Microwave Engineering + Applications OPGandhi



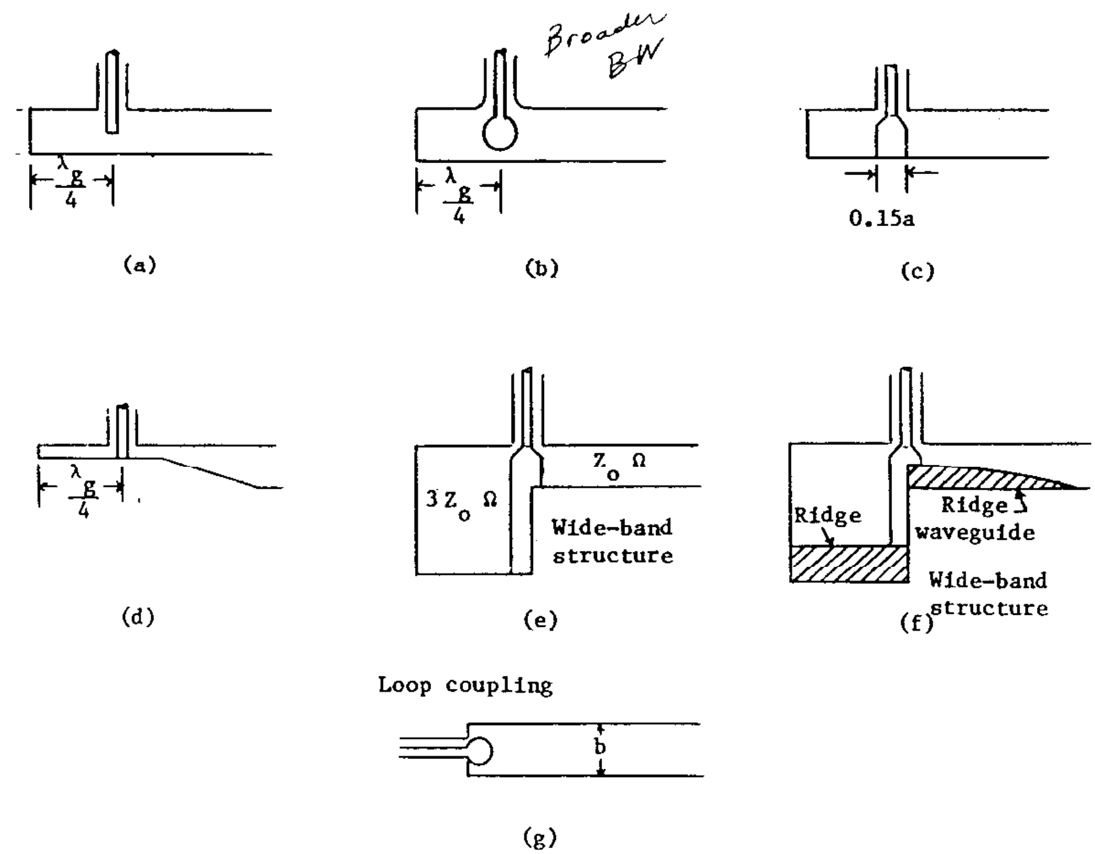


Fig. 6.9. Various types of coaxial to waveguide transformers. [Source: T. K. Ishii [2].]

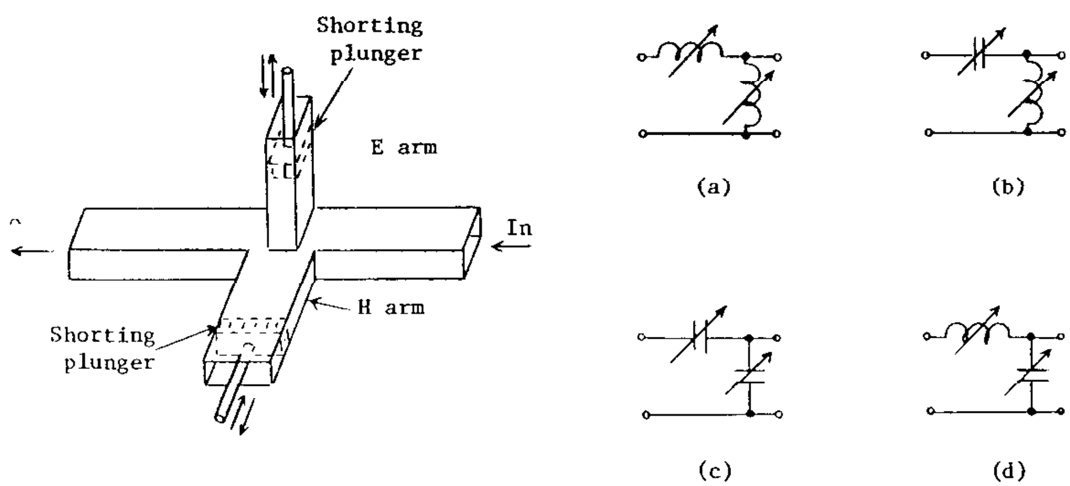


Fig. 6.10. E-H tuner and possible equivalent circuits. [Source: T. K. Ishii [2].]

Linear EM Fields
for Engineers
G.E. Miner

$$-\frac{\partial \Phi}{\partial y}\hat{y} = -\frac{V_0}{d}\hat{y} = E_y\hat{y} \qquad (9\text{-}22)$$

lly if the top plate were positive ($V_0 > 0$). Of course, the actual voltage varies sinusoidally.

From Eq. 9-14, with $\sigma_d = 0$, we have the propagation constant

$$\gamma_{\text{TEM}}^2 = -\omega^2\mu\epsilon \qquad \text{or} \qquad \gamma_{\text{TEM}} = j\omega\sqrt{\mu\epsilon} = j\beta = jk \qquad (9\text{-}23)$$

If $\epsilon$ is complex (lossy dielectric) then $\beta = \text{Im}\{\gamma_{\text{TEM}}\}$. The total electric field intensity is constructed using Eq. 9-11 with $E_z$ and $E_x = 0$:

$$\vec{E}(x, y, z) = -\frac{V_0}{d}e^{-j\omega\sqrt{\mu\epsilon}z}\hat{y} = -\frac{V_0}{d}e^{-jkz}\hat{y} \qquad (9\text{-}24)$$

Using one of Maxwell's equations (Faraday's law),

$$\begin{aligned}\vec{H}(x, y, z) &= j\frac{1}{\omega\mu}\nabla \times \vec{E}(x, y, z) = j\frac{1}{\omega\mu}\left(-\frac{\partial E_y}{\partial z}\hat{x}\right) \\ &= -j\frac{1}{\omega\mu}\left(-\frac{V_0}{d}\right)(-jk)e^{-jkz}\hat{x} = \frac{V_0 k}{\omega\mu d}e^{-jkz}\hat{x} = \frac{V_0}{\eta d}e^{-jkz}\hat{x}\end{aligned} \qquad (9\text{-}25)$$

Th … s. This also agrees with what one
mi … ıe coefficient of this last equation
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Excitation of
Modes

$$= \frac{V_0}{\eta d} \qquad (9\text{-}26)$$

wl … hms.

bc … ıtrinsic impedance of the medium
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… inary for any frequency.

Suppose we want to excite TEM propagation by means other than an applied voltage source. One method is to place a small element into the region that has a strong field component along the direction or directions of the field solutions $\vec{E}$ and $\vec{H}$. For example, if we want to couple to the electric field, the element must have a significant $\hat{y}$ electric field component. This could be done by inserting the center conductor of a coax into the region as shown in Figure 9-2$a$. Note that the voltage difference between the coax inner and outer conductors would produce a field (qualitatively) as indicated. There is a strong

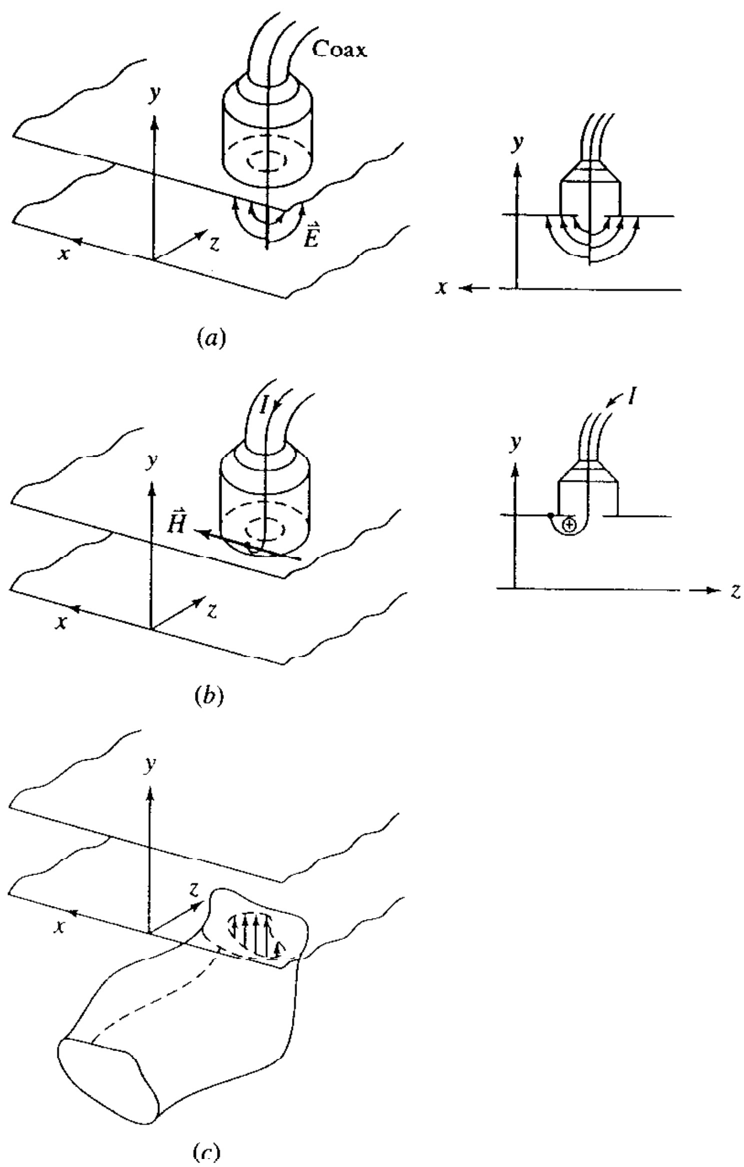



**Figure 9-2.** Methods for exciting TEM modes between parallel planes. (*a*) Electric field probe upling. (*b*) Magnetic field loop coupling. (*c*) Aperture coupling.

vertical ($\hat{y}$) component. This element is called a *probe coupler*. The outer conductor of the connector is physically connected to the plate and the center conductor passes through a small hole.

Coupling to the magnetic field is accomplished by causing current to flow such that the current generates a magnetic field component in the direction required by the mode equations. In this case we need an $\hat{x}$ component of $\vec{H}$. This can be done as shown in Figure 9-2*b*. Here, the center conductor is inserted through a small hole, bent into a loop, and physically connected to the top plate on the underside. The right-hand rule shows that within the loop there is a strong $\hat{x}$ magnetic-field component. This element is called a *loop coupler.*

A third way of coupling into the system is by generating either the electric or magnetic field over a hole in a plate or some other device. In a following section we will see that a rectangular cross section guide can generate just such a field structure. This coupling mechanism is shown in Figure 9-2*c*. A hole (aperture) is cut in the generating structure and placed by the plates so that the electric field is in the proper direction. This element is called an *aperture coupler.*

It turns out that electron beams and electrical discharges can also be used to generate the waves, but these will not be described here.

The guide wavelength is defined as the distance in the $z$ direction required to obtain $2\pi$ radians phase shift. We denote this by $\lambda_g$. This is easily obtained from the exponent of Eq. 9-24 or Eq. 9-25. The definition requires that for $z = \lambda_g$

$$k\lambda_g = 2\pi \tag{9-27}$$

Then

$$\lambda_g = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{1}{f\sqrt{\mu\epsilon}}\ \text{m} \tag{9-28}$$

We can obtain the "characteristic impedance" in the $z$ direction using Eq. 7-67, where $\hat{r} \sim \hat{z}$, for the wave impedance:

$$Z^w_{z,\text{TEM}} = \frac{(\vec{E} \times \vec{H}^*) \cdot \hat{z}}{(\hat{z} \times \vec{H}) \cdot (\hat{z} \times \vec{H})^*} = \frac{\dfrac{+V_0^2}{\eta d^2}\,\hat{z}\cdot\hat{z}}{\dfrac{V_0}{\eta d}\,e^{-jkz}\hat{y} \cdot \dfrac{V_0}{\eta d}\,e^{jkz}\hat{y}} = \eta\ \Omega \tag{9-29}$$

This could also have been obtained by taking the ratio of $E_t$ to $H_t$. However, to use this approach requires careful attention to the sign. If either component is negative we must negate the ratio to get the correct wave impedance. In this case the electric field is in the $-\hat{y}$ direction so we have

$$Z^w_{z,\text{TEM}} = -\frac{E_y}{H_x} = -\frac{-\dfrac{V_0}{d}}{\dfrac{V_0}{\eta d}} = \eta$$

Now that we have "$Z_0$" and the wavelength we can use the Smith chart to solve planar waveguide transmission problems.

The propagation velocities are now easily evaluated. For the phase velocity, using $\beta = k$ for TEM:

$$V_{p,\text{TEM}} = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}\ \text{m/s} \tag{9-30}$$

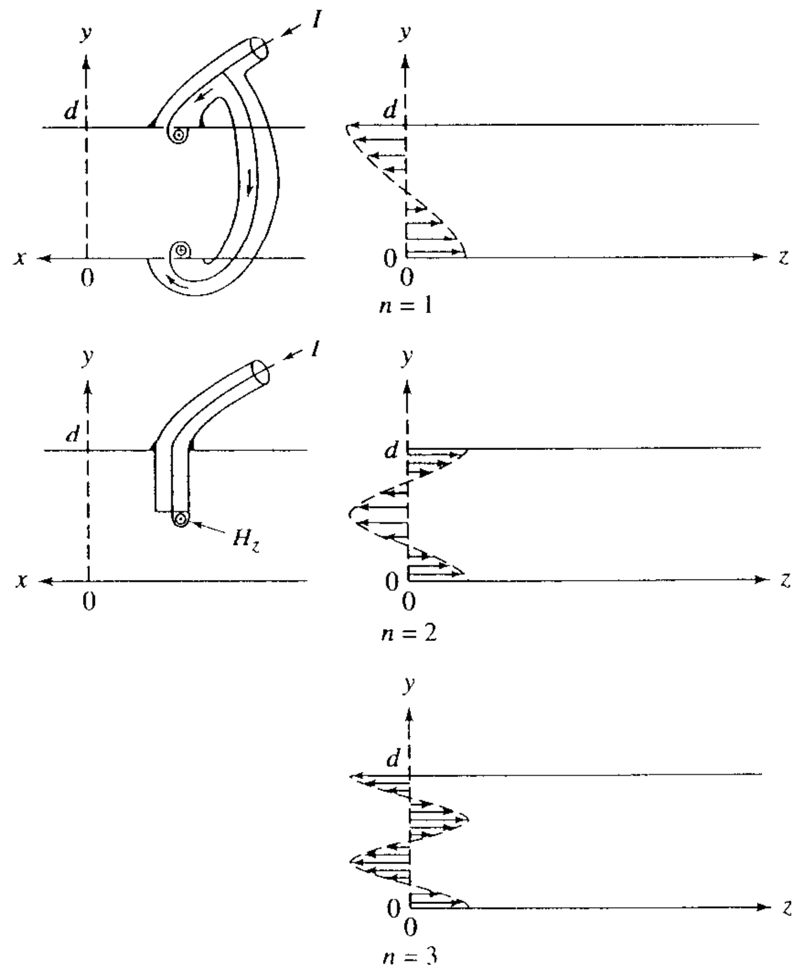
Since the phase velocity is independent of frequency the system is dispersionless.

$TM_n$ have an interesting relationship. If we multiply the wave impedances together we obtain

$$Z^w_{z,TM} Z^w_{z,TE} = \eta^2 = \frac{\mu}{\epsilon} \tag{9-115}$$

Since the results for $TE_n$ and $TM_m$ modes are modes essentially identical (except for the wave impedances) how do we know which mode type will propagate? The answer is in how we excite the system. To obtain TM modes we excited the system so that a strong $E_z$ was produced. Similarly, to obtain TE modes we introduce a strong $H_z$ component. The amplitude patterns for $H_z$, modes $n = 1, 2, 3$, are shown in Figure 9-17. Loop couplings for $n = 1$ and $n = 2$ are also shown. For a mode to propagate it must be excited in the appropriate way *and* be at a frequency above cutoff. Examples 9-71 and 9-72 apply to TE modes as well.

**Figure 9-17.** Field amplitude variations of $H_z$ for the first three modes for $TE_n$ modes between parallel planes. Loop couplings for $n = 1$ and $n = 2$ shown.



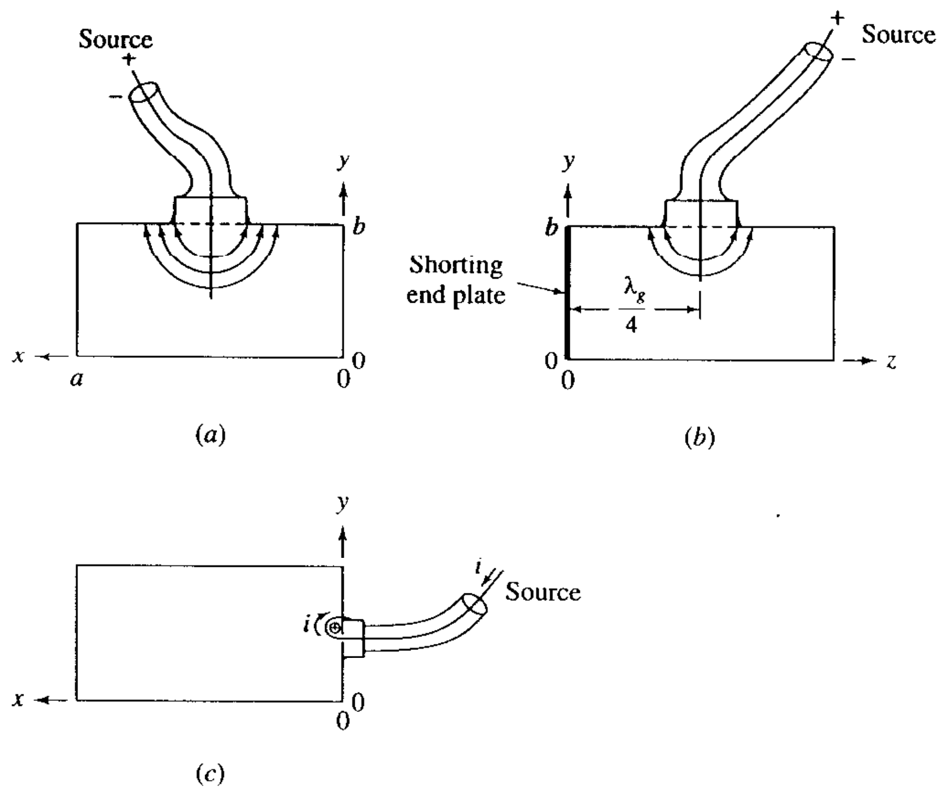



**Figure 9-21.** Exciting the $TE_{10}$ mode in rectangular waveguides. (*a*) Probe coupling. (*b*) Coax to waveguide adaptor. (*c*) Loop coupling.

slit that is parallel to the $z$ axis and then placing the probe on a movable carriage. The output of the coax is then applied to a suitable detector.

Expressions for the group and phase velocities are obtained directly from the definitions. Alternate forms that use the radical expressions developed earlier are possible. The results are

$$V_p^{\mathrm{TE}} = \frac{\omega}{\beta_{\mathrm{TE}}} = \frac{v_0}{\sqrt{1 - \dfrac{\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2}{\omega^2\mu\epsilon}}} = \frac{v_0}{\sqrt{1 - \left(\dfrac{\lambda_0}{\lambda_{c,\mathrm{TE}}}\right)^2}}$$

$$= \frac{v_0}{\sqrt{1 - \left(\dfrac{\omega_{c,\mathrm{TE}}}{\omega}\right)^2}} \text{ m/s} \tag{9-143}$$

$$V_g^{\mathrm{TE}} = \frac{d\omega}{d\beta_{\mathrm{TE}}} = v_0\sqrt{1 - \dfrac{\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2}{\omega^2\mu\epsilon}} = v_0\sqrt{1 - \left(\dfrac{\lambda_0}{\lambda_{c,\mathrm{TE}}}\right)^2}$$

$$= v_0\sqrt{1 - \left(\dfrac{\omega_{c,\mathrm{TE}}}{\omega}\right)^2} \text{ m/s} \tag{9-144}$$

**9-6.4d** Using the time domain equations determine the $z$ location in View 2 ($TE_{11}$ mode) that corresponds to View 1 at $t = 0$. See Figure 9-30.

**9-6.4e** Determine the equation for the surface current at $\rho = b$ for the $TE_{11}$ mode. Using the time domain equation verify view 3 in Figure 9-30. For $t = 0$ where is $z = 0$ in this view?

**9-6.4f** Design a coaxial waveguide that has a cutoff frequency of 2.8 GHz and for which other low-order modes will not propagate below 5 GHz. Can the TEM mode propagate?

**9-6.4g** Design loop and probe coupling methods for exciting the $TE_{11}$ mode.

## 9-7 Aperture (Slot) Coupling

The techniques for loop and probe coupling discussed in the preceding section assumed that waveguide excitation was to be accomplished using a coaxial feed line. There are important cases where it is desirable to couple directly between two waveguide structures. For example, if one wanted to sample the field in a main guide using another waveguide section, it would be advantageous to couple the guides together using a common "hole," or aperture, between them. A waveguide directional coupler is an example of this.

A microwave frequency meter provides another example of waveguide-to-waveguide coupling. The frequency meter is often a section of circular waveguide formed into a resonant cavity (see Section 9-11). A sample of the main waveguide energy is coupled by an aperture into the resonant cavity. When the cavity dimensions are such that resonance occurs energy is absorbed by the cavity. This small energy decrease can be observed at the main guide output.

The aperture coupling concepts will be qualitative only. Quantitative details are to be found in [14].

There are two types of aperture coupling: *electric* and *magnetic*. The type is determined by the orientation of the aperture with respect to the fields present. Of course since the fields are varying with time we always have both $\vec{E}$ and $\vec{H}$, but one component will be dominant in the description of the coupling process. Whichever description is used, the key idea is that the physical orientation of the devices to be coupled must be such that the modes in each device have the coupled field component in common.

Since an aperture extends across a finite extent of the wall any field component in the aperture will have a field component present for several modes. However, if the operating frequency is below cutoff for all modes except one, only one mode will propagate. Thus the operating frequency is also an important consideration.

*Electric field coupling* has the easiest explanation when the electric field is either normal to the surface of the aperture or parallel to the surface of the aperture. When the slot is cut normal to the electric field lines, there are no surface charges to provide the termination of the line. The electric field lines then pass through the slot and induce terminating charges on external metal surfaces. This is the situation shown in Figure 9-31. In Figure 9-31*b* the field lines move through and then propagate away from the slot. See Exercise 9-7a for the electric field parallel to the aperture.

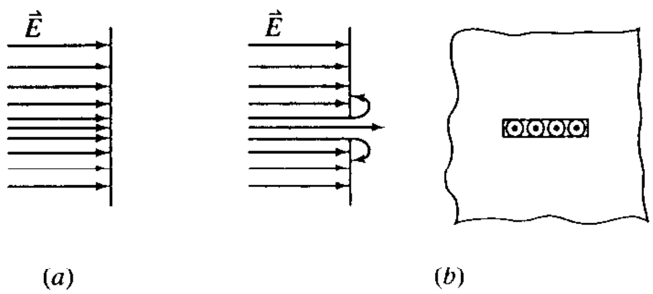



**Figure 9-31.** Coupling of electric field lines through an aperture. $\vec{E}$ Normal to plane of aperture.

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**Example 9-8**

Develop a method for coupling a $TE_{10}$ mode in one rectangular waveguide to a $TE_{10}$ mode in a second identical waveguide. This device is useful when we want to sample the field in the first guide.

We cut the apertures in the guides so that $E_y$ is normal to the aperture planes. Also, the slots are cut so that they have minimal interference with the surface current $\vec{J}_s$, requiring that the slots run along the $z$ direction. The final configuration is shown in Figure 9-32 slot number 1. Note that there are several ways that the slots could be cut as indicated in the figure. However, Case 1 shown provides minimum current interference.

It turns out that the distance $d$ is important. It should be about $\lambda_g/4$ for optimum coupling since the part of the wave going to the left hits the short and returns back to the aperture to reinforce the coupled field.

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*Magnetic field coupling* is the dominant effect when the aperture is oriented so that its longest sides are normal to the surface current or, equivalently, when they are parallel to the surface magnetic field lines. This situation is illustrated physically in Figure 9-33.

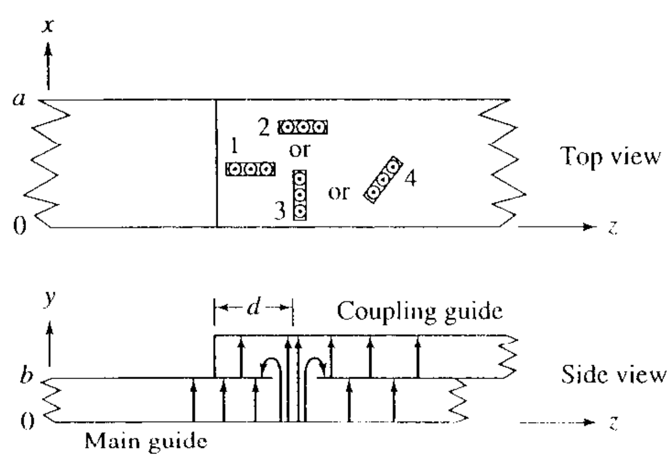


**Figure 9-32.**

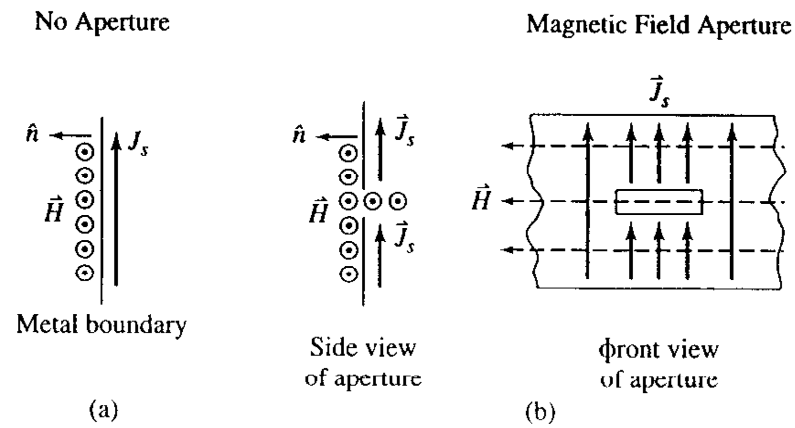


**Figure 9-33.** Magnetic coupling to aperture. (*a*) Metal surface boundary conditions. (*b*) Aperture orientation for maximum magnetic field coupling.

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**Example 9-9**

Develop a method for coupling the $TE_{10}$ mode in a rectantangular waveguide to the $TM_{01}$ mode in a cylindrical waveguide. Use magnetic field coupling.

The design procedure will be explained using Figure 9-34. Figure 9-34*a* shows the field pattern for the $TM_{01}$ mode in a cylindrical guide. The longitudinal ($z$ axis) view is from the *outside* of the guide and shows the magnetic field and surface current density on the inner wall surface. The surface normal is into the page ($-\rho$ direction). To produce magnetic coupling the aperture must be oriented as shown. Note that one could also use the electric field concept here since the electric field is normal to the plane of the aperture. However, the connection to the rectangular guide will be at an electric field minimum so the process will be predominantly magnetic.

The rectangular $TE_{10}$ mode field pattern is shown in Figure 9-34*b*. On the vertical side wall narrow dimension the electric field would be zero ($E_y$ is tangential $\vec{E}$ there). If we cut the aperture as shown in the longitudinal view we produce magnetic coupling. Now if we line up the two apertures as shown in Figure 9-34*c* the magnetic field components would couple. Note that the electric field lines in the cylindrical guide have the incorrect orientation to produce a $y$ component of electric field in the rectangular guide.

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**Exercises**

**9-7.0a** Draw a figure like that of Figure 9-30 where $\vec{E}$ is parallel to the aperture. Explain the dominant field direction on the other side of the aperture. Explain why an aperture size on the order of $\lambda_g/8$ is desired for good coupling. Consider $E_{tan} = 0$.

**9-7.0b** Develop a method for coupling the rectangular $TE_{10}$ mode into a cylindrical $TM_{01}$ mode using electric field coupling. Give two methods. Note you can couple $E_y$ in two ways.

**9-7.0c** If two rectangular waveguides are joined along the narrow ($y = b$) sides, what modes could be coupled with an electric coupling aperture? Draw diagrams.

**9-7.0d** The two systems shown in Figure 9-35 are intended to couple from rectangular $TE_{10}$ to cylindrical $TM_{01}$. Comment on the relative electric field amplitudes.