3.5 MICROSTRIP LINE STRUCTURES

Microstrip line may well be the most popular transmission line structure. Ease of fabrication by photographic techniques and a good range of impedances and couplings allow it to be used for a wide variety of circuit components. Harold Wheeler developed a planar transmission line (two coplanar strips) which could be rolled up in 1936 and a stripline-like structure in 1942. Flat coaxial transmission line was used by V.H. Rumsey and H.W. Jamieson in WWII. Coaxial cable was first adapted to a flat configuration using printed circuit techniques by Barrett. This then evolved into stripline after WWII. The first use of the microstrip-line configuration was reported by engineers at the Federal Telecommunications Research Laboratories (a division of ITT) sometime after 1949 [2].

REFERENCES


3.5.1 Microstrip Line

This is probably the most common and most analyzed transmission line structure. It is easy to use and has a good range of practical impedances. Equations have been calculated by an incredible variety of techniques both analytic and computational.
Improvements in Schneider's \( \varepsilon_{\text{ef}} \) made by Hamnerstad and Bakkadi [22] are given here for \( w / h < 1.0 \):

\[
\varepsilon_{\text{ef}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ \left( 1 + \frac{12 h}{w} \right)^{-0.5} + 0.04 \left( 10 - \frac{w}{h} \right) \right]
\] (3.5.12)

and for \( w / h > 1.0 \):

\[
\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1.0}{2} + \frac{\varepsilon_r - 1.0}{2} \left( 1 - \frac{12.0 h}{w} \right)^{-0.5}
\] (3.5.13)

The equations for \( \varepsilon_{\text{eff}} \) are accurate to within 1% for:

- \( \varepsilon \leq 16 \) (\( \leq 2\% \) error \( \varepsilon \leq 16 \))
- \( 0.05 \leq \frac{w}{h} \leq 20.0 \) (\( \leq 2\% \) error \( \varepsilon \leq 16 \))
- \( 0 \leq h \leq 0.13 \)

The thickness of the trace can be corrected by relating it to an equivalent change in the width. Owens and Potok [40] examined a number of formulas for this correction and found that Wheeler's is the most accurate:

\[
\frac{\Delta w}{w} = \frac{1.0}{11.2} \left[ \frac{4}{\pi} \left( \frac{1}{\pi} + \frac{1}{w / t + 1} \right) \right] \] (3.5.14)

\[
w = w + \Delta w
\] (3.5.15)

\[
\Delta \gamma = \Delta w \left( \frac{1.0 + 1.0 / \varepsilon_r}{2.0} \right)
\] (3.5.16)

Error in \( \Delta \gamma \) is less than 2% for any \( \varepsilon_r, \gamma \).

### 3.5.1.1 Frequency Dependences of Microstrip Line

The frequency dependency (dispersion) of the microstrip line's \( \varepsilon_{\text{eff}} \) can be calculated with (see also [35]):

\[
\varepsilon_{\text{eff}}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff}}(f = 0)}{1.0 + P(f)}
\] (3.5.17)

\[
P(f) = P_1 P_2 (0.1844 - P_3 P_4) \times (10.0 f h)
\] (3.5.18)

\[
P_1 = 0.27418 + \left[ 0.615 + \frac{0.525}{(1.0 + 0.157 f h)^{20}} \right] \frac{w}{h} - 0.06583 \ e^{-8.7513 w / h}
\] (3.5.19)

\[
P_2 = 0.3362 \left[ 1.0 - e^{-0.3441 \varepsilon_r} \right]
\] (3.5.20)

\[
P_3 = 0.036; e^{-4.6 w / h} \left[ 1.0 - e^{- (f h / 3.87)^{4.97}} \right]
\] (3.5.21)

\[
P_4 = 1.0 - 2.751 \left[ 1.0 - e^{- (f h / 15.916)^{3}} \right]
\] (3.5.22)

where \( f \) is in GHz and \( h \) is in \( \text{cm} \). The accuracy of this correction is better than 0.6% for

- \( 0.1 \leq w / h \leq 100.0 \)
- \( 1.3 \leq \varepsilon_r \leq 20.0 \)
- \( 0 \leq h / \lambda \leq 0.13 \)

Atwater [2] compares several corrections for \( \varepsilon_{\text{eff}}(f) \) with actual measurement data from eight published papers and found the above correction to be the best (by an admitted small margin). His version of the equation is a rearranged equivalent.

### 3.5.1.2 Conductor Losses

Conductor losses are a result of several contributing factors: the actual conductivity of the material, the frequency-dependent skin effect losses, and the surface roughness losses.
caused by the lengthened path at the surface. Conductor losses due to skin effect and the metal's conductivity are given in [53]. For \( w/h \leq 1 \):

\[
\alpha_c = \frac{10}{\pi \ln 10.0} \frac{1.0}{Z_0 c} \frac{h}{Z_0 c} \frac{1.0 + \frac{h}{w} \frac{\partial w}{\partial x}}{1.0 + \frac{\partial w}{\partial x}} \quad \text{(dB/ unit length)}
\]

For \( w/h \geq 1 \):

\[
\alpha_c = \frac{Z_0 R_{g}}{720.0 \pi^2 h \ln 10.0} \left[ 1.0 + \frac{0.14 h^2}{w^2} + 6.0 \frac{h^2}{w^2} \left( 1.0 - \frac{h}{w} \right) \right] \times \left( 1.0 + \frac{w}{h} - \frac{\partial w}{\partial x} \right) \quad \text{(dB/ unit length)}
\]

where \( \alpha_c \leq \frac{1}{2 \pi} \)

\[
\frac{\partial w}{\partial x} = \frac{1.0}{\pi} \ln \frac{4.0 \pi w}{t}
\]

for \( w/h \geq \frac{1}{2 \pi} \)

\[
\frac{\partial w}{\partial x} = \frac{1.8}{\pi} \ln \frac{2.0 t}{\varphi}
\]

These equations are valid for ground and strip conductivities the same and

\( i << h \)

\( i < h/2.0 \)

\( \partial w/\partial x > 1.0 \)

Pucel et al. [42] also give equations for the conduction losses.

### 3.5.1.2.1 Effect of Ground Plane on Conductor Losses

The ground plane's resistance can be a significant factor as frequency increases. At low frequencies the current in the ground plane is spread over the full width of the ground and the current is evenly distributed in the center conductor. Because of this, the ground plane contributes little resistance relative to the center conductor. As frequency increases the current in the line begins to concentrate in the sides and bottom of the trace. Simultaneously, the ground current begins to concentrate in the area directly under the strip, increasing the effective resistance. Faraji-Dana and Chow [16] give a closed-form equation suitable for design calculations. The resistance of the ground plane is \( R_g \)

\[
R_g = 0.55 R_{dc} \sqrt{\frac{\tau}{20}} \sqrt{\frac{t}{w}} \left( 1.0 - e^{-w'/1.2} \right) P
\]

where

\[
w' = w/h
\]

\[
F = \sqrt{2.0} \mu \sigma f w t
\]

\( R_{dc} \) is the dc resistance of the signal conductor

The total ac resistance for use in the microstrip line model \( s R_i \).

\[
R_i = R_g + R_g
\]

### 3.5.1.2.2 Effect of Conductor Surface Roughness

The conductor surface analyzed in the formulas above are perfectly smooth. The physical processes which make real world conductors create scratches, bumps, and random bumps. See Tanaka and Okida [62] for some scanning microscope photographs of typical conductor surfaces.

Conductor surface roughness losses are discussed thoroughly in Sanderson [53]. Sanderson also reports on a technique used for calculating the roughness effect on the losses, \( Z_k \) and \( V_p \). Higher than expected losses were experienced in stainless steel conductors that can be modeled by a lumped resistance. Good agreement between the predicted performance and the experimental measurements was found from 20 MHz to 100 GHz in the other cases. Saac [50] gives a plot from Lending [32] for losses as a function of surface roughness. In [20], an approximate formula is given for the conductor losses due to roughness:

\[
\alpha_c = \alpha_{c,0} \left[ 1.0 + \frac{2.0}{\pi} \tan^{-1} \left[ 1.40 \left( \frac{\Delta h}{h} \right) \right] \right]
\]

where

\( \alpha_{c,0} \) is the conductor losses calculated for a perfectly smooth conductor; equations (3.5.1.12) and (3.5.1.13)

\[
\alpha_c = \frac{10}{\pi \ln 10.0} \frac{1.0}{Z_0 c} \frac{h}{Z_0 c} \frac{1.0 + \frac{h}{w} \frac{\partial w}{\partial x}}{1.0 + \frac{\partial w}{\partial x}} \quad \text{(dB/ unit length)}
\]
\[ \Delta = \text{therms surface roughness} \]

\[ \delta = \text{the skin depth} \]

The results of a number of references [20, 32, 39, 60, 65] are summarized in Figure 3.5.1.2 below. This figure shows good agreement between Morgan's data and the Goldfarb close-form equation.

**Figure 3.5.1.2.21: Microstrip Line Losses vs. Surface Roughness**

\[
\alpha_r = 60 \left( \frac{2 \pi \hbar}{\lambda_0} \right)^2 F(\varepsilon_{\text{eff}})
\]

where \( F(\varepsilon_{\text{eff}}) \) is:

\[
F(\varepsilon_{\text{eff}}) = \frac{\varepsilon_{\text{eff}} + 1.0}{\varepsilon_{\text{eff}}} - \frac{(\varepsilon_{\text{eff}} - 1.0)^2}{2.0 \varepsilon_{\text{eff}}} \log \left( \frac{\sqrt{\varepsilon_{\text{eff}} + 1.0}}{\sqrt{\varepsilon_{\text{eff}} - 1.0}} \right), \text{ for an open-circuited line}
\]

\[
F(\varepsilon_{\text{eff}}) = 10 - \frac{(\varepsilon_{\text{eff}} - 1.0)}{2.0 \sqrt{\varepsilon_{\text{eff}}}} \log \left( \frac{\sqrt{\varepsilon_{\text{eff}} + 1.0}}{\sqrt{\varepsilon_{\text{eff}} - 1.0}} \right), \text{ for a matched line}
\]

### 3.5.1.5 Higher Order Mode in Microstrip Line

The microstrip line structure is not strictly TEM; other modes may propagate. The lowest order TE1 mode frequency is [69]:

\[
f_c = \frac{c}{4.0 \hbar \sqrt{\varepsilon_r - 1.0}}
\]

These non-TEM modes are not to be confused with waveguide modes which can be propagated when an microstrip line is placed in a metallic enclosure for shielding. These modes are launched or stimulated by radiation or discontinuities in the transmission line. These modes can be suppressed by decreasing the enclosure dimensions, adding shorting posts, lossy films, or by damping materials. See [71, 72] for more information.

### 3.5.1.6 Magnetic Substrates

Magnetic substrates are useful for building voltage programmable transmission lines. This is useful, for example, in constructing programmable phase shifters. Magnetic substrates are analyzed in [42, 43] which propose the following modifications to the above:

\[
Z_0 = \varepsilon_0 \mu_0 (\varepsilon_r = 1, \mu_r = 1, h, w, t) \sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}} \quad (\Omega)
\]

\[
\mu_{\text{eff}}(w / h, \lambda) = \frac{1.0}{\varepsilon_{\text{eff}} \mu(w / h, 1.0 / \mu)}
\]