$$k_1 = \frac{\sinh\left(\frac{\pi a}{h}\right)}{\sinh\left(\frac{\pi b}{h}\right)} \qquad k_1' = \sqrt{1.0 - k_1} \qquad (3.4.11.4)$$

$$k_2 = \frac{\tanh\left(\frac{\pi a}{h_1}\right)}{\tanh\left(\frac{\pi b}{h}\right)} \qquad k_2' = \sqrt{1.0 - k_2} \qquad (3.4.11.5)$$

ıese equations are valid to X-band for usual dimensions of PCBs, hybrids, and ICs.

REFERENCES:

] Ghione, Giovanni, and Carlo U. Naldi, "Coplanar Waveguides for MMIC Applications: Effect of Upper Shielding Conductor Backing, Finite-Extent Ground Planes, and Line-to-Line Coupling," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-35, No. 3, March 1987, pp. 260–267.

Design Handbook
Brian C. Wadell

3.5 MICROSTRIP LINE STRUCTURES

Microstrip line may well be the most popular transmission line structure. Ease of fabrication by photolithographic techniques and a good range of impedances and couplings allow it to be used for a wide variety of circuit components. Harold Wheeler developed a planar transmission line (two coplanar strips) which could be rolled up in 1936 and a stripline-like structure in 1942. Flat coaxial transmission line was used by V.H. Rumsey and H.W. Jamieson in WWII. Coaxial cable was first adapted to a flat configuration using printed circuit techniques by Barrett. This then evolved into stripline after WWII. The first use of the microstrip line configuration was reported by engineers at the Federal Telecommunications Research Laboratories (a division of ITT) sometime after 1949 [2].

REFERENCES

[1] Ayer, D.R., and C.A. Wheeler, "The Evolution of Strip Transmission Line," *Microwave Journal*, Vol. 12, No. 5, May 1969, pp. 31–40.

[2] Barrett, Robert M., "Microwave Printed Circuits—A Historical Survey," *IRE Transactions on Microwave Theory and Techniques*, Vol. MTT-3, No. 2, March 1955, pp. 1–9.

[3] Howe, Harlan, Jr., "Microwave Integrated Circuits—An Historical Perspective," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-32, No. 9, September 1984, pp. 991–996.

3.5.1 Microstrip Line

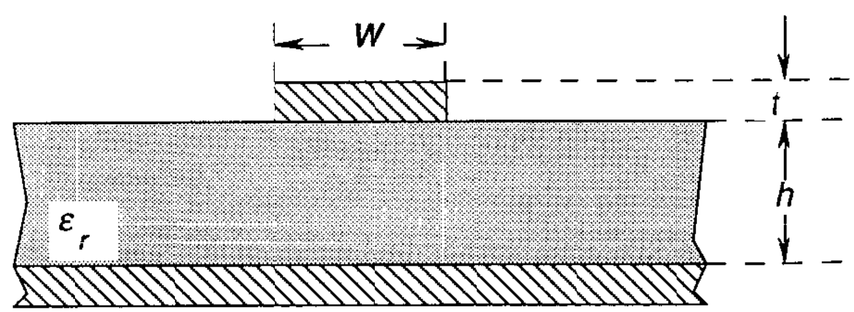

Figure 3.5.1.1: Microstrip Line

This is probably the most common and most analyzed transmission line structure. It is easy to use and has a good range of practical impedances. Equations have been calculated ·h an incredible variety of techniques both analytic and computational. A ˉˉʼTRAN program, MSTRIP2, is commonly encountered in references as a tool for checking the results of new analysis and synthesis equations. It is a numerical analysis program that assumes quasistatic conditions, zero thickness strips, and perfect conductivity. It is also assumed that the dielectric thickness and trace widths are thin relative to a wavelength.

Bogatin [10] experimentally compared various calculation techniques for this structure and recommends using the Wheeler equations with Schneider's ε_{eff}.

$$Z_0 = \frac{\eta_0}{2.0\sqrt{2.0}\,\pi\sqrt{\varepsilon_r + 1.0}} \ln\left\{1.0 + \frac{4.0\,h}{w'}\left[\frac{14.0 + 8.0/\varepsilon_r}{11.0}\,\frac{4.0\,h}{w'} + \sqrt{\left(\frac{14.0 + 8.0/\varepsilon_r}{11.0}\right)^2\left(\frac{4.0\,h}{w'}\right)^2 + \frac{1.0+1.0/\varepsilon_r}{2.0}\,\pi^2}\,\right]\right\} \quad (\Omega) \qquad (3.5.1.1)$$

Improvements in Schneider's ε_{eff} made by Hammerstad and Bekkadal [22] are given here. For $w/h \le 1.0$:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + \frac{12\,h}{w}\right)^{-0.5} + 0.04\left(1.0 - \frac{w}{h}\right)^2\right] \qquad (3.5.1.2)$$

and for $w/h \ge 1$:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1.0}{2.0} + \frac{\varepsilon_r - 1.0}{2.0}\left(1 + \frac{12.0\,h}{w}\right)^{-0.5} \qquad (3.5.1.3)$$

The equations for ε_{eff} are accurate to within 1% for:

$\varepsilon_r \le 16$ ($< 2\%$ error $\varepsilon_r > 16$)

$0.05 \le \frac{w}{h} \le 20.0$ ($< 2\%$ error $\frac{w}{h} < 0.05$)

The thickness of the trace can be corrected for by relating it to an equivalent change in the width. Owens and Potok [40] examined a number of formulas for this correction and show that Wheeler's is the most accurate:

$$\frac{\Delta w}{t} = \frac{1.0}{\pi}\ln\left[\frac{4\,e}{\sqrt{(t/h)^2 + \left(\frac{1/\pi}{w/t + 1.1}\right)^2}}\right] \qquad (3.5.1.4)$$

$$w' = w + \Delta w' \qquad (3.5.1.5)$$

$$\Delta w' = \Delta w\left(\frac{1.0 + 1.0/\varepsilon_r}{2.0}\right) \qquad (3.5.1.6)$$

Error in Z_0 is less than 2% for any ε_r, w.

3.5.1.1 Frequency Dependencies of Microstrip Line

The frequency dependency (dispersion) of the microstrip line's ε_{eff} can be calculated with (see also [35]:)

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}(f=0)}{1.0 + P(f)} \qquad (3.5.1.7)$$

$$P(f) = P_1P_2[(0.1844 + P_3P_4) \times (10.0\,f\,h)]$$

$$P_1 = 0.27488 + \left[0.6315 + \frac{0.525}{(1.0 + 0.157\,f\,h)^{20}}\right]\frac{w}{h} - 0.065683\,e^{-8.7513w/h} \qquad (3.5.1.8)$$

$$P_2 = 0.33622\left[1.0 - e^{-.03442\,\varepsilon_r}\right] \qquad (3.5.1.9)$$

$$P_3 = 0.0363\,e^{-4.6w/h}\left[1.0 - e^{-(f\,h/3.87)^{4.97}}\right] \qquad (3.5.1.10)$$

$$P_4 = 1.0 + 2.751\left[1.0 - e^{-(\varepsilon_r/15.916)^8}\right] \qquad (3.5.1.11)$$

where f is in GHz and h is in cm. The accuracy of this correction is better than 0.6% for

$0.1 \le w/h \le 100.0$

$1.0 \le \varepsilon_r \le 20.0$

$0 \le h/\lambda \le 0.13$

Atwater [2] compares several corrections for $\varepsilon_{eff}(f)$ with actual measurement data from eight published papers and found the above correction to be the best (by an admittedly small margin). His version of the equation is a rearranged equivalent.

3.5.1.2 Conductor Losses

Conductor losses are a result of several contributing factors: the actual conductance of material, the frequency-dependent skin effect losses, and the surface ro[illegible]s losses caused by the lengthened path at the surface. Conductor losses due to skin effect and the metal's conductivity are given in [53]. For $w/h \le 1$:

$$\alpha_c = \frac{10.0\, R_s}{\pi \ln 10.0} \frac{\left(\frac{8.0\, h}{w} - \frac{w}{4\, h}\right)\left(1.0 + \frac{h}{w} + \frac{h}{w}\frac{\partial w}{\partial t}\right)}{h\, Z_0\, e^{Z_0/60}} \quad \text{(dB/ unit length)} \qquad (3.5.1.12)$$

MAIN CONDUCTOR - CONDUCTIVITY & SKIN EFFECT LOSSES

For $w/h \ge 1$:

$$\alpha_c = \frac{Z_0\, R_s}{720.0\, \pi^2\, h \ln 10.0}\left[1.0 + \left(\frac{0.44\, h^2}{w^2}\right) + \frac{6.0\, h^2}{w^2}\left(1.0 - \frac{h}{w}\right)^5\right]$$

$$\times\left(1.0 + \frac{w}{h} + \frac{\partial w}{\partial t}\right) \quad \text{(dB/ unit length)} \qquad (3.5.1.13)$$

where for $\frac{w}{h} \le \frac{1}{2\,\pi}$

$$\frac{\partial w}{\partial t} = \left(\frac{1.0}{\pi}\right) \ln \frac{4.0\, \pi\, w}{t} \qquad (3.5.1.14)$$

for $\frac{w}{h} \ge \frac{1}{2\,\pi}$

$$\frac{\partial w}{\partial t} = \left(\frac{1.0}{\pi}\right) \ln \frac{2.0\, h}{t} \qquad (3.5.1.15)$$

These equations are valid for ground and strip conductivities the same and

$t << h$

$t < h / 2.0$

$\partial w / \partial t > 1.0$

Pucel *et al.* [43] also give equations for the conduction losses.

.5.1.2.1 *Effect of Ground Plane on Conductor Losses*

The ground plane's resistance can be a significant factor as frequency increases. At ow frequencies the current in the ground plane is spread over the full width of the ground lane and the current is evenly distributed in the center conductor. Because of this, the round plane cont s little resistance relative to the center conductor. As frequency

increases the current in the line begins to concentrate in the sides and bottom of the trace. Simultaneously, the ground current begins to concentrate in the area directly under the strip, increasing the effective resistance. Faraji-Dana and Chow [16] give a closed-form equation suitable for design calculations. The resistance of the ground plane is R_g

$$R_g = 0.55\, R_{dc} \sqrt{\frac{\pi}{2.0}} \sqrt{\frac{t}{w}} \left(1.0 - e^{-w'/1.2\,\pi}\right) P \qquad (3.5.1.16)$$

where

$$w' = w / h \qquad (3.5.1.17)$$

$$P = \sqrt{2.0\, \mu\, \sigma\, f\, w\, t} \qquad (3.5.1.18)$$

R_{dc} = the dc resistance of the signal conductor

The total ac resistance for use in the microstrip line model is R_t.

$$R_t = R_s + R_g \qquad (3.5.1.19)$$

3.5.1.2.2 *Effect of Conductor Surface Roughness*

The conductor surface analyzed in the formulas above are perfectly smooth. The physical processes which make real world conductors create scratches, bumps, and random bumps. See Tanaka and Okada [62] for some scanning microscope photographs of typical conductor surfaces.

Conductor surface roughness losses are discussed thoroughly in Sanderson [53]. Sanderson also reports on a technique used for calculating the roughness effect on the losses, Z_0, and v_p. Higher than expected losses were experienced in stainless steel conductors that can be modeled by a lumped resistance. Good agreement between the predicted performance and the experimental measurements was found from 200 MHz to 10 GHz in the other cases. Saad [50] gives a plot from Lending [32] for losses as a function of surface roughness. In [20], an approximate formula is given for the conductor losses due to roughness:

$$\alpha_c = \alpha_{c,0}\left\{1.0 + \frac{2.0}{\pi}\tan^{-1}\left[1.40\left(\frac{\Delta}{\delta}\right)^2\right]\right\} \qquad (3.5.1.20)$$

MORE SIGNIF. FOR HIGHER FREQS.

where

$\alpha_{c,0}$ = the conductor losses calculated for a perfectly smooth conductor; equations (3.5.1.12) and (3.5.1.13)

Δ = the rms surface roughness

δ = the skin depth

The results of a number of references [20, 32, 39, 60, 66] are summarized in Figure 3.5.1.2 below. This figure shows good agreement between Morgan's data and the Goldfarb closed form equation.

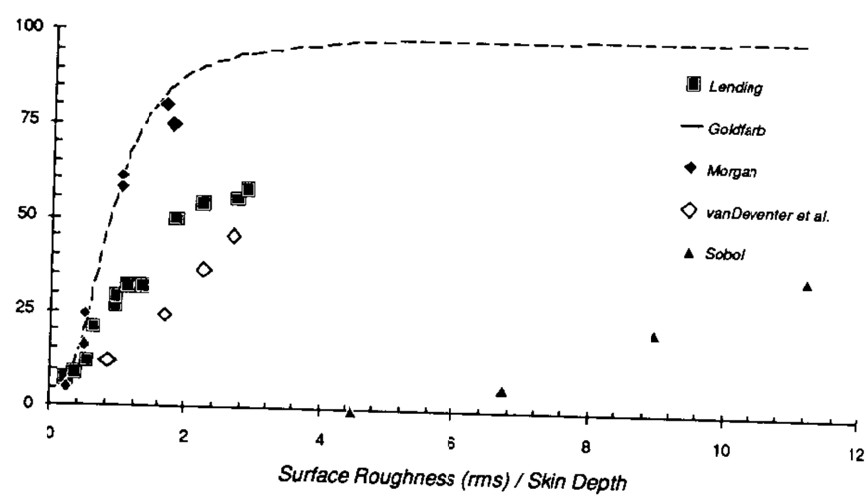

Figure 3.5.1.2.2.1: Microstrip Line Losses vs. Surface Roughness

3.5.1.3 Dielectric Losses

Dielectric losses are given in Schneider [50]:

$$\alpha_d = \frac{20.0\ \pi}{\ln\ (10.0)} \frac{q \tan \delta}{\lambda} \quad \text{(dB / unit length)} \qquad (3.5.1.21)$$

where q is the filling factor and λ_g is the wavelength in microstrip line both previously defined.

3.5.1.4 Radiation Losses

Radiative losses were discussed in [1]. The equations are:

$$\alpha_r = 60 \left(\frac{2\ \pi\ h}{\lambda_0} \right)^2 F(\varepsilon_{eff}) \qquad (3.5.1.22)$$

where $F(\varepsilon_{eff})$ is:

$$F(\varepsilon_{eff}) = \frac{\varepsilon_{eff} + 1.0}{\varepsilon_{eff}} - \frac{(\varepsilon_{eff} - 1.0)^2}{2.0\ \varepsilon_{eff}^{3/2}} \log \left(\frac{\sqrt{\varepsilon_{eff}} + 1.0}{\sqrt{\varepsilon_{eff}} - 1.0} \right) \text{, for an open-circuited line} \qquad (3.5.1.23)$$

$$F(\varepsilon_{eff}) = 1.0 - \frac{(\varepsilon_{eff} - 1.0)}{2.0 \sqrt{\varepsilon_{eff}}} \log \left(\frac{\sqrt{\varepsilon_{eff}} + 1.0}{\sqrt{\varepsilon_{eff}} - 1.0} \right) \text{, for a matched line} \qquad (3.5.1.24)$$

3.5.1.5 Higher Order Modes in Microstrip Line

The microstrip line structure is not strictly TEM; other modes may propagate. The lowest order TE_1 mode frequency is [69]:

$$f_c = \frac{c}{4.0\ h \sqrt{\varepsilon_r - 1.0}} \qquad (3.5.1.25)$$

These non-TEM modes are not to be confused with waveguide modes which can be propagated when a microstrip line is placed in a metallic enclosure for shielding. These modes are launched or stimulated by radiation or discontinuities in the transmission line. These modes can be suppressed by decreasing the enclosure dimensions, adding shorting posts, lossy films, or by damping materials. See [71, 72] for more information.

3.5.1.6 Magnetic Substrates

Magnetic substrates are useful for building voltage programmable transmission lines. This is useful, for example, in constructing programmable phase shifters. Magnetic substrates are analyzed in [42, 43] which propose the following modifications to the above:

$$Z_0 = Z_0(\varepsilon_r = 1, \mu_r = 1, h, w, t) \sqrt{\frac{\mu_{eff}}{\varepsilon_{eff}}} \quad (\Omega) \qquad (3.5.1.26)$$

$$\mu_{eff}(w / h, \mu) = \frac{1.0}{\varepsilon_{eff}\ (w\ /\ h,\ 1.0\ /\ \mu)} \qquad (3.5.1.27)$$