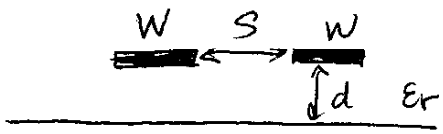
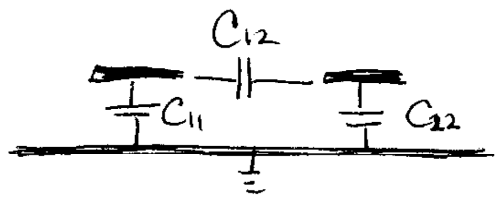
Begin of Coupled Lines:

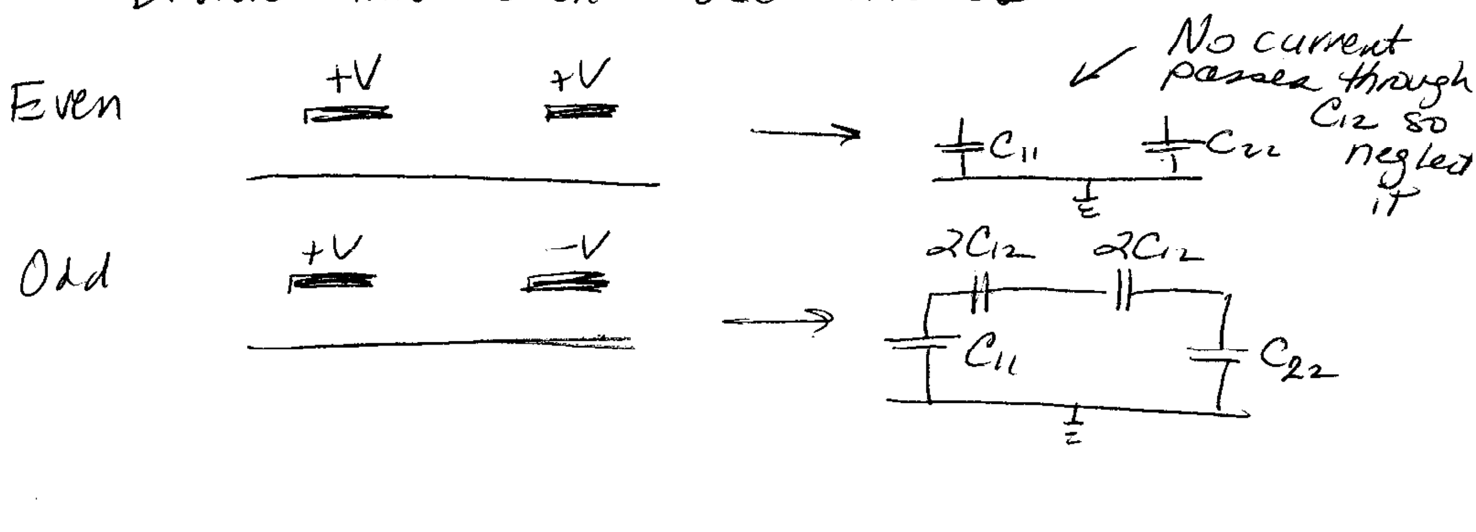


Assume TEM Propagation (perfect for stripline, not perfect for microstrip)

Model Capacitive Coupling



Divide Into Even & Odd Modes



For Even Mode

$$C_e = C_{11} = C_{12} \quad \text{(if lines are same size)}$$

$$Z_{oe} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{v_p C_e}$$

For Odd Mode

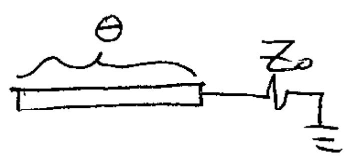
$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12} \quad \text{(if same size line)}$$

$$Z_{oo} = \frac{1}{v_p C_o}$$

Any source: Superposition of even + odd modes

To find actual values of Z_{oe} and Z_{oo}, need to find capacitances. This is generally done numerically — see handout.

Coupler Design



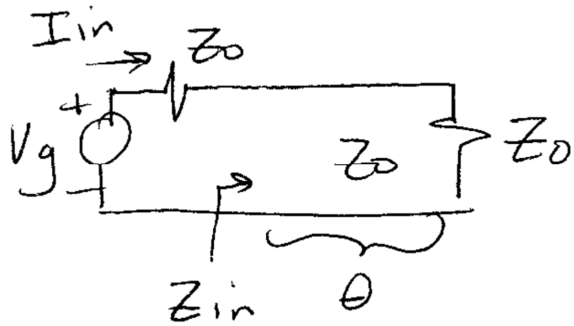
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}$$

$$Z_{in}^e = Z_{oe} \frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta}$$

$$Z_{in}^o = Z_{oo} \frac{Z_o + jZ_{oo}\tan\theta}{Z_{oo} + jZ_o\tan\theta}$$

$$V_1^o = V_g \frac{Z_{in}^o}{Z_{in}^o + Z_o} \qquad V_1^e = V_g \frac{Z_{in}^e}{Z_{in}^e + Z_o}$$

$$I_1^o = \frac{V_g}{Z_{in}^o + Z_o} \qquad I_1^e = \frac{V_g}{Z_{in}^e + Z_o}$$



$$Z_{in} = Z_o + 2\left(\frac{Z_{in}^o Z_{in}^e - Z_o^2}{Z_{in}^e + Z_{in}^o + 2Z_o}\right)$$

Want $Z_{in} = Z_o$ (matched)

Let $(Z_o)^2 = (Z_{in}^o Z_{in}^e)$

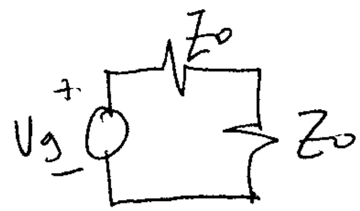
this defined impedances of lines

$$Z_{in}^e = Z_{oe} \frac{\sqrt{Z_{oo}} + j\sqrt{Z_{oe}} \tan\theta}{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}} \tan\theta}$$

$$Z_{in}^o = Z_{oo} \frac{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}} \tan\theta}{\sqrt{Z_{oe}} + j\sqrt{Z_{oe}} \tan\theta}$$

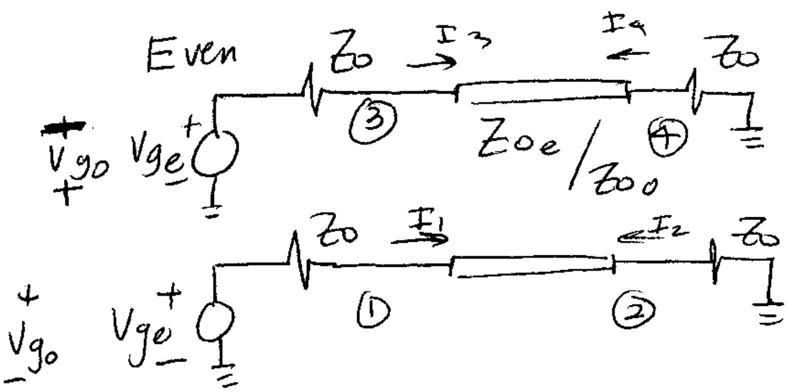
$$Z_{in} = Z_o$$

For matched input $Z_{in} = Z_0$



$$V_1^o + V_1^e = V_1 = \frac{V_g}{2} + \frac{V_g}{2} = V_g$$

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o$$



$$= V_g\left[\frac{Z_{in}^e}{Z_{in}^e + Z_0} - \frac{Z_{in}^o}{Z_{in}^o + Z_0}\right]$$

Midband Coupling Coeffecient V_3 / V_g

$$C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

100% Isolation $\quad V_4 = V_4^e + V_4^o = 0$

$$V_3 = V \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta}$$

$$V_2 = V \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

$$\frac{V_2}{V_g} = -j\sqrt{1-C^2} \qquad \frac{V_3}{V} = C$$