RF Circuit Design
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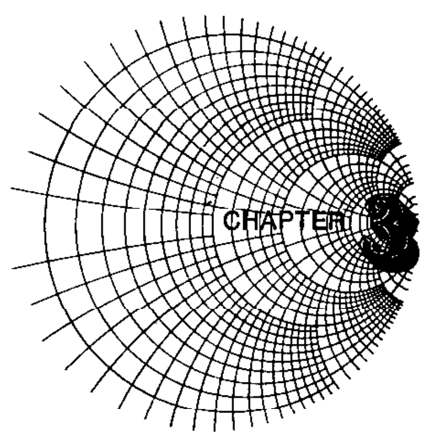


FILTER DESIGN

Filters occur so frequently in the instrumentation and communications industries that no book covering the field of rf circuit design could be complete without at least one chapter devoted to the subject. Indeed, entire books have been written on the art of filter design alone, so this single chapter cannot possibly cover all aspects of all types of filters. But it will familiarize you with the characteristics of four of the most commonly used filters and will enable you to design very quickly and easily a filter that will meet, or exceed, most of the common filter requirements that you will encounter.

We will cover Butterworth, Chebyshev, and Bessel filters in all of their common configurations: *low-pass, high-pass, bandpass,* and *bandstop.* We will learn how to take advantage of the attenuation characteristics unique to each type of filter. Finally, we will learn how to design some very powerful filters in as little as 5 minutes by merely looking through a catalog to choose a design to suit your needs.

BACKGROUND

In Chapter 2, the concept of resonance was explored and we determined the effects that component value changes had on resonant circuit operation. You should now be somewhat familiar with the methods that are used in analyzing passive resonant circuits to find quantities, such as loaded Q, insertion loss, and bandwidth. You should also be capable of designing one- or two-resonator circuits for any loaded Q desired (or, at least, determine why you cannot). Quite a few of the filter applications that you will encounter, however, cannot be satisfied with the simple bandpass arrangement given in Chapter 2. There are occasions when, instead of passing a certain band of frequencies while rejecting frequencies above and below (bandpass), we would like to attenuate a small band of frequencies while passing all others. This type of filter is called, appropriately enough, a *bandstop filter.* Still other requirements call for a low-pass or high-pass response. The characteristic curves for these responses are shown in Fig. 3-1. The low-pass filter will allow all signals below a certain cutoff frequency to pass while attenuating all others. A high-pass filter's response is the mirror-image of the low-pass response and attenuates all signals below a certain cutoff frequency while allowing those above cutoff to pass. These types of response simply cannot be handled very well with the two-resonator bandpass designs of Chapter 2.

In this chapter, we will use the low-pass filter as our workhorse, as all other responses will be derived from it. So let's take a quick look at a simple low-pass filter and examine its characteristics. Fig. 3-2 is an example of a very simple *two-pole,* or *second-order* low-pass filter. The order of a filter is determined by the slope of the attenuation curve it presents in the stopband. A second-order filter is one whose rolloff is a function of the frequency squared, or 12 dB per octave. A third-order filter causes a rolloff that is proportional to frequency cubed, or 18 dB per octave. Thus, the order of a filter can be equated with the number of significant reactive elements that it presents to the source as the signal deviates from the passband.

The circuit of Fig. 3-2 can be analyzed in much the same manner as was done in Chapter 2. For instance, an examination of the effects of loaded Q on the response would yield the family of curves shown in Fig. 3-3. Surprisingly, even this circuit configuration can cause a peak in the response. This is due to the fact that at some frequency, the inductor and capacitor will become resonant and, thus, peak the response if the loaded Q is high enough. The resonant frequency can be determined from

$$F_r = \frac{1}{2\pi\sqrt{LC}} \qquad \text{(Eq. 3-1)}$$

For low values of loaded Q, however, no response peak will be noticed.

The loaded Q of this filter is dependent upon the individual Q's of the series leg and the shunt leg where, assuming perfect components,

$$Q_1 = \frac{X_L}{R_s} \qquad \text{(Eq. 3-2)}$$

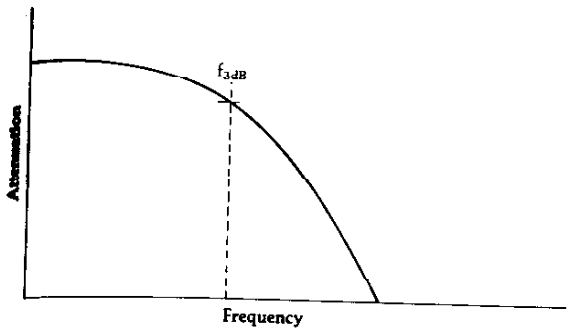
and,

$$Q_2 = \frac{R_L}{X_c} \qquad \text{(Eq. 3-3)}$$

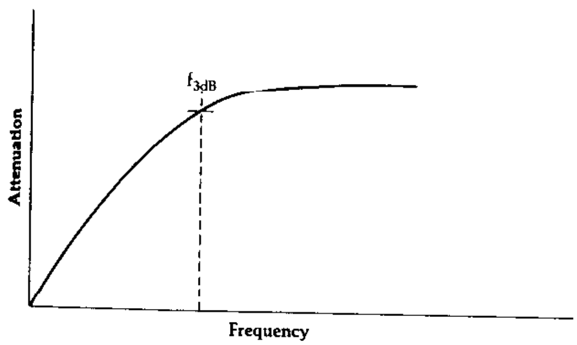
and the total Q is:

$$Q_{total} = \frac{Q_1 Q_2}{Q_1 + Q_2} \qquad \text{(Eq. 3-4)}$$

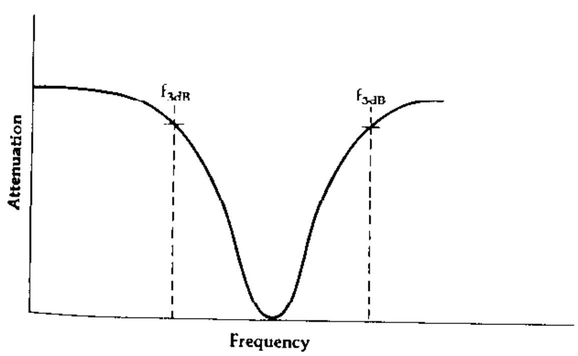
If the total Q of the circuit is greater than about 0.5, then for optimum transfer of power from the source to the load, Q_1 should equal Q_2. In this case, at the peak frequency, the response will approach 0-dB insertion loss. If the total Q of the network is less than about 0.5, there will be no peak in the response and, for optimum transfer of power, R_s should equal R_L. The peaking of the filter's response is commonly called ripple (defined in Chapter 2) and can vary consider-



(A) *Low-pass.*



(B) *High-pass.*



(C) *Bandstop.*

Fig. 3-1. Typical filter response curves.

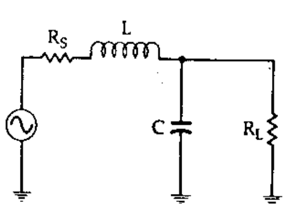


Fig. 3-2. A simple low-pass filter.

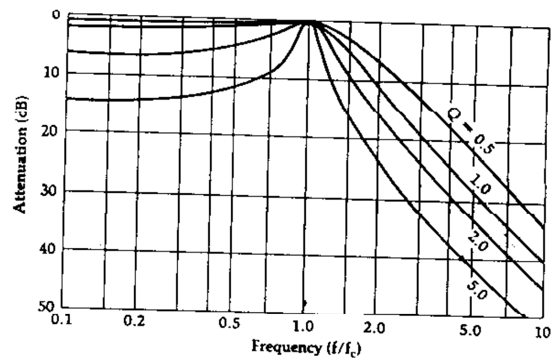


Fig. 3-3. Typical two-pole filter response curves.

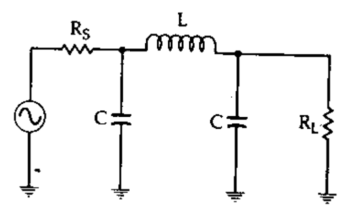


Fig. 3-4. Three-element low-pass filter.

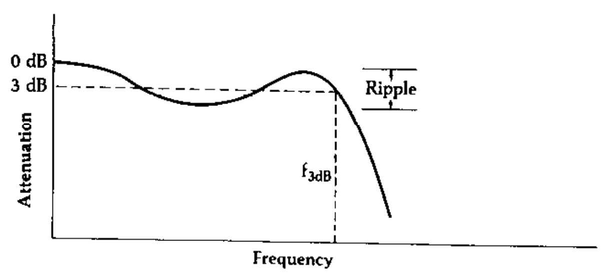


Fig. 3-5. Typical response of a three-element low-pass filter.

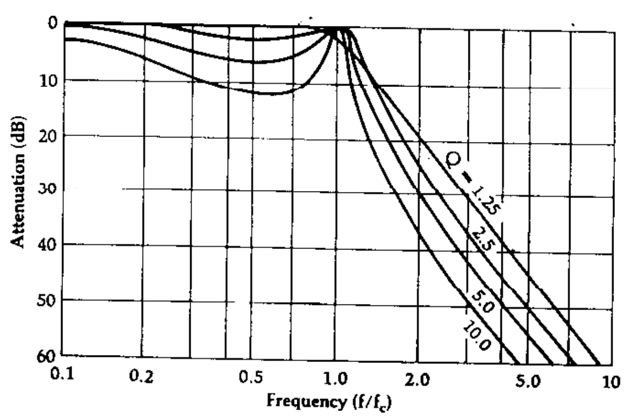


Fig. 3-6. Curves showing frequency response vs. loaded Q for three-element low-pass filters.

ably from one filter design to the next depending on the application. As shown, the two-element filter exhibits only one response peak at the edge of the passband.

It can be shown that the number of peaks within the passband is directly related to the number of elements in the filter by:

$$\text{Number of Peaks} = N - 1$$

where,

N = the number of elements.

Thus, the three-element low-pass filter of Fig. 3-4 should exhibit two response peaks as shown in Fig. 3-5. This is true only if the loaded Q is greater than one. Typical response curves for various values of loaded Q for the circuit given in Fig. 3-4 are shown in Fig. 3-6. For all odd-order networks, the response at dc and at the upper edge of the passband approaches 0 dB with dips in the response between the two frequencies. All even-order networks will produce an insertion loss at dc equal to the amount of passband ripple in dB. Keep in mind, however, that either of these two networks, if designed for low values of loaded Q, can be made to exhibit little or no passband ripple. But, as you can see from Figs. 3-3 and 3-6, the elimination of passband ripple can be made only at the expense of bandwidth. The smaller the ripple that is allowed, the wider the bandwidth becomes and, therefore, selectivity suffers. Optimum flatness in the passband occurs when the loaded Q of the three-element circuit is equal to one (1). Any value of loaded Q that is less than one will cause the response to roll off noticeably even at very low frequencies, within the defined passband. Thus, not only is the selectivity poorer but the passband insertion loss is too. In an application where there is not much signal to begin with, an even further decrease in signal strength could be disastrous.

Now that we have taken a quick look at two representative low-pass filters and their associated responses, let's discuss filters in general:

1. High-Q filters tend to exhibit a far greater initial slope toward the stopband than their low-Q counterparts with the same number of elements. Thus, at any frequency in the stopband, the attenuation will be greater for a high-Q filter than for one with a lower Q. The penalty for this improvement is the increase in passband ripple that must occur as a result.
2. Low-Q filters tend to have the flattest passband response but their initial attenuation slope at the band edge is small. Thus, the penalty for the reduced passband ripple is a decrease in the *initial* stopband attenuation.
3. As with the resonant circuits discussed in Chapter 2, the source and load resistors loading a filter will have a profound effect on the Q of the filter and, therefore, on the passband ripple and shape factor of the filter. If a filter is inserted between two resistance values for which it was not designed, the performance will suffer to an extent, depending upon the degree of error in the terminating impedance values.
4. The *final* attenuation slope of the response is dependent upon *the order of the network*. The order of the network is equal to the number of reactive elements *in the low-pass filter*. Thus, a second-order network (2 elements) falls off at a final attenuation slope of 12 dB per octave, a third-order network (3 elements) at the rate of 18 dB per octave, and so on, with the addition of 6 dB per octave per element.

MODERN FILTER DESIGN

Modern filter design has evolved through the years from a subject known only to specialists in the field (because of the advanced mathematics involved) to a practical well-organized catalog of ready-to-use circuits available to anyone with a knowledge of eighth grade level math. In fact, an average individual with absolutely no prior practical filter design experience should be able to sit down, read this chapter, and within 30 minutes be able to design a practical high-pass, low-pass, bandpass, or bandstop filter to his specifications. It sounds simple and it is—once a few basic rules are memorized.

The approach we will take in all of the designs in this chapter will be to make use of the myriad of normalized *low-pass prototypes* that are now available to the designer. The actual design procedure is, therefore, nothing more than determining your requirements and, then, finding a filter in a catalog which satisfies these requirements. Each normalized element value is then scaled to the frequency and impedance you desire and, then, transformed to the type of response (bandpass, high-pass, bandstop) that you wish. With practice, the procedure becomes very simple and soon you will be defining and designing filters.

The concept of normalization may at first seem foreign to the person who is a newcomer to the field of filter design, and the idea of transforming a low-pass filter into one that will give one of the other three types of responses might seem absurd. The best advice I can give (to anyone not familiar with these practices and who might feel a bit skeptical at this point) is to press on. The only way to truly realize the beauty and simplicity of this approach is to try a few actual designs. Once you try a few, you will be hooked, and any other approach to filter design will suddenly seem tedious and unnecessarily complicated.

NORMALIZATION AND THE LOW-PASS PROTOTYPE

In order to offer a catalog of useful filter circuits to the electronic filter designer, it became necessary to standardize the presentation of the material. Obviously, in practice, it would be extremely difficult to compare the performance and evaluate the usefulness of two filter networks if they were operating under two totally different sets of circumstances. Similarly, the presentation of any comparative design information for filters, if not standardized, would be totally useless. This concept of standardization or *normalization*, then, is merely a tool used by filter experts to present all filter design and performance information in a manner useful to circuit designers. Normalization assures the designer of the capability of comparing the performance of any two filter types when given the same operating conditions.

All of the catalogued filters in this chapter are low-pass filters normalized for a cutoff frequency of one radian per second (0.159 Hz) and for source and load resistors of one ohm. A characteristic response of such a filter is shown in Fig. 3-7. The circuit used to generate this response is called the *low-pass prototype*.

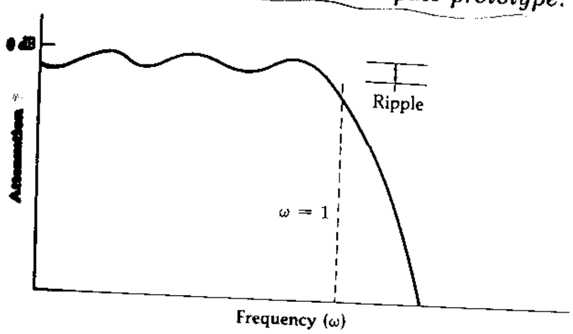


Fig. 3-7. Normalized low-pass response.

Obviously, the design of a filter with such a low cutoff frequency would require component values much larger than those we are accustomed to working with; capacitor values would be in farads rather than microfarads and picofarads, and the inductor values would be in henries rather than in microhenries and nanohenries. But once we choose a suitable low-pass prototype from the catalog, we can change the impedance level and cutoff frequency of the filter to any value we wish through a simple process called *scaling*. The net result of this process is a practical filter design with realizable component values.

FILTER TYPES

Many of the filters used today bear the names of the men who developed them. In this section, we will take a look at three such filters and examine their attenuation characteristics. Their relative merits will be discussed and their low-pass prototypes presented. The filter types discussed will include the Butterworth, Chebyshev, and Bessel responses.

Butterworth Response

The Butterworth filter is a medium-Q filter that is used in designs which require the amplitude response of the filter to be as flat as possible. The Butterworth response is the flattest passband response available and contains no ripple. The typical response of such a filter might look like that of Fig. 3-8.

Since the Butterworth response is only a medium-Q filter, its initial attenuation steepness is not as good as some filters but it is better than others. This characteristic often causes the Butterworth response to be called a middle-of-the-road design.

The attenuation of a Butterworth filter is given by

$$A_{dB} = 10 \log\left[1 + \left(\frac{\omega}{\omega_c}\right)^{2n}\right] \qquad \text{(Eq. 3-5)}$$

where,

ω = the frequency at which the attenuation is desired,

ω_c = the cutoff frequency ($\omega_{3\,dB}$) of the filter,

n = the number of elements in the filter.

If Equation 3-5 is evaluated at various frequencies for various numbers of elements, a family of curves is generated which will give a very good graphical representation of the attenuation provided by any order of filter at any frequency. This information is illustrated in Fig. 3-9. Thus, from Fig. 3-9, a 5-element (fifth order) Butterworth filter will provide an attenuation of approximately 30 dB at a frequency equal to

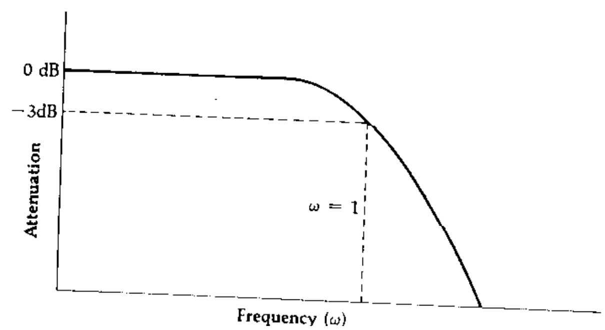


Fig. 3-8. The Butterworth response.

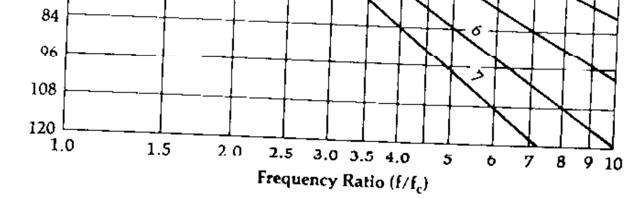


Fig. 3-9. Attenuation characteristics for Butterworth filters.

twice the cutoff frequency of the filter. Notice here that the frequency axis is normalized to ω/ω_c and the graph begins at the cutoff (−3 dB) point. This graph is extremely useful as it provides you with a method of determining, at a glance, the order of a filter needed to meet a given attenuation specification. A brief example should illustrate this point (Example 3-1).

EXAMPLE 3-1

How many elements are required to design a Butterworth filter with a cutoff frequency of 50 MHz, if the filter must provide at least 50 dB of attenuation at 150 MHz?

Solution

The first step in the solution is to find the ratio of $\omega/\omega_c = f/f_c$.

$$\frac{f}{f_c} = \frac{150\text{ MHz}}{50\text{ MHz}} = 3$$

Thus, at 3 times the cutoff frequency, the response must be down by at least 50 dB. Referring to Fig. 3-9, it is seen very quickly that a minimum of 6 elements is required to meet this design goal. At an f/f_c of 3, a 6-element design would provide approximately 57 dB of attenuation, while a 5-element design would provide only about 47 dB, which is not quite good enough.

The element values for a normalized Butterworth low-pass filter operating between equal 1-ohm terminations (source and load) can be found by

$$A_k = 2 \sin \frac{(2k-1)\pi}{2n}, k = 1,2, \ldots n \qquad \text{(Eq. 3-6)}$$

where,

n is the number of elements,

A_k is the k-th reactance in the ladder and may be either an inductor or capacitor.

The term $(2k-1)\pi/2n$ is in radians. We can use Equation 3-6 to generate our first entry into the catalog of low-pass prototypes shown in Table 3-1. The placement of each component of the filter is shown immediately above and below the table.

The rules for interpreting Butterworth tables are simple. The schematic shown above the table is used whenever the ratio R_S/R_L is calculated as the design criteria. The table is read from the top down. Alternately, when R_L/R_S is calculated, the schematic below the table is used. Then, the element designators in the table are read from the bottom up. Thus, a four-element low-pass prototype could appear as shown in Fig. 3-10. Note here that the element values not given in Table 3-1 are simply left out of the prototype ladder network. The 1-ohm load resistor is then placed directly across the output of the filter.

Remember that the cutoff frequency of each filter is 1 radian per second, or 0.159 Hz. Each capacitor value given is in farads, and each inductor value is in henries. The network will later be scaled to the impedance and frequency that is desired through a simple multiplication and division process. The component values will then appear much more realistic.

Occasionally, we have the need to design a filter that will operate between two unequal terminations as shown in Fig. 3-11. In this case, the circuit is normal-

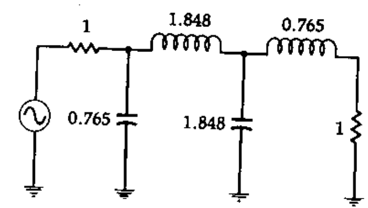
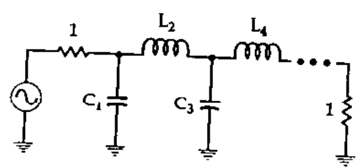
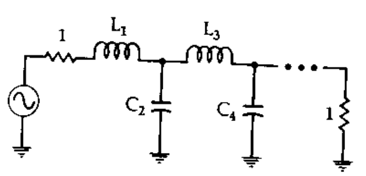


Fig. 3-10. A four-element Butterworth low-pass prototype circuit.

Table 3-1. Butterworth Equal Termination Low-Pass Prototype Element Values ($R_S = R_L$)



n	C_1	L_2	C_3	L_4	C_5	L_6	C_7
2	1.414	1.414					
3	1.000	2.000	1.000				
4	0.765	1.848	1.848	0.765			
5	0.618	1.618	2.000	1.618	0.618		
6	0.518	1.414	1.932	1.932	1.414	0.518	
7	0.445	1.247	1.802	2.000	1.802	1.247	0.445
n	L_1	C_2	L_3	C_4	L_5	C_6	L_7



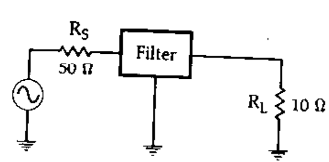


Fig. 3-11. Unequal terminations.

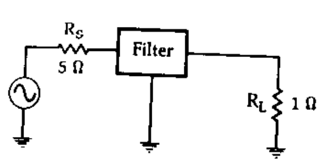
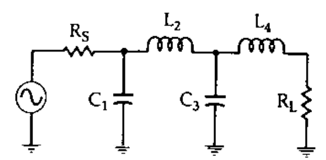


Fig. 3-12. Normalized unequal terminations.

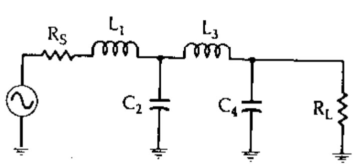
ed for a load resistance of 1 ohm, while taking what left for the source resistance. Dividing both the load and source resistor by 10 will yield a load resistance of 1 ohm and a source resistance of 5 ohms as shown in Fig. 3-12. We can use the normalized terminating resistors to help us find a low-pass prototype circuit.

Table 3-2 is a list of Butterworth low-pass prototype values for various ratios of source to load impedance (R_S/R_L). The schematic shown above the table is used when R_S/R_L is calculated, and the element values are read down from the top of the table.

Table 3-2A. Butterworth Low-Pass Prototype Element Values



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.111	1.035	1.835		
	1.250	0.849	2.121		
	1.429	0.697	2.439		
	1.667	0.566	2.828		
	2.000	0.448	3.346		
	2.500	0.342	4.095		
	3.333	0.245	5.313		
	5.000	0.156	7.707		
	10.000	0.074	14.814		
	∞	1.414	0.707		
3	0.900	0.808	1.633	1.599	
	0.800	0.844	1.384	1.926	
	0.700	0.915	1.165	2.277	
	0.600	1.023	0.965	2.702	
	0.500	1.181	0.779	3.261	
	0.400	1.425	0.604	4.064	
	0.300	1.838	0.440	5.363	
	0.200	2.669	0.284	7.910	
	0.100	5.167	0.138	15.455	
	∞	1.500	1.333	0.500	
4	1.111	0.466	1.592	1.744	1.469
	1.250	0.388	1.695	1.511	1.811
	1.429	0.325	1.862	1.291	2.175
	1.667	0.269	2.103	1.082	2.613
	2.000	0.218	2.452	0.883	3.187
	2.500	0.169	2.986	0.691	4.009
	3.333	0.124	3.883	0.507	5.338
	5.000	0.080	5.684	0.331	7.940
	10.000	0.039	11.094	0.162	15.642
	∞	1.531	1.577	1.082	0.383
n	R_L/R_S	L_1	C_2	L_3	C_4



Alternately, when R_L/R_S is calculated, the schematic below the table is used while reading up from the bottom of the table to get the element values (Example 3-2).

EXAMPLE 3-2

Find the low-pass prototype value for an n = 4 Butterworth filter with unequal terminations: R_S = 50 ohms, R_L = 100 ohms.

Solution

Normalizing the two terminations for R_L = 1 ohm will yield a value of R_S = 0.5. Reading down from the top of Table 3-2, for an n = 4 low-pass prototype value, we see that there is no R_S/R_L = 0.5 ratio listed. Our second choice, then, is to take the value of R_L/R_S = 2, and read up from the bottom of the table while using the schematic below the table as the form for the low-pass prototype values. This approach results in the low-pass prototype circuit of Fig. 3-13.

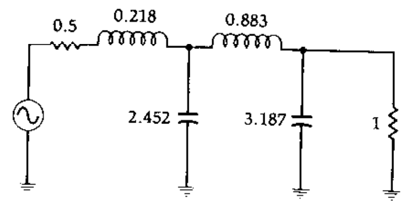


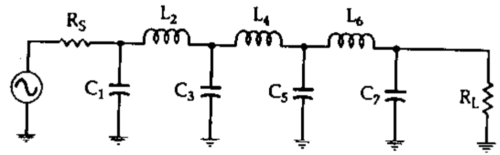
Fig. 3-13. Low-pass prototype circuit for Example 3-2.

Obviously, all possible ratios of source to load resistance could not possibly fit on a chart of this size. This, of course, leaves the potential problem of not being able to find the ratio that you need for a particular design task. The solution to this dilemma is to simply choose a ratio which most closely matches the ratio you need to complete the design. For ratios of 100:1 or so, the best results are obtained if you assume this value to be so high for practical purposes as to be infinite. Since, in these instances, you are only approximating the ratio of source to load resistance, the filter derived will only approximate the response that was originally intended. This is usually not too much of a problem.

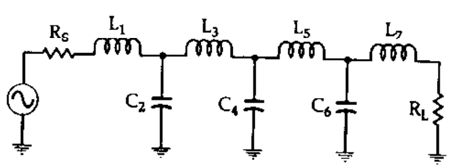
The Chebyshev Response

The Chebyshev filter is a high-Q filter that is used when: (1) a steeper initial descent into the stopband is required, and (2) the passband response is no longer required to be flat. With this type of requirement, ripple can be allowed in the passband. As more ripple is introduced, the initial slope at the beginning of the stopband is increased and produces a more rectangular attenuation curve when compared to the rounded Butterworth response. This comparison is made in Fig. 3-14. Both curves are for n = 3 filters. The Chebyshev response shown has 3 dB of passband ripple and produces a 10 dB improvement in stopband attenuation over the Butterworth filter.

Table 3-2B. Butterworth Low-Pass Prototype Element Values



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	0.900	0.442	1.027	1.910	1.756	1.389		
	0.800	0.470	0.866	2.061	1.544	1.738		
	0.700	0.517	0.731	2.285	1.333	2.108		
	0.600	0.586	0.609	2.600	1.126	2.552		
	0.500	0.686	0.496	3.051	0.924	3.133		
	0.400	0.838	0.388	3.736	0.727	3.965		
	0.300	1.094	0.285	4.884	0.537	5.307		
	0.200	1.608	0.186	7.185	0.352	7.935		
	0.100	3.512	0.091	14.095	0.173	15.710		
	∞	1.545	1.694	1.382	0.894	0.309		
6	1.111	0.289	1.040	1.322	2.054	1.744	1.335	
	1.250	0.245	1.116	1.126	2.239	1.550	1.688	
	1.429	0.207	1.236	0.957	2.499	1.346	2.062	
	1.667	0.173	1.407	0.801	2.858	1.143	2.509	
	2.000	0.141	1.653	0.654	3.369	0.942	3.094	
	2.500	0.111	2.028	0.514	4.141	0.745	3.931	
	3.333	0.082	2.656	0.379	5.433	0.552	5.280	
	5.000	0.054	3.917	0.248	8.020	0.363	7.922	
	10.000	0.026	7.705	0.122	15.786	0.179	15.738	
	∞	1.553	1.759	1.553	1.202	0.758	0.259	
7	0.900	0.299	0.711	1.404	1.489	2.125	1.727	1.296
	0.800	0.322	0.606	1.517	1.278	2.334	1.546	1.652
	0.700	0.357	0.515	1.688	1.091	2.618	1.350	2.028
	0.600	0.408	0.432	1.928	0.917	3.005	1.150	2.477
	0.500	0.480	0.354	2.273	0.751	3.553	0.951	3.064
	0.400	0.590	0.278	2.795	0.592	4.380	0.754	3.904
	0.300	0.775	0.206	3.671	0.437	5.761	0.560	5.258
	0.200	1.145	0.135	5.427	0.287	8.526	0.369	7.908
	0.100	2.257	0.067	10.700	0.142	16.822	0.182	15.748
	∞	1.558	1.799	1.659	1.397	1.055	0.656	0.223
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



The attenuation of a Chebyshev filter can be found by making a few simple but tiresome calculations, and can be expressed as:

$$A_{dB} = 10 \log \left[1 + \epsilon^2 C_n^2 \left(\frac{\omega}{\omega_c}\right)'\right] \qquad \text{(Eq. 3-7)}$$

where,

$C_n^2 \left(\frac{\omega}{\omega_c}\right)'$ is the Chebyshev polynomial to the order n evaluated at $\left(\frac{\omega}{\omega_c}\right)'$.

The Chebyshev polynomials for the first seven orders are given in Table 3-3. The parameter ϵ is given by:

$$\epsilon = \sqrt{10^{R_{dB}/10} - 1} \qquad \text{(Eq. 3-8)}$$

where,

R_{dB} is the passband ripple in decibels.

Note that $\left(\frac{\omega}{\omega_c}\right)'$ is not the same as $\left(\frac{\omega}{\omega_c}\right)$. The quantity $\left(\frac{\omega}{\omega_c}\right)'$ can be found by defining another parameter:

$$B = \frac{1}{n} \cosh^{-1}\left(\frac{1}{\epsilon}\right) \qquad \text{(Eq. 3-9)}$$

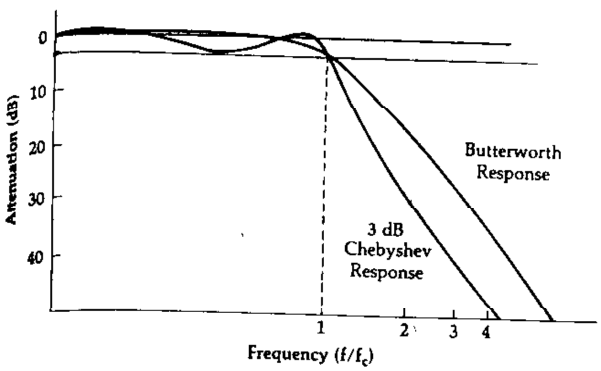


Fig. 3-14. Comparison of three-element Chebyshev and Butterworth responses.

Table 3-3. Chebyshev Polynomials to the Order n

n	*Chebyshev Polynomial*
1	$\frac{\omega}{\omega_c}$
2	$2\left(\frac{\omega}{\omega_c}\right)^2 - 1$
3	$4\left(\frac{\omega}{\omega_c}\right)^3 - 3\left(\frac{\omega}{\omega_c}\right)$
4	$8\left(\frac{\omega}{\omega_c}\right)^4 - 8\left(\frac{\omega}{\omega_c}\right)^2 + 1$
5	$16\left(\frac{\omega}{\omega_c}\right)^5 - 20\left(\frac{\omega}{\omega_c}\right)^3 + 5\left(\frac{\omega}{\omega_c}\right)$
6	$32\left(\frac{\omega}{\omega_c}\right)^6 - 48\left(\frac{\omega}{\omega_c}\right)^4 + 18\left(\frac{\omega}{\omega_c}\right)^2 - 1$
7	$64\left(\frac{\omega}{\omega_c}\right)^7 - 112\left(\frac{\omega}{\omega_c}\right)^5 + 56\left(\frac{\omega}{\omega_c}\right)^3 - 7\left(\frac{\omega}{\omega_c}\right)$

where,

n = the order of the filter,
ϵ = the parameter defined in Equation 3-8,
$\cosh^{-1}$ = the inverse hyperbolic cosine of the quantity in parentheses.

Finally, we have:

$$\left(\frac{\omega}{\omega_c}\right)' = \left(\frac{\omega}{\omega_c}\right)\cosh A \qquad \text{(Eq. 3-10)}$$

where,

$\left(\frac{\omega}{\omega_c}\right)$ = the ratio of the frequency of interest to the cutoff frequency,
cosh = the hyperbolic cosine.

If your calculator does not have hyperbolic and inverse hyperbolic functions, they can be manually determined from the following relations:

$$\cosh x = 0.5(e^x + e^{-x})$$

and

$$\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$$

The preceding equations yield families of attenuation curves, each classified according to the amount of ripple allowed in the passband. Several of these families of curves are shown in Figs. 3-15 through 3-18, and include 0.01-dB, 0.1-dB, 0.5-dB, and 1.0-dB ripple. Each curve begins at $\omega/\omega_c = 1$, which is the normalized cutoff, or 3-dB frequency. The passband ripple is, therefore, not shown.

If other families of attenuation curves are needed with different values of passband ripple, the preceding Chebyshev equations can be used to derive them. The problem in Example 3-3 illustrates this.

Obviously, performing the calculations of Example 3-3 for various values of ω/ω_c, ripple, and filter order is a very time-consuming chore unless a programmable calculator or computer is available to do most of the work for you.

The low-pass prototype element values corresponding to the Chebyshev responses of Figs. 3-15 through 3-18 are given in Tables 3-4 through 3-7. Note that the Chebyshev prototype values could not be separated into two distinct sets of tables covering the equal and

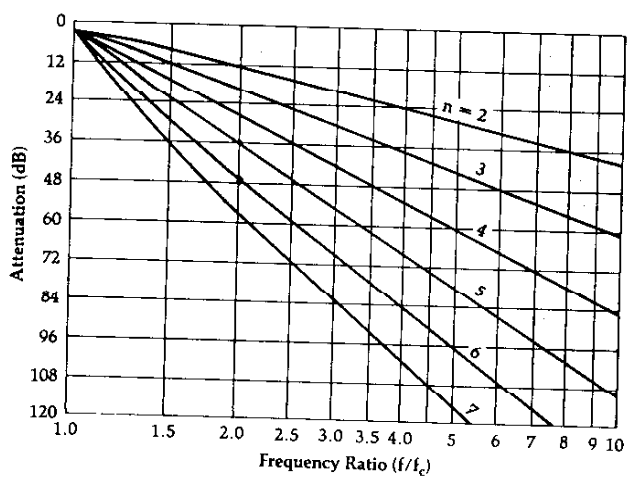


Fig. 3-15. Attenuation characteristics for a Chebyshev filter with 0.01-dB ripple.

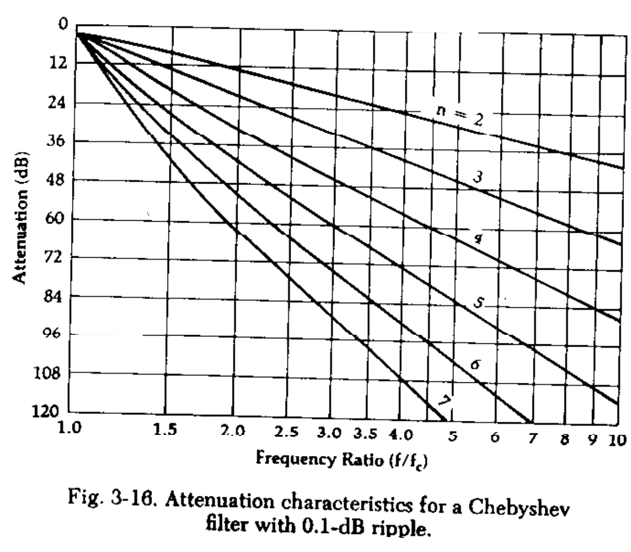


Fig. 3-16. Attenuation characteristics for a Chebyshev filter with 0.1-dB ripple.

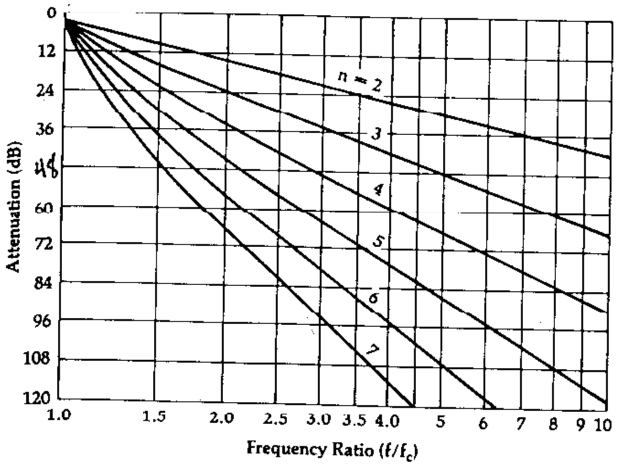


Fig. 3-17. Attenuation characteristics for a Chebyshev filter with 0.5-dB ripple.

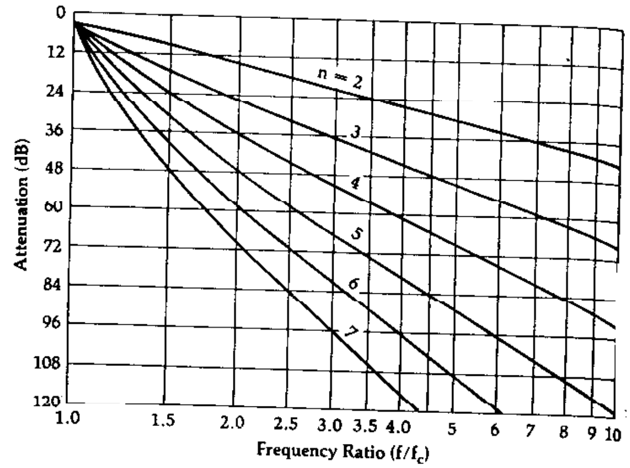


Fig. 3-18. Attenuation characteristics for a Chebyshev filter with 1-dB ripple.

unequal termination cases, as was done for the Butterworth prototypes. This is because the even order ($n = 2, 4, 6, \ldots$) Chebyshev filters cannot have equal terminations. The source and load must always be different for proper operation as shown in the tables.

EXAMPLE 3-3

Find the attenuation of a 4-element, 2.5-dB ripple, low-pass Chebyshev filter at $\omega/\omega_c = 2.5$.

Solution

First evaluate the parameter:

$$\epsilon = \sqrt{10^{2.5/10} - 1}$$
$$= 0.882$$

Next, find B.

$$B = \tfrac{1}{4}\left[\cosh^{-1}\left(\frac{1}{0.882}\right)\right]$$
$$= 0.1279$$

Then, $(\omega/\omega_c)'$ is:

$$(\omega/\omega_c)' = 2.5 \cosh .1279$$
$$= 2.5204$$

Finally, we evaluate the fourth order ($n = 4$) Chebyshev polynomial at $(\omega/\omega_c)' = 2.52$.

$$C_n\left(\frac{\omega}{\omega_c}\right) = 8\left(\frac{\omega}{\omega_c}\right)^4 - 8\left(\frac{\omega}{\omega_c}\right)^2 + 1$$
$$= 8(2.5204)^4 - 8(2.5204)^2 + 1$$
$$= 273.05$$

We can now evaluate the final equation.

$$A_{dB} = 10 \log_{10}\left[1 + \epsilon^2 C_n^2\left(\frac{\omega}{\omega_c}\right)'\right]$$
$$= 10 \log_{10}[1 + (0.882)^2(273.05)^2]$$
$$= 47.63 \text{ dB}$$

Thus, at an ω/ω_c of 2.5, you can expect 47.63 dB of attenuation for this filter.

The rules used for interpreting the Butterworth tables apply here also. The schematic shown above the table is used, and the element designators are read down from the top, when the ratio R_S/R_L is calculated as a design criteria. Alternately, with R_L/R_S calculations, use the schematic given below the table and read the element designators upwards from the bottom of the table. Example 3-4 is a practice problem for use in understanding the procedure.

EXAMPLE 3-4

Find the low-pass prototype values for an $n = 5$, 0.1-dB ripple, Chebyshev filter if the source resistance you are designing for is 50 ohms and the load resistance is 250 ohms.

Solution

Normalization of the source and load resistors yields an $R_s/R_L = 0.2$. A look at Table 3-5, for a 0.1-dB ripple filter with an $n = 5$ and an $R_s/R_L = 0.2$, yields the circuit values shown in Fig. 3-19.

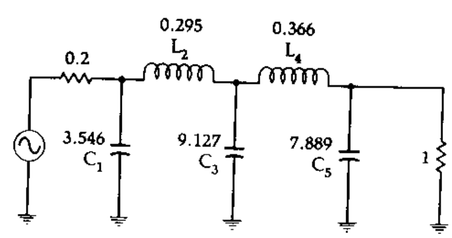
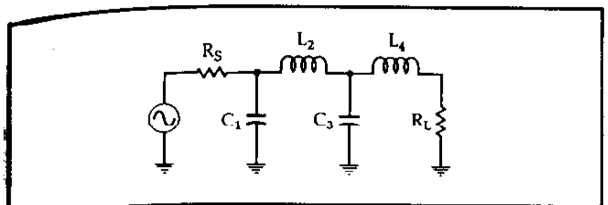


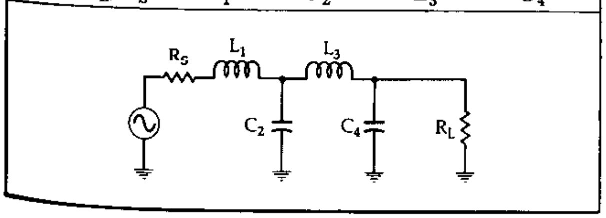
Fig. 3-19. Low-pass prototype circuit for Example 3-4.

It should be mentioned here that equations could have been presented in this section for deriving the element values for the Chebyshev low-pass prototypes. The equations are extremely long and tedious, however, and there would be little to be gained from their presentation.

Table 3-4A. Chebyshev Low-Pass Element Values for 0.01-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.101	1.347	1.483		
	1.111	1.247	1.595		
	1.250	0.943	1.997		
	1.429	0.759	2.344		
	1.667	0.609	2.750		
	2.000	0.479	3.277		
	2.500	0.363	4.033		
	3.333	0.259	5.255		
	5.000	0.164	7.650		
	10.000	0.078	14.749		
	∞	1.412	0.742		
3	1.000	1.181	1.821	1.181	
	0.900	1.092	1.660	1.480	
	0.800	1.097	1.443	1.806	
	0.700	1.160	1.228	2.165	
	0.600	1.274	1.024	2.598	
	0.500	1.452	0.829	3.164	
	0.400	1.734	0.645	3.974	
	0.300	2.216	0.470	5.280	
	0.200	3.193	0.305	7.834	
	0.100	6.141	0.148	15.390	
	∞	1.501	1.433	0.591	
4	1.100	0.950	1.938	1.761	1.046
	1.111	0.854	1.946	1.744	1.165
	1.250	0.618	2.075	1.542	1.617
	1.429	0.495	2.279	1.334	2.008
	1.667	0.398	2.571	1.128	2.461
	2.000	0.316	2.994	0.926	3.045
	2.500	0.242	3.641	0.729	3.875
	3.333	0.174	4.727	0.538	5.209
	5.000	0.112	6.910	0.352	7.813
	10.000	0.054	13.469	0.173	15.510
	∞	1.529	1.694	1.312	0.523
n	R_L/R_S	L_1	C_2	L_3	C_4



The Bessel Filter

The initial stopband attenuation of the Bessel filter is very poor and can be approximated by:

$$A_{dB} = 3\left(\frac{\omega}{\omega_c}\right)^2 \qquad \text{(Eq. 3-11)}$$

This expression, however, is not very accurate above an ω/ω_c that is equal to about 2. For values of ω/ω_c greater than 2, a straight-line approximation of 6 dB per octave per element can be made. This yields the family of curves shown in Fig. 3-20.

But why would anyone deliberately design a filter with very poor initial stopband attenuation characteristics? The Bessel filter was originally optimized to obtain a *maximally flat group delay* or *linear phase* characteristic in the filter's passband. Thus, selectivity or stopband attenuation is not a primary concern when dealing with the Bessel filter. In high- and medium-Q filters, such as the Chebyshev and Butterworth filters, the phase response is extremely nonlinear over the filter's passband. This phase nonlinearity results in distortion of wideband signals due to the widely varying time delays associated with the different spectral components of the signal. Bessel filters, on the other hand, with their maximally flat (constant) group delay are able to pass wideband signals with a minimum of distortion, while still providing *some* selectivity.

The low-pass prototype element values for the Bessel filter are given in Table 3-8. Table 3-8 tabulates the prototype element values for various ratios of source to load resistance.

FREQUENCY AND IMPEDANCE SCALING

Once you specify the filter, choose the appropriate attenuation response, and write down the low-pass prototype values, the next step is to transform the prototype circuit into a usable filter. Remember, the cutoff frequency of the prototype circuit is 0.159 Hz ($\omega = 1$ rad/sec), and it operates between a source and load resistance that are normalized so that $R_L = 1$ ohm.

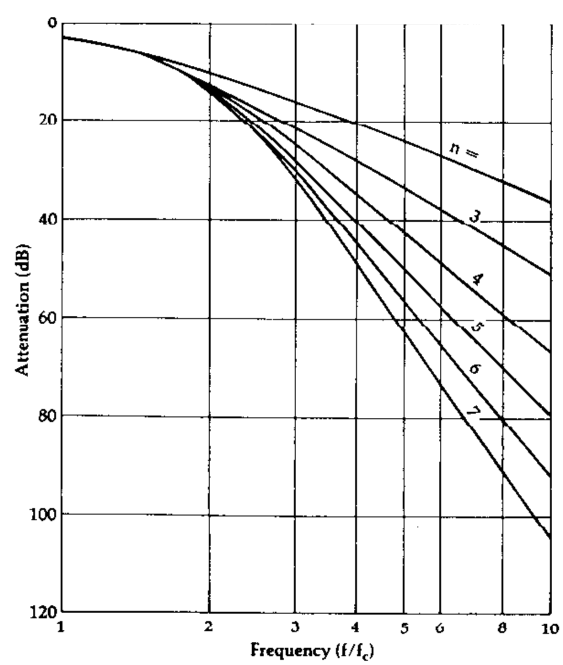
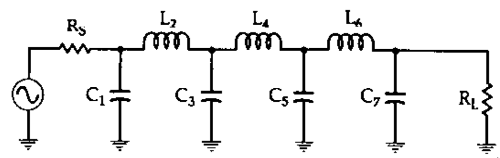
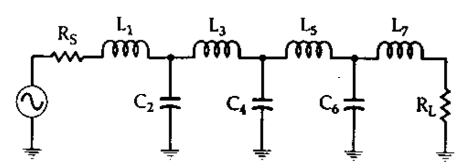


Fig. 3-20. Attenuation characteristics of Bessel filters.

Table 3-4B. Chebyshev Low-Pass Element Values for 0.01-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	0.977	1.685	2.037	1.685	0.977		
	0.900	0.880	1.456	2.174	1.641	1.274		
	0.800	0.877	1.235	2.379	1.499	1.607		
	0.700	0.926	1.040	2.658	1.323	1.977		
	0.600	1.019	0.863	3.041	1.135	2.424		
	0.500	1.166	0.699	3.584	0.942	3.009		
	0.400	1.398	0.544	4.403	0.749	3.845		
	0.300	1.797	0.398	5.772	0.557	5.193		
	0.200	2.604	0.259	8.514	0.368	7.826		
	0.100	5.041	0.127	16.741	0.182	15.613		
	∞	1.547	1.795	1.645	1.237	0.488		
6	1.101	0.851	1.796	1.841	2.027	1.631	0.937	
	1.111	0.760	1.782	1.775	2.094	1.638	1.053	
	1.250	0.545	1.864	1.489	2.403	1.507	1.504	
	1.429	0.436	2.038	1.266	2.735	1.332	1.899	
	1.667	0.351	2.298	1.061	3.167	1.145	2.357	
	2.000	0.279	2.678	0.867	3.768	0.954	2.948	
	2.500	0.214	3.261	0.682	4.667	0.761	3.790	
	3.333	0.155	4.245	0.503	6.163	0.568	5.143	
	5.000	0.100	6.223	0.330	9.151	0.376	7.785	
	10.000	0.048	12.171	0.162	18.105	0.187	15.595	
	∞	1.551	1.847	1.790	1.598	1.190	0.469	
7	1.000	0.913	1.595	2.002	1.870	2.002	1.595	0.913
	0.900	0.816	1.362	2.089	1.722	2.202	1.581	1.206
	0.800	0.811	1.150	2.262	1.525	2.465	1.464	1.538
	0.700	0.857	0.967	2.516	1.323	2.802	1.307	1.910
	0.600	0.943	0.803	2.872	1.124	3.250	1.131	2.359
	0.500	1.080	0.650	3.382	0.928	3.875	0.947	2.948
	0.400	1.297	0.507	4.156	0.735	4.812	0.758	3.790
	0.300	1.669	0.372	5.454	0.546	6.370	0.568	5.148
	0.200	2.242	0.242	8.057	0.360	9.484	0.378	7.802
	0.100	4.701	0.119	15.872	0.178	18.818	0.188	15.652
	∞	1.559	1.867	1.866	1.765	1.563	1.161	0.456
n	R_S/R_L	L_1	C_2	L_3	C_4	L_5	C_6	L_7



The transformation is affected through the following formulas:

$$C = \frac{C_n}{2\pi f_c R} \qquad \text{(Eq. 3-12)}$$

and

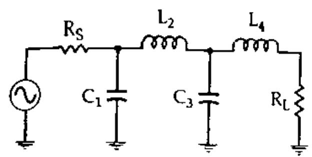
$$L = \frac{RL_n}{2\pi f_c} \qquad \text{(Eq. 3-13)}$$

where,

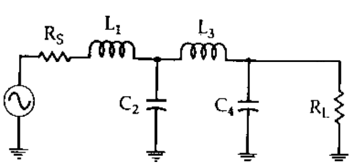
C = the final capacitor value,
L = the final inductor value,
C_n = a low-pass prototype element value,
L_n = a low-pass prototype element value,
R = the final load resistor value,
f_c = the final cutoff frequency.

The normalized low-pass prototype source resistor must also be transformed to its final value by multiplying it by the final value of the load resistor (Example 3-5). Thus, the ratio of the two always remains the same.

3-5A. Chebyshev Low-Pass Prototype Element Values for 0.1-dB Ripple



R_S/R_L	C_1	L_2	C_3	L_4
1.355	1.209	1.638		
1.429	0.977	1.982		
1.667	0.733	2.489		
2.000	0.560	3.054		
2.500	0.417	3.827		
3.333	0.293	5.050		
5.000	0.184	7.426		
10.000	0.087	14.433		
∞	1.391	0.819		
1.000	1.433	1.594	1.433	
0.900	1.426	1.494	1.622	
0.800	1.451	1.356	1.871	
0.700	1.521	1.193	2.190	
0.600	1.648	1.017	2.603	
0.500	1.853	0.838	3.159	
0.400	2.186	0.660	3.968	
0.300	2.763	0.486	5.279	
0.200	3.942	0.317	7.850	
0.100	7.512	0.155	15.466	
∞	1.513	1.510	0.716	
1.355	0.992	2.148	1.585	1.341
1.429	0.779	2.348	1.429	1.700
1.667	0.576	2.730	1.185	2.243
2.000	0.440	3.227	0.967	2.856
2.500	0.329	3.961	0.760	3.698
3.333	0.233	5.178	0.560	5.030
5.000	0.148	7.607	0.367	7.614
10.000	0.070	14.887	0.180	15.230
∞	1.511	1.768	1.455	0.673
n R_L/R_S	L_1	C_2	L_3	C_4



The process for designing a low-pass filter is a very nple one which involves the following procedure:

Define the response you need by specifying the required attenuation characteristics at selected frequencies.

Normalize the frequencies of interest by dividing them by the cutoff frequency of the filter. This step forces your data to be in the same form as that of the attenuation curves of this chapter, where the 3-dB point on the curve is:

$$\frac{f}{f_c} = 1$$

EXAMPLE 3-5

Scale the low-pass prototype values of Fig. 3-19 (Example 3-4) to a cutoff frequency of 50 MHz and a load resistance of 250 ohms.

Solution

Use Equations 3-12 and 3-13 to scale each component as follows:

$$C_1 = \frac{3.546}{2\pi(50 \times 10^6)(250)} = 45 \text{ pF}$$

$$C_3 = \frac{9.127}{2\pi(50 \times 10^6)(250)} = 116 \text{ pF}$$

$$C_6 = \frac{7.889}{2\pi(50 \times 10^6)(250)} = 100 \text{ pF}$$

$$L_2 = \frac{(250)(0.295)}{2\pi(50 \times 10^6)} = 235 \text{ nH}$$

$$L_4 = \frac{(250)(0.366)}{2\pi(50 \times 10^6)} = 291 \text{ nH}$$

The source resistance is scaled by multiplying its normalized value by the final value of the load resistor.

$$R_{s(final)} = 0.2(250) = 50 \text{ ohms}$$

The final circuit appears in Fig. 3-21.

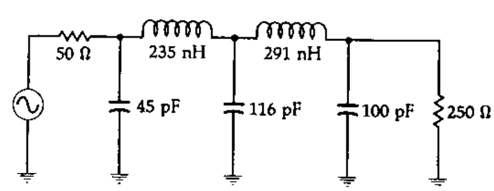
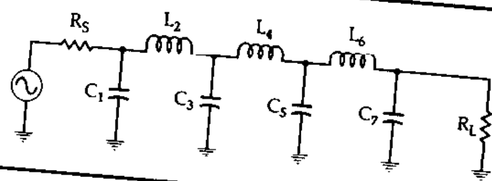


Fig. 3-21. Low-pass filter circuit for Example 3-5.

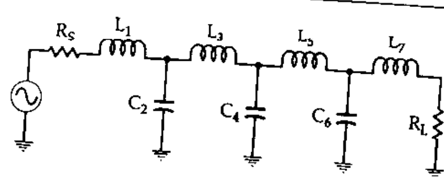
3. Determine the maximum amount of ripple that you can allow in the passband. Remember, the greater the amount of ripple allowed, the more selective the filter is. Higher values of ripple may allow you to eliminate a few components.
4. Match the normalized attenuation characteristics (Steps 1 and 2) with the attenuation curves provided in this chapter. Allow yourself a small "fudge-factor" for good measure. This step reveals the minimum number of circuit elements that you can get away with—given a certain filter type.
5. Find the low-pass prototype values in the tables.
6. Scale all elements to the frequency and impedance of the final design.

Example 3-6 diagrams the process of designing a low-pass filter using the preceding steps.

Table 3-5B. Chebyshev Low-Pass Prototype Element Values for 0.1-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.301	1.556	2.241	1.556	1.301		
	0.900	1.285	1.433	2.380	1.488	1.488		
	0.800	1.300	1.282	2.582	1.382	1.738		
	0.700	1.358	1.117	2.868	1.244	2.062		
	0.600	1.470	0.947	3.269	1.085	2.484		
	0.500	1.654	0.778	3.845	0.913	3.055		
	0.400	1.954	0.612	4.720	0.733	3.886		
	0.300	2.477	0.451	6.196	0.550	5.237		
	0.200	3.546	0.295	9.127	0.366	7.889		
	0.100	6.787	0.115	17.957	0.182	15.745		
	∞	1.561	1.807	1.766	1.417	0.651		
6	1.355	0.942	2.080	1.659	2.247	1.534	1.277	
	1.429	0.735	2.249	1.454	2.544	1.405	1.029	
	1.667	0.542	2.600	1.183	3.064	1.185	2.174	
	2.000	0.414	3.068	0.958	3.712	0.979	2.794	
	2.500	0.310	3.765	0.749	4.651	0.778	3.645	
	3.333	0.220	4.927	0.551	6.195	0.580	4.996	
	5.000	0.139	7.250	0.361	9.261	0.384	7.618	
	10.000	0.067	14.220	0.178	18.427	0.190	15.350	
	∞	1.534	1.884	1.831	1.749	1.394	0.638	
7	1.000	1.262	1.520	2.239	1.680	2.239	1.520	1.262
	0.900	1.242	1.395	2.361	1.578	2.397	1.459	1.447
	0.800	1.255	1.245	2.548	1.443	2.624	1.362	1.697
	0.700	1.310	1.083	2.819	1.283	2.942	1.233	2.021
	0.600	1.417	0.917	3.205	1.209	3.384	1.081	2.444
	0.500	1.595	0.753	3.764	0.928	4.015	0.914	3.018
	0.400	1.885	0.593	4.618	0.742	4.970	0.738	3.855
	0.300	2.392	0.437	6.054	0.556	6.569	0.557	5.217
	0.200	3.428	0.286	8.937	0.369	9.770	0.372	7.890
	0.100	6.570	0.141	17.603	0.184	19.376	0.180	15.813
	∞	1.575	1.858	1.921	1.827	1.734	1.379	0.631
n	R_S/R_L	L_1	C_2	L_3	C_4	L_5	C_6	L_7



HIGH-PASS FILTER DESIGN

Once you have learned the mechanics of low-pass filter design, high-pass design becomes a snap. You can use all of the attenuation response curves presented, thus far, for the low-pass filters by simply inverting the f/f_c axis. For instance, a 5-element, 0.1–dB-ripple Chebyshev *low-pass filter* will produce an attenuation of about 60 dB at an f/f_c of 3 (Fig. 3-16). If you were working instead with a *high-pass filter* of the same size and type, you could still use Fig. 3-16 to tell you that at an f/f_c of 1/3 (or, $f_c/f = 3$) a 5-element, 0.1-dB-ripple Chebyshev *high-pass filter* will also produce an attenuation of 60 dB. This is obviously more convenient than having to refer to more than one set of curves.

After finding the response which satisfies all of the requirements, the next step is to simply refer to the tables of low-pass prototype values and copy down the prototype values that are called for. High-pass values for the elements are then obtained directly from the low-pass prototype values as follows (refer to Fig. 3-24):

EXAMPLE 3-6

Design a low-pass filter to meet the following specifications:

$f_c = 35$ MHz,
Response greater than 60 dB down at 105 MHz,
Maximally flat passband—no ripple,
$R_s = 50$ ohms,
$R_L = 500$ ohms.

Solution

The need for a maximally flat passband automatically indicates that the design must be a Butterworth response. The first step in the design process is to normalize everything. Thus,

$$\frac{R_s}{R_L} = \frac{50}{500} = 0.1$$

Next, normalize the frequencies of interest so that they may be found in the graph of Fig. 3-9. Thus, we have:

$$\frac{f_{60dB}}{f_{3dB}} = \frac{105 \text{ MHz}}{35 \text{ MHz}} = 3$$

We next look at Fig. 3-9 and find a response that is down at least 60 dB at a frequency ratio of $f/f_c = 3$. Fig. 3-9 indicates that it will take a minimum of 7 elements to provide the attenuation specified. Referring to the catalog of Butterworth low-pass prototype values given in Table 3-2 yields the prototype circuit of Fig. 3-22.

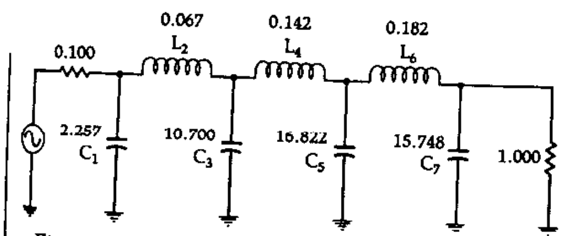


Fig. 3-22. Low-pass prototype circuit for Example 3-6.

We then scale these values using Equations 3-12 and 3-13. The first two values are worked out for you.

$$C_1 = \frac{2.257}{2\pi(35 \times 10^6)500} = 21 \text{ pF}$$

$$L_2 = \frac{(500)(0.067)}{2\pi(35 \times 10^6)} = 152 \text{ nH}$$

Similarly,

$C_3 = 97$ pF,
$C_5 = 153$ pF,
$C_7 = 143$ pF,
$L_4 = 323$ nH,
$L_6 = 414$ nH,
$R_s = 50$ ohms,
$R_L = 500$ ohms.

The final circuit is shown in Fig. 3-23.

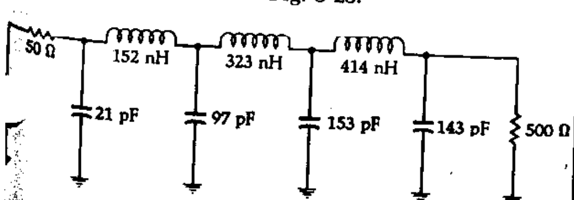
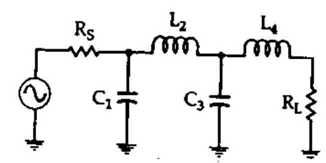
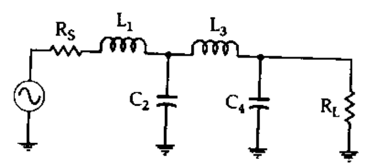


Fig. 3-23. Low-pass filter circuit for Example 3-6.

Table 3-6A. Chebyshev Low-Pass Prototype Element Values for 0.5-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.984	0.983	1.950		
	2.000	0.909	2.103		
	2.500	0.564	3.165		
	3.333	0.375	4.411		
	5.000	0.228	6.700		
	10.000	0.105	13.322		
	∞	1.307	0.975		
3	1.000	1.864	1.280	1.834	
	0.900	1.918	1.209	2.026	
	0.800	1.997	1.120	2.237	
	0.700	2.114	1.015	2.517	
	0.500	2.557	0.759	3.436	
	0.400	2.985	0.615	4.242	
	0.300	3.729	0.463	5.576	
	0.200	5.254	0.309	8.225	
	0.100	9.890	0.153	16.118	
	∞	1.572	1.518	0.932	
4	1.984	0.920	2.586	1.304	1.826
	2.000	0.845	2.720	1.238	1.985
	2.500	0.516	3.766	0.869	3.121
	3.333	0.344	5.120	0.621	4.480
	5.000	0.210	7.708	0.400	6.987
	10.000	0.098	15.352	0.194	14.262
	∞	1.436	1.889	1.521	0.913
n	R_L/R_S	L_1	C_2	L_3	C_4

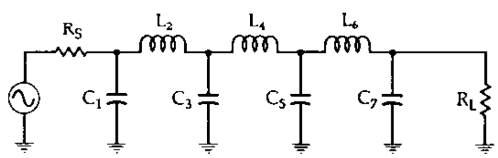


Simply replace each filter element with an element of the opposite type and with a reciprocal value. Thus, L_1 of Fig. 3-24B is equal to $1/C_1$ of Fig. 3-24A. Likewise, $C_2 = 1/L_2$ and $L_3 = 1/C_3$.

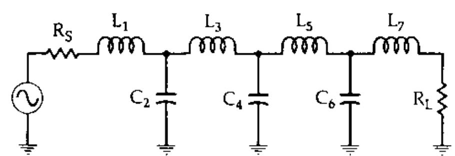
Stated another way, if the low-pass prototype indicates a capacitor of 1.181 farads, then, use an inductor with a value of $1/1.181 = 0.847$ henry, instead, for a high-pass design. However, the source and load resistors should not be altered.

The transformation process results in an attenuation characteristic for the high-pass filter that is an exact mirror image of the low-pass attenuation characteristic. The ripple, if there is any, remains the same and the magnitude of the slope of the stopband (or pass-

Table 3-6B. Chebyshev Low-Pass Prototype Element Values for 0.5-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.807	1.303	2.691	1.303	1.807		
	0.900	1.854	1.222	2.849	1.238	1.970		
	0.800	1.926	1.126	3.060	1.157	2.185		
	0.700	2.035	1.015	3.353	1.058	2.470		
	0.600	2.200	0.890	3.765	0.942	2.861		
	0.500	2.457	0.754	4.367	0.810	3.414		
	0.400	2.870	0.609	5.296	0.664	4.245		
	0.300	3.588	0.459	6.871	0.508	5.625		
	0.200	5.064	0.306	10.054	0.343	8.367		
	0.100	9.556	0.153	19.647	0.173	16.574		
	∞	1.630	1.740	1.922	1.514	0.903		
6	1.984	0.905	2.577	1.368	2.713	1.299	1.796	
	2.000	0.830	2.704	1.291	2.872	1.237	1.956	
	2.500	0.506	3.722	0.890	4.109	0.881	3.103	
	3.333	0.337	5.055	0.632	5.699	0.635	4.481	
	5.000	0.206	7.615	0.406	8.732	0.412	7.031	
	10.000	0.096	15.186	0.197	17.681	0.202	14.433	
7	1.000	1.790	1.296	2.718	1.385	2.718	1.296	1.790
	0.900	1.835	1.215	2.869	1.308	2.883	1.234	1.953
	0.800	1.905	1.118	3.076	1.215	3.107	1.155	2.168
	0.700	2.011	1.007	3.364	1.105	3.416	1.058	2.455
	0.600	2.174	0.882	3.772	0.979	3.852	0.944	2.848
	0.500	2.428	0.747	4.370	0.838	2.289	0.814	3.405
	0.400	2.835	0.604	5.295	0.685	5.470	0.669	4.243
	0.300	3.546	0.455	6.867	0.522	7.134	0.513	5.635
	0.200	5.007	0.303	10.049	0.352	10.496	0.348	8.404
	0.100	9.456	0.151	19.649	0.178	20.631	0.176	16.665
	∞	1.646	1.777	2.031	1.789	1.924	1.503	0.895
n	R_S/R_L	L_1	C_2	L_3	C_4	L_5	C_6	L_7



band) skirts remains the same. Example 3-7 illustrates the design of high-pass filters.

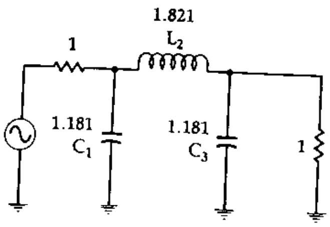
A closer look at the filter designed in Example 3-7 reveals that it is symmetric. Indeed, all filters given for the equal termination class are symmetric. The equal termination class of filter thus yields a circuit that is easier to design (fewer calculations) and, in most cases, cheaper to build for a high-volume product, due to the number of equal valued components.

THE DUAL NETWORK

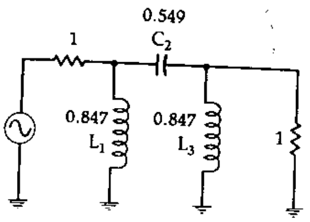
Thus far, we have been referring to the group of low-pass prototype element value tables presented and, then, we choose the schematic that is located either above or below the tables for the form of th filter that we are designing, depending on the valu of R_L/R_S. Either form of the filter will produce exactl the same attenuation, phase, and group-delay charac teristics, and each form is called the *dual* of the othe

Any filter network in a ladder arrangement, such a the ones presented in this chapter, can be change into its dual form by application of the following rules

1. Change all inductors to capacitors, and vice-verse without changing element values. Thus, 3 henrie becomes 3 farads.
2. Change all resistances into conductances, and vice versa, with the value unchanged. Thus, 3 ohms be comes 3 mhos, or ⅓ ohm.



(A) Low-pass prototype circuit.



(B) Equivalent high-pass prototype circuit.

Fig. 3-24. Low-pass to high-pass filter transformation.

3. Change all shunt branches to series branches, and vice-versa.
4. Change all elements in series with each other into elements that are in parallel with each other.
5. Change all voltage sources into current sources, and vice-versa.

Fig. 3-26 shows a ladder network and its dual representation.

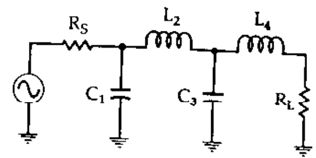
Dual networks are convenient, in the case of equal terminations, if you desire to change the topology of the filter without changing the response. It is most often used, as shown in Example 3-7, to eliminate an unnecessary inductor which might have crept into the design through some other transformation process. Inductors are typically more lower-Q devices than capacitors and, therefore, exhibit higher losses. These losses tend to cause insertion loss, in addition to generally degrading the overall performance of the filter. The number of inductors in any network should, therefore, be reduced whenever possible.

A little experimentation with dual networks having unequal terminations will reveal that you can quickly get yourself into trouble if you are not careful. This is especially true if the load and source resistance are a design criteria and cannot be changed to suit the needs of your filter. Remember, when the dual of a network with unequal terminations is taken, then, the terminations *must*, by definition, change value as shown in Fig. 3-26.

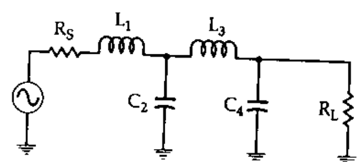
BANDPASS FILTER DESIGN

The low-pass prototype circuits and response curves in this chapter can also be used in the design of bandpass filters. This is done through a simple transformation process similar to what was done in the high-pass case.

Table 3-7A. Chebyshev Low-Pass Prototype Element Values for 1.0-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4
2	3.000	0.572	3.132		
	4.000	0.365	4.600		
	8.000	0.157	9.658		
	∞	1.213	1.109		
3	1.000	2.216	1.088	2.216	
	0.500	4.431	0.817	2.216	
	0.333	6.647	0.726	2.216	
	0.250	8.862	0.680	2.216	
	0.125	17.725	0.612	2.216	
	∞	1.652	1.460	1.108	
4	3.000	0.653	4.411	0.814	2.535
	4.000	0.452	7.083	0.612	2.848
	8.000	0.209	17.164	0.428	3.281
	∞	1.350	2.010	1.488	1.106
n	R_L/R_S	L_1	C_2	L_3	C_4



The most difficult task awaiting the designer of a bandpass filter, if the design is to be derived from the low-pass prototype, is in specifying the bandpass attenuation characteristics in terms of the low-pass response curves. A method for doing this is shown by the curves in Fig. 3-27. As you can see, when a low-pass design is transformed into a bandpass design, the attenuation bandwidth ratios remain the same. This means that a low-pass filter with a 3-dB cutoff frequency, or a bandwidth of 2 kHz, would transform into a bandpass filter with a 3-dB bandwidth of 2 kHz. If the response of the low-pass network were down 30 dB at a frequency or bandwidth of 4 kHz ($f/f_c = 2$), then the response of the bandpass network would be down 30 dB at a bandwidth of 4 kHz. Thus, the normalized f/f_c axis of the low-pass attenuation curves becomes a ratio of bandwidths rather than frequencies, such that:

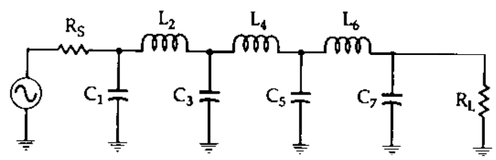
$$\frac{BW}{BW_c} = \frac{f}{f_c} \qquad \text{(Eq. 3-14)}$$

where,

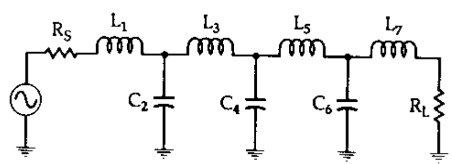
BW = the bandwidth at the required value of attenuation,

BW_c = the 3-dB bandwidth of the bandpass filter.

Table 3-7B. Chebyshev Low-Pass Prototype Element Values for 1.0-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	2.207	1.128	3.103	1.128	2.207		
	0.500	4.414	0.565	4.653	1.128	2.207		
	0.333	6.622	0.376	6.205	1.128	2.207		
	0.250	8.829	0.282	7.756	1.128	2.207		
	0.125	17.657	0.141	13.961	1.128	2.207		
	∞	1.721	1.645	2.061	1.493	1.103		
6	3.000	0.679	3.873	0.771	4.711	0.969	2.406	
	4.000	0.481	5.644	0.476	7.351	0.849	2.582	
	8.000	0.227	12.310	0.198	16.740	0.726	2.800	
	∞	1.378	2.007	1.690	2.074	1.494	1.102	
7	1.000	2.204	1.131	3.147	1.194	3.147	1.131	2.204
	0.500	4.408	0.566	6.293	0.895	3.147	1.131	2.204
	0.333	6.612	0.377	9.441	0.796	3.147	1.131	2.204
	0.250	8.815	0.283	12.588	0.747	3.147	1.131	2.204
	0.125	17.631	0.141	25.175	0.671	3.147	1.131	2.204
	∞	1.741	1.677	2.155	1.703	2.079	1.494	1.102
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



Often a bandpass response is not specified, as in Example 3-8. Instead, the requirements are often given as attenuation values at specified frequencies as shown by the curve in Fig. 3-28. In this case, you must transform the stated requirements into information that takes the form of Equation 3-14. As an example, consider Fig. 3-28. How do we convert the data that is given into the bandwidth ratios we need? Before we can answer that, we have to find f_3. Use the following method.

The frequency response of a bandpass filter exhibits geometric symmetry. That is, it is only symmetric when plotted on a logarithmic scale. The center frequency of a geometrically symmetric filter is given by the formula:

$$f_o = \sqrt{f_a f_b} \qquad \text{(Eq. 3-15)}$$

where f_a and f_b are any two frequencies (one above and one below the passband) having equal attenuation. Therefore, the center frequency of the response curve shown in Fig. 3-28 must be

$$f_o = \sqrt{(45)(75)}\ \text{MHz}$$
$$= 58.1\ \text{MHz}$$

We can use Equation 3-15 again to find f_3.

$$58.1 = \sqrt{f_3(125)}$$

or,

$$f_3 = 27\ \text{MHz}$$

Now that f_3 is known, the data of Fig. 3-28 can put into the form of Equation 3-14.

$$\frac{BW_{40\,dB}}{BW_{3\,dB}} = \frac{125\ \text{MHz} - 27\ \text{MHz}}{75\ \text{MHz} - 45\ \text{MHz}}$$
$$= 3.27$$

To find a low-pass prototype curve that will satisfy these requirements, simply refer to any of the pertinent graphs presented in this chapter and find a response which will provide 40 dB of attenuation at an f/f_c of 3.27. (A fourth-order or better Butterworth filter will do quite nicely.)

The actual transformation from the low-pass to the bandpass configuration is accomplished by resonating each low-pass element with an element of the opposite type and of the same value. All shunt elements of the low-pass prototype circuit become parallel-resonant

EXAMPLE 3-7

Design an LC high-pass filter with an f_c of 60 MHz and minimum attenuation of 40 dB at 30 MHz. The source and load resistance are equal at 300 ohms. Assume that a 0.5-dB passband ripple is tolerable.

Solution

First, normalize the attenuation requirements so that the low-pass attenuation curves may be used.

$$\frac{f}{f_c} = \frac{30\ \text{MHz}}{60\ \text{MHz}} = 0.5$$

Inverting, we get:

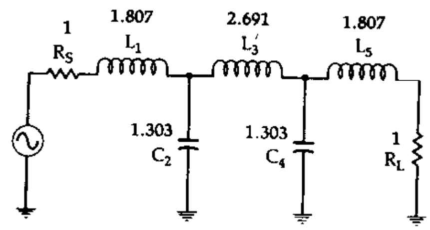
$$\frac{f_c}{f} = 2$$

Next, select a normalized low-pass filter that offers at least 40 dB attenuation at a ratio of $f_c/f = 2$. Reference to Fig. [illegible] attenuation response of 0.5–dB-ripple Chebyshev filters indicates that a normalized n = 5 Chebyshev will provide the needed attenuation. Table 3-6 contains the element values for the corresponding network. The normalized low-pass prototype circuit is shown in Fig. 3-25A. Note that the schematic below Table 3-6B was chosen as the low-pass prototype circuit rather than the schematic above the table. The reason for doing this will become obvious after the next step. Keep in mind, however, that the ratio of R_s/R_L is the same as the ratio of R_L/R_s, and is unity. Therefore, it does not matter which form is used for the prototype circuit.

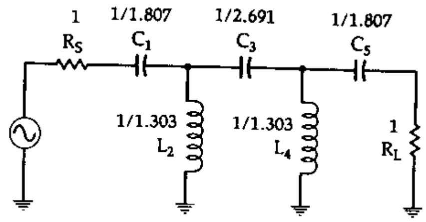
Next, transform the low-pass circuit to a high-pass network by replacing each inductor with a capacitor, and vice-versa, using reciprocal element values as shown in Fig. 3-25B. Note here that had we begun with the low-pass prototype circuit shown above Table 3-6B, this transformation would have yielded a filter containing three inductors rather than the two shown in Fig. 3-25B. The object in any of these filter designs is to reduce the number of inductors in the final design. More on this later.

The final step in the design process is to scale the network in both impedance and frequency using Equations 3-12 and 3-13. The first two calculations are done for you.

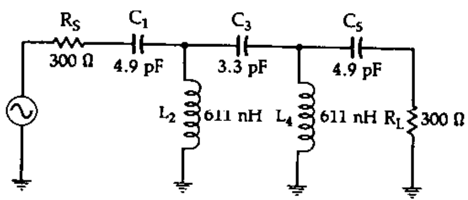
$$C_1 = \frac{\frac{1}{1.807}}{2\pi(60 \times 10^6)(300)} = 4.9\ \text{pF}$$



(*A*) *Normalized low-pass filter circuit.*



(*B*) *High-pass transformation.*



(*C*) *Frequency and impedance-scaled filter circuit.*

Fig. 3-25. High-pass filter design for Example 3-7.

$$L_2 = \frac{300\left(\frac{1}{1.303}\right)}{2\pi(60 \times 10^6)} = 611\ \text{nH}$$

The remaining values are:

$C_3 = 3.3$ pF
$C_5 = 4.9$ pF
$L_4 = 611$ nH

The final filter circuit is given in Fig. 3-25C.

EXAMPLE 3-8

Find the Butterworth low-pass prototype circuit which, when transformed, would satisfy the following bandpass filter requirements:

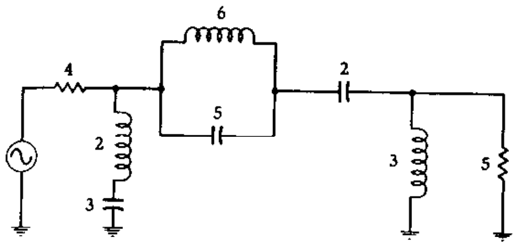
$$BW_{3dB} = 2\ \text{MHz}$$
$$BW_{40dB} = 6\ \text{MHz}$$

Solution

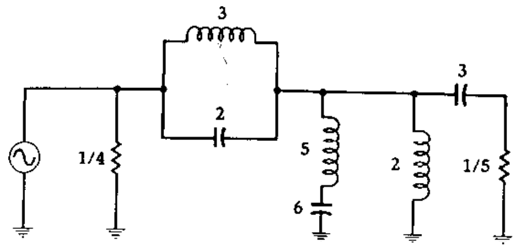
Note that we are not concerned with the center frequency of the bandpass response just yet. We are only concerned with the relationship between the above requirements and the low-pass response curves. Using Equation 3-14, we have:

$$\frac{BW}{BW_c} = \frac{f}{f_c} = \frac{BW_{40dB}}{BW_{3dB}} = \frac{6\ \text{MHz}}{2\ \text{MHz}} = 3$$

Therefore, turn to the Butterworth response curves shown in Fig. 3-9 and find a prototype value that will provide 40 dB of attenuation at an $f/f_c = 3$. The curves indicate a 5-element Butterworth filter will provide the needed attenuation.

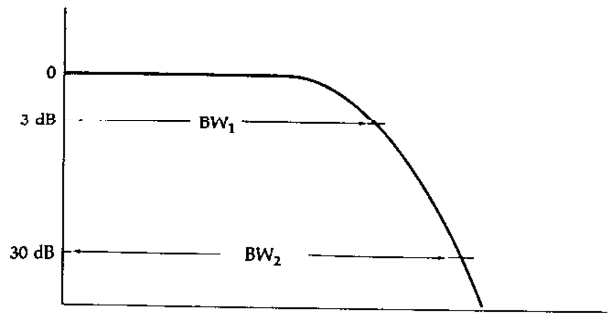


(A) A representative ladder network.

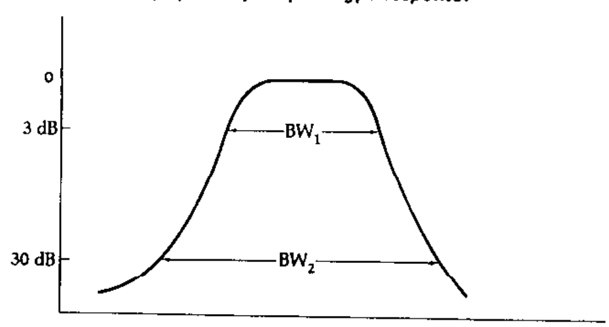


(B) Its dual form.

Fig. 3-26. Duality.



(A) Low-pass prototype response.



(B) Bandpass response.

Fig. 3-27. Low-pass to bandpass transformation bandwidths.

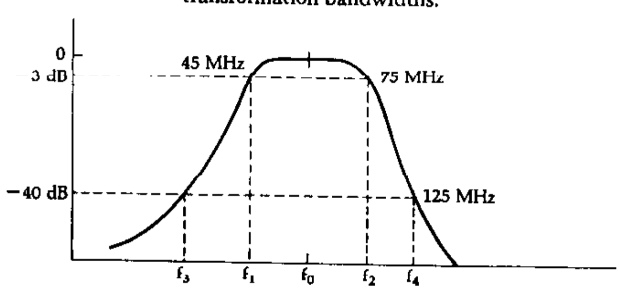
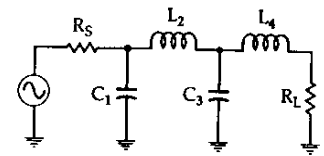
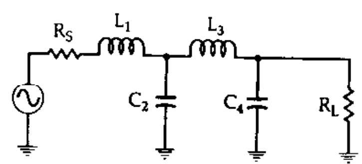


Fig. 3-28. Typical bandpass specifications.

Table 3-8A. Bessel Low-Pass Prototype Element Values



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.000	0.576	2.148		
	1.111	0.508	2.310		
	1.250	0.443	2.510		
	1.429	0.380	2.764		
	1.667	0.319	3.099		
	2.000	0.260	3.565		
	2.500	0.203	4.258		
	3.333	0.149	5.405		
	5.000	0.097	7.688		
	10.000	0.047	14.510		
	∞	1.362	0.454		
3	1.000	0.337	9.701	2.203	
	0.900	0.371	0.865	2.375	
	0.800	0.412	0.761	2.587	
	0.700	0.466	0.658	2.858	
	0.600	0.537	0.558	3.216	
	0.500	0.635	0.459	3.714	
	0.400	0.783	0.362	4.457	
	0.300	1.028	0.267	5.689	
	0.200	1.518	0.175	8.140	
	0.100	2.983	0.086	15.470	
	∞	1.463	0.843	0.293	
4	1.000	0.233	0.673	1.082	2.240
	1.111	0.209	0.742	0.967	2.414
	1.250	0.184	0.829	0.853	2.630
	1.429	0.160	0.941	0.741	2.907
	1.667	0.130	1.089	0.630	3.273
	2.000	0.112	1.295	0.520	3.782
	2.500	0.089	1.604	0.412	4.543
	3.333	0.066	2.117	0.306	5.805
	5.000	0.043	3.142	0.201	8.319
	10.000	0.021	6.209	0.099	15.837
	∞	1.501	0.978	0.613	0.211
n	R_L/R_S	L_1	C_2	L_3	C_4



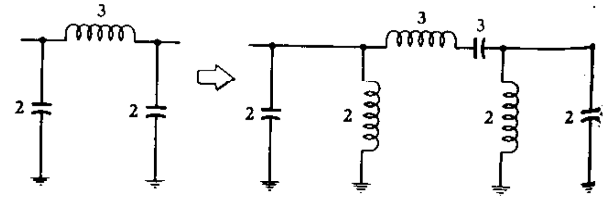
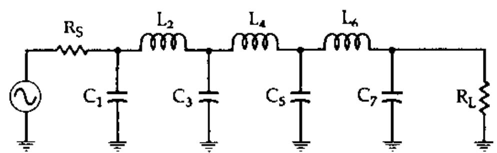


Fig. 3-29. Low-pass to bandpass circuit transformation

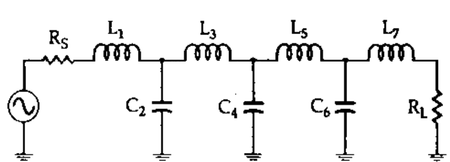
circuits, and all series elements become series-resonant circuits. This process is illustrated in Fig. 3-30.19

To complete the design, the transformed filter

Table 3-8B. Bessel Low-Pass Prototype Element Values



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	0.174	0.507	0.804	1.111	2.258		
	0.900	0.193	0.454	0.889	0.995	2.433		
	0.800	0.215	0.402	0.996	0.879	2.650		
	0.700	0.245	0.349	1.132	0.764	2.927		
	0.600	0.284	0.298	1.314	0.651	3.295		
	0.500	0.338	0.247	1.567	0.538	3.808		
	0.400	0.419	0.196	1.946	0.427	4.573		
	0.300	0.555	0.146	2.577	0.317	5.843		
	0.200	0.825	0.096	3.835	0.210	8.375		
	0.100	1.635	0.048	7.604	0.104	15.949		
	∞	1.513	1.023	0.753	0.473	0.162		
6	1.000	0.137	0.400	0.639	0.854	1.113	2.265	
	1.111	0.122	0.443	0.573	0.946	0.996	2.439	
	1.250	0.108	0.496	0.508	1.060	0.881	2.655	
	1.429	0.094	0.564	0.442	1.207	0.767	2.933	
	1.667	0.080	0.655	0.378	1.402	0.653	3.300	
	2.000	0.067	0.782	0.313	1.675	0.541	3.812	
	2.500	0.053	0.973	0.249	2.084	0.429	4.577	
	3.333	0.040	1.289	0.186	2.763	0.319	5.847	
	5.000	0.026	1.289	0.123	4.120	0.211	8.378	
	10.000	0.013	3.815	0.061	8.186	0.105	15.951	
	∞	1.512	1.033	0.813	0.607	0.379	0.129	
7	1.000	0.111	0.326	0.525	0.702	0.869	1.105	2.266
	0.900	0.122	0.292	0.582	0.630	0.963	0.990	2.440
	0.800	0.137	0.259	6.652	0.559	1.080	0.875	2.656
	0.700	0.156	0.226	0.743	0.487	1.231	0.762	2.932
	0.600	0.182	0.193	0.863	0.416	1.431	0.649	3.298
	0.500	0.217	0.160	1.032	0.346	1.711	0.537	3.809
	0.400	0.270	0.127	1.285	0.276	2.130	0.427	4.572
	0.300	0.358	0.095	1.705	0.206	2.828	0.318	5.836
	0.200	0.534	0.063	2.545	0.137	4.221	0.210	8.362
	0.100	1.061	0.031	5.062	0.068	8.397	0.104	15.917
	∞	1.509	1.029	0.835	0.675	0.503	0.311	0.105
n	R_S/R_L	L_1	C_2	L_3	C_4	L_5	C_6	L_7



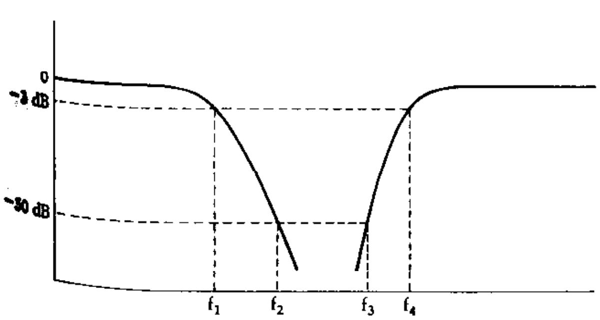


Fig. 3-30. Typical band-rejection filter curves.

then frequency- and impedance-scaled using the following formulas. For the parallel-resonant branches,

$$C = \frac{C_n}{2\pi RB} \qquad \text{(Eq. 3-16)}$$

$$L = \frac{RB}{2\pi f_o^2 L_n} \qquad \text{(Eq. 3-17)}$$

and, for the series-resonant branches,

$$C = \frac{B}{2\pi f_o^2 C_n R} \qquad \text{(Eq. 3-18)}$$

$$L = \frac{RL_n}{2\pi B} \qquad \text{(Eq. 3-19)}$$

where, in all cases,

R = the final load impedance,
B = the 3-dB bandwidth of the final design,
f_o = the geometric center frequency of the final design,
L_n = the normalized inductor *bandpass* element values,
C_n = the normalized capacitor *bandpass* element values.

Example 3-9 furnishes one final example of the procedure for designing a bandpass filter.

SUMMARY OF THE BANDPASS FILTER DESIGN PROCEDURE

1. Transform the bandpass requirements into an equivalent low-pass requirement using Equation 3-14.
2. Refer to the low-pass attenuation curves provided in order to find a response that meets the requirements of Step 1.
3. Find the corresponding low-pass prototype and write it down.
4. Transform the low-pass network into a bandpass configuration.
5. Scale the bandpass configuration in both impedance and frequency using Equations 3-16 through 3-19.

BAND-REJECTION FILTER DESIGN

Band-rejection filters are very similar in design approach to the bandpass filter of the last section. Only, in this case, we want to *reject* a certain group of frequencies as shown by the curves in Fig. 3-30.

The band-reject filter lends itself well to the low-pass prototype design approach using the same procedures as were used for the bandpass design. First, define the bandstop requirements in terms of the low-pass attenuation curves. This is done by using the inverse of Equation 3-14. Thus, referring to Fig. 3-30, we have:

$$\frac{BW_c}{BW} = \frac{f_4 - f_1}{f_3 - f_2}$$

This sets the attenuation characteristic that is needed and allows you to read directly off the low-pass attenuation curves by substituting BW_c/BW for f_c/f on the normalized frequency axis. Once the number of elements that are required in the low-pass prototype circuit is determined, the low-pass network is transformed into a band-reject configuration as follows:

> Each shunt element in the low-pass prototype circuit is replaced by a shunt *series-resonant circuit,* and each series-element is replaced by a series *parallel-resonant circuit.*

EXAMPLE 3-9

Design a bandpass filter with the following requirements:

$f_o = 75$ MHz
$BW_{3dB} = 7$ MHz
$BW_{45dB} = 35$ MHz
Passband Ripple = 1 dB
$R_s = 50$ ohms
$R_L = 100$ ohms

Solution

Using Equation 3-14:

$$\frac{BW_{45dB}}{BW_{3dB}} = \frac{35}{7} = 5$$

Substitute this value for f/f_c in the low-pass attenuation curves for the 1-dB-ripple Chebyshev response shown in Fig. 3-18. This reveals that a 3-element filter will provide about 50 dB of attenuation at an $f/f_c = 5$, which is more than adequate. The corresponding element values for this filter can be found in Table 3-7 for an $R_S/R_L = 0.5$ and an $n = 3$. This yields the low-pass prototype circuit of Fig. 3-32A which is transformed into the bandpass prototype circuit of Fig. 3-32B. Finally, using Equations 3-16 through 3-19, we obtain the final circuit that is shown in Fig. 3-32C. The calculations follow. Using Equations 3-16 and 3-17:

$$C_1 = \frac{4.431}{2\pi(100)(7 \times 10^6)} = 1007 \text{ pF}$$

$$L_1 = \frac{(100)(7 \times 10^6)}{2\pi(75 \times 10^6)^2(4.431)} = 4.47 \text{ nH}$$

Using Equations 3-18 and 3-19:

$$C_2 = \frac{7 \times 10^6}{2\pi(75 \times 10^6)^2(0.817)100} = 2.4 \text{ pF}$$

$$L_2 = \frac{(100)(0.817)}{2\pi(7 \times 10^6)} = 1.86\ \mu\text{H}$$

Similarly,

$C_3 = 504$ pF
$L_3 = 8.93$ nH

This is shown in Fig. 3-31. Note that both elements in each of the resonant circuits have the same normalized value.

Once the prototype circuit has been transformed into its band-reject configuration, it is then scaled in impedance and frequency using the following formulas. For all series-resonant circuits:

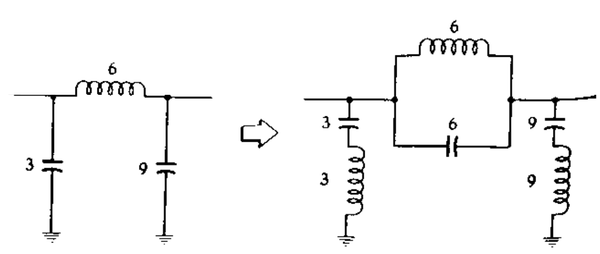
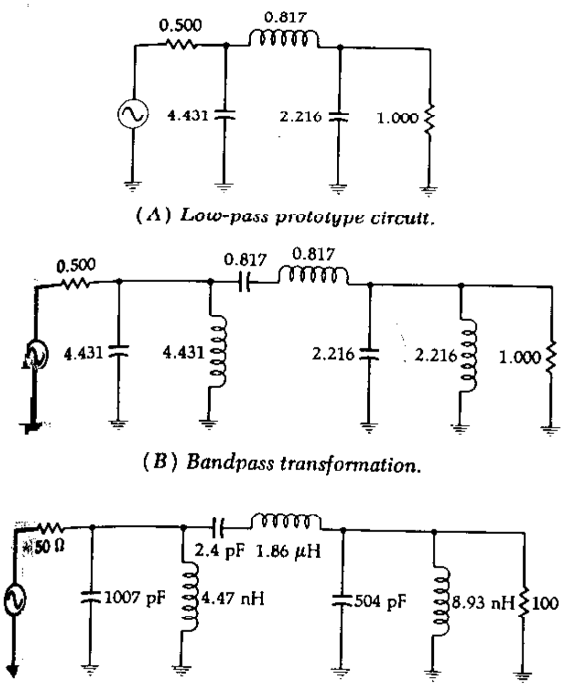


Fig. 3-31. Low-pass to band-reject transformation.



(*A*) *Low-pass prototype circuit.*

(*B*) *Bandpass transformation.*

(*C*) *Final circuit with frequency and impedance scaled.*

Fig. 3-32. Bandpass filter design for Example 3-9.

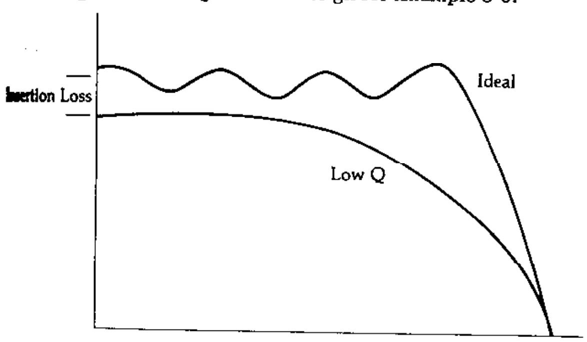


Fig. 3-33. The effect of finite-Q elements on filter response.

$$C = \frac{C_n}{2\pi RB} \qquad \text{(Eq. 3-20)}$$

$$L = \frac{RB}{2\pi f_o^2 L_n} \qquad \text{(Eq. 3-21)}$$

For all parallel-resonant circuits:

$$C = \frac{B}{2\pi f_o^2 RC_n} \qquad \text{(Eq. 3-22)}$$

$$L = \frac{RL_n}{2\pi B} \qquad \text{(Eq. 3-23)}$$

where, in all cases,

B = the 3-dB bandwidth,
R = the final load resistance,
f_o = the geometric center frequency,
C_n = the normalized capacitor band-reject element value,
L_n = the normalized inductor band-reject element value.

THE EFFECTS OF FINITE Q

Thus far in this chapter, we have assumed the inductors and capacitors used in the designs to be lossless. Indeed, all of the response curves presented in this chapter are based on that assumption. But we know from our previous study of Chapters 1 and 2 that even though capacitors can be approximated as having infinite Q, inductors cannot, and the effects of the finite-Q inductor must be taken into account in any filter design.

The use of finite element Q in a design intended for lossless elements causes the following unwanted effects (refer to Fig. 3-33):

1. Insertion loss of the filter is increased whereas the final stopband attenuation does not change. The relative attenuation between the two is decreased.
2. At frequencies in the vicinity of cutoff (f_c), the response becomes more rounded and usually results in an attenuation greater than the 3 dB that was originally intended.
3. Ripple that was designed into the passband will be reduced. If the element Q is sufficiently low, ripple will be totally eliminated.
4. For band-reject filters, the attenuation in the stopband becomes finite. This, coupled with an increase in *passband* insertion loss, decreases the relative attenuation significantly.

Regardless of the gloomy predictions outlined above, however, it is possible to design filters, using the approach outlined in this chapter, that very closely resemble the ideal response of each network. The key is to use the highest-Q inductors available for the given task. Table 3-9 outlines the recommended minimum element-Q requirements for the filters presented in this chapter. Keep in mind, however, that anytime a low-Q component is used, the actual attenuation response of the network strays from the ideal response to a degree depending upon the element Q. It is, therefore, highly recommended that you make it a habit to use only the highest-Q components available.

Table 3-9. Filter Element-Q Requirements

Filter Type	*Minimum Element Q Required*
Bessel	3
Butterworth	15
0.01-dB Chebyshev	24
0.1-dB Chebyshev	39
0.5-dB Chebyshev	57
1-dB Chebyshev	75

The insertion loss of the filters presented in this chapter can be calculated in the same manner as was used in Chapter 2. Simply replace each reactive element with resistor values corresponding to the Q of the element and, then, exercise the voltage division rule from source to load.