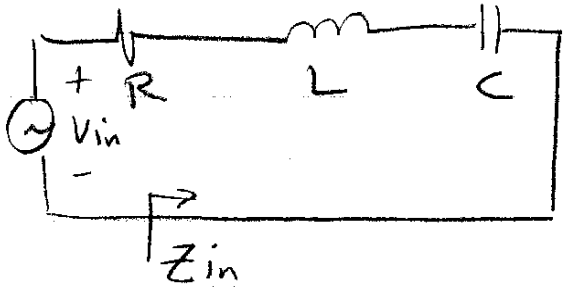
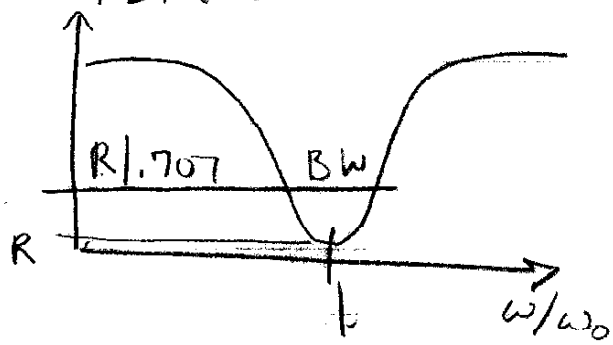
Microwave Resonators Ch 6.1

1. Series Resonant Circuit





$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} |I|^2 Z_{in} = \frac{1}{2} Z_{in} \left| \frac{V_{in}}{Z_{in}} \right|^2 = \frac{1}{2} |I|^2 (R + j\omega L + 1/j\omega C)$$

$$P_{loss}\ (\text{in } R) = \frac{1}{2} |I|^2 R$$

W_m (@ av magnetic stored energy) $= \frac{1}{4} |I|^2 L$

W_e (@ av electric stored energy) $= \frac{1}{4} |V_c|^2 C = \frac{1}{4} |I^2| \frac{1}{\omega^2 C}$

At Resonance $W_m = W_e$

What is the impedance @ resonance?

$$Z_{in} = \frac{P_{in}}{\frac{1}{2}|I|^2} = \frac{P_{loss} + 2j\omega (W_m - W_e)}{\frac{1}{2}|I|^2}$$

At resonance $Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$

Find resonance frequency ω_0 from $W_m = W_e$

$$\frac{1}{4} |I|^2 L = \frac{1}{4} |I|^2 \frac{1}{\omega_0^2 C} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Quality Factor

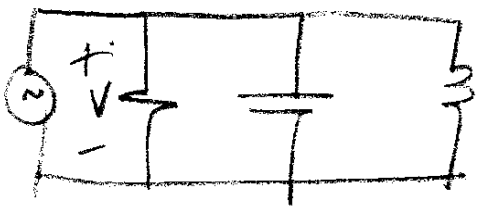
$$Q \equiv \omega \frac{\text{av energy stored}}{\text{energy loss/sec}} = \omega \frac{W_m + W_e}{P_{loss}}$$

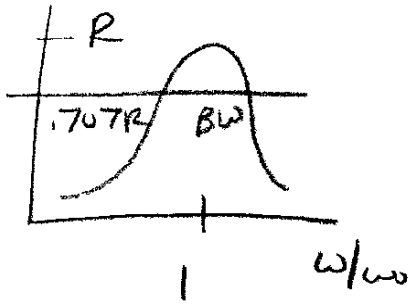
At resonance

$$Q = \omega_0 \frac{2W_m}{P_{loss}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$\uparrow R \rightarrow \downarrow Q$ ($\uparrow Q$ ☹) BW = 1/Q

2. Parallel resonance





$$Z_{in} = \left(\frac{1}{R} + j\omega C + \frac{1}{j\omega L}\right)^{-1}$$

$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} Z_{in} |I|^2 = \frac{1}{2} |V|^2 \frac{1}{Z_{in}^*}$$

$$= \frac{1}{2} |V|^2 \left(\frac{1}{R} - j\omega C + \frac{j}{\omega L}\right)$$

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R}$$

$$W_e = \frac{1}{4} |V|^2 C \qquad W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{2} |V|^2 \frac{1}{\omega^2 L}$$

$$P_{in} = P_{loss} + 2j\omega (W_m - W_e)$$

$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega (W_m - W_e)}{\frac{1}{2}|I|^2}$$

@ resonance $W_m = W_e$ $Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 \frac{2W_m}{P_{loss}} = \frac{R}{\omega_0 L} = \omega_0 RC$$

$$BW = 1/Q$$

Loaded Q

Unloaded (above) no load on ckt
Loaded (below) Load on ckt

External Q:

$$Q_e = \begin{cases} \frac{\omega_0 L}{R_L} & \text{series ckt} \\ \frac{R_L}{\omega_0 L} & \text{parallel} \end{cases}$$

Loaded Q

$$Q_L = \left(\frac{1}{Q_e} + \frac{1}{Q} \right)^{-1}$$