ECE 6130 Impedance and Admittance Matrices and S-Parameters

Text Sections: 4.2, 4.3

Describe Z and S matrices, how to compute them, and how to convert between them. See for example Chapter 4, Problems 7,9

**Impedance Matrix:**

DRAW an N-port network.

Impedance matrix is used to model V and I relations for all ports.

\[ Z_{ij} = \frac{V_i}{I_j} \text{ with } I_k = 0 \text{ for } k \neq j \]

1) Open all other ports except j
2) Drive port j with current \( I_j \)
3) Read \( V_i \)
4) Compute \( Z_{ij} \)

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{1N} \\
Z_{21} & Z_{22} & Z_{2N} \\
\vdots & \vdots & \vdots \\
Z_{N1} & Z_{N2} & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
\]

OR: \( V = ZI \)

**Admittance Matrix:**

\( I = YV \)

\( Y = Z^{-1} \) (matrices are inverses of each other)

**Reciprocal Network:**

\( Z_{ij} = Z_{ji} \)

Examples of reciprocal networks: any R,L,C network
Examples of non-reciprocal networks: transistors, amplifiers, attenuators

**Lossless Network:**

Real \( (Z_{ij}) = 0 \) \( \ll Z_{ij} \) is strictly imaginary (change of phase, but no attenuation)

**EXAMPLE:** T-Network
Find $Z_{11}$:

$I_2 = 0$; $V_1 = I_1 (R_a + R_c)$; $Z_{11} = \frac{V_1}{I_1} = R_a + R_c$

Find $Z_{12}$:

$I_1 = 0$; $V_2 = I_2 (Z_b + Z_c)$; $V_1 = V_2 \frac{Z_c}{Z_b + Z_c}$; $Z_{12} = \frac{V_1}{I_2} = Z_c$

Find $Z_{21}$:

$I_2 = 0$; $V_1 = I_1 (Z_a + Z_c)$; $V_2 = V_1 \frac{Z_c}{Z_a + Z_c}$; $Z_{21} = V_2 / I_1 = Z_c = Z_{12}$

Find $Z_{22}$:

$I_1 = 0$; $V_2 = I_2 (Z_b + Z_c)$; $Z_{22} = V_2 / I_2 = Z_b + Z_c$

Scattering Matrix (S-parameters)

$$
\begin{bmatrix}
V_{1}^-
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{1N} \\
S_{21} & S_{22} & S_{2N}
\end{bmatrix}
\begin{bmatrix}
V_{1}^+ \\
V_{2}^+ \\
V_{N}^+
\end{bmatrix}
$$

Where

$S_{ij} = V_{i}^- / V_{j}^+$ when $V_{k}^+ = 0$ for $k \neq j$

1) Terminate all ports except $j$ with matched load.
2) Drive port $j$ with $V_{j}^+$
3) Measure reflected voltage $V_{i}^-$ on port $i$.

EXAMPLE: 3dB attenuator

Find $S_{11}$:

$Z_2 = 50$ ohms; $Z_{in(port 1)} = 8.56 + (141.8 \parallel (8.56 + 50)) = 50$ ohms

$V_1^- = 0$ (no reflection)

$S_{11} = V_{1}^- / V_{1}^+ = 0$
Find S22:
Circuit is symmetric.
$S_{22} = S_{11}$

Find S12: