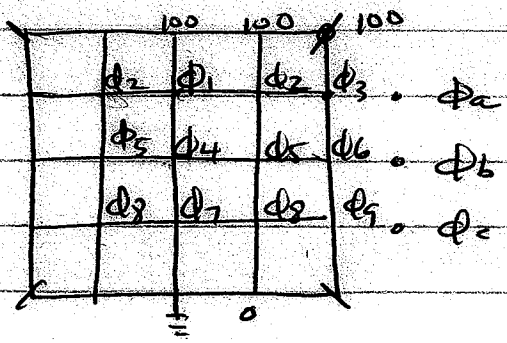
$5.3 = \frac{21.2}{4}$ ← 21.2

Row 2: $(1)\phi_1 + 9.6\phi_3 + 100\left(\frac{10.6}{2}\right) - 4\left(\frac{10.6}{2}\right)\phi_2 = 0$ (5.3)

$\phi_1 = 59.15$
$\phi_2 = 36.6$
$\phi_3 = 19.47$
$\phi_4 = 41.28$
$\phi_5 = 59.64$

Let $h = .25_m$

#3



$\frac{\partial V}{\partial x} = 25\ V/m$

$q_v = 0$

	1	2	3	4	5	6	7	8	9		
1	-4	2	0	1	0	0	0	0	0	ϕ_1	-100
2	1	-4	1	0	1	0	0	0	0	ϕ_2	-100
3	0	(2)	-4	0	0	1	0	0	0	ϕ_3	-112.5
4	1	0	0	-4	2	0	1	0	0	ϕ_4	0
5	0	1	0	1	-4	1	0	1	0	ϕ_5	0
6	0	0	1	0	2	-4	0	0	1	ϕ_6	-12.5
7	0	0	0	1	0	0	-4	2	0	ϕ_7	0
8	0	0	0	0	1	0	1	-4	1	ϕ_8	0
9	0	0	0	0	0	1	0	2	-4	ϕ_9	-12.5

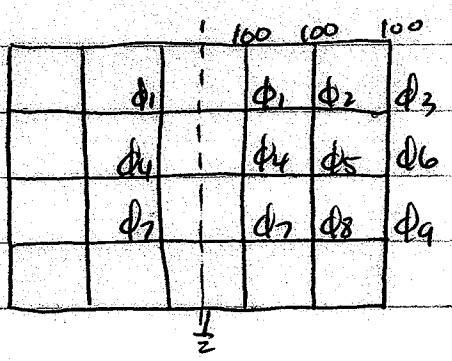
$\frac{\partial V}{\partial x} = \left(\frac{\phi_a - \phi_2}{2h}\right) = 25\ V/m$; $\phi_a = 50h + \phi_2$

$h^2(100 + \phi_2 + \phi_6 + \phi_a - 4\phi_3) = -q_v/\varepsilon = 0$

$100 + \phi_2 + \phi_6 + 50(0.25) + \phi_2 - 4\phi_3 = 0$

$\phi_4 =$ [illegible]

#4



$\frac{\partial V}{\partial x} = 25 V/m$

	1	2	3	4	5	6	7	8	9			
1	-3	1	0	1	0	0	0	0	0	ϕ_1	=	-100
2	1	-4	1	0	1	0	0	0	0	ϕ_2		-100
3	0	2	-4	0	0	1	0	0	0	ϕ_3		-112.5
4	1	0	0	-3	1	0	1	0	0	ϕ_4		0
5	0	1	0	1	-4	1	0	1	0	ϕ_5		0
6	0	0	1	0	2	-4	0	0	1	ϕ_6		-12.5
7	0	0	0	1	0	0	-3	1	0	ϕ_7		0
8	0	0	0	0	1	0	1	-4	1	ϕ_8		0
9	0	0	0	0	0	1	0	2	-4	ϕ_9		-12.5

Row 1: $100 + \phi_1 + \phi_4 + \phi_2 - 4\phi_1 = 0$

$\phi_4 =$ [illegible] = center