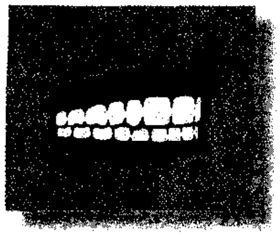

Microwave Bytes

Coupler Talk

■ **Steve Cripps**

The directional coupler is such a familiar item in microwave engineering that we all probably think we know what they do and can do. In these days, where the distinction between microwave and analog techniques is becoming ever more indistinct, the directional coupler seems to survive as an exclusive tool on the microwave side of the divide, a surviving artifact of the bygone days when microwave engineering had a closer affiliation to Maxwell than to Kirchoff. Where coaxial lines and macroscopic dimensions are standard fare, such useful devices can be readily realized, albeit with a few tricks if optimum performance is required. Such would not appear to be the case, however, in the more modern RFIC environment.

The most common use for a coupler is a directional power monitor. Although a somewhat mundane application, it is a very useful one. Recently I was asked if such a function could be translated to a more modern RFIC environment. In considering this question, it occurred to me that I might not have quite as firm a grasp of the underlying theory and principles as I had previously thought and that there may be important practical applications for these devices that lie outside of the normal laboratory measurement function.

Steve Cripps is with The Mangold House, Hare Lane, Buckland St. Mary Chard TA20 3JS Somerset, U.K., stevehywave@aol.com.

On revising my knowledge of couplers, it seemed like a useful first step to find a book that did the basic analysis of two coupled transmission lines. To my considerable surprise, none of the various books on my shelf had such a section, at least not to the point of deriving the full four port S-parameter matrix for the general case where the odd and even mode velocities are not assumed to be equal and the length not necessarily a quarter wave. There are enough microwave books that are not on my shelf, and I'm sure some may have this analysis. But, in the end, I had to dig quite deeply into my archives to find a paper by Napoli and Hughes [1], which satisfied my needs admirably. Taking a generalized pair of coupled lines, with ports designated as shown in Figure 1, the S-parameters work out to be

$$S_{11} = \frac{\rho_e}{2}\left(1 - \frac{1-\rho_e^2}{e^{2j\beta_e l} - \rho_e^2}\right) + \frac{\rho_o}{2}\left(1 - \frac{1-\rho_o^2}{e^{2j\beta_o l} - \rho_o^2}\right), \tag{1a}$$

$$S_{21} = \frac{\left(1-\rho_e^2\right)e^{j\beta_e l}}{2\left(e^{2j\beta_e l} - \rho_e^2\right)} + \frac{\left(1-\rho_o^2\right)e^{j\beta_o l}}{2\left(e^{2j\beta_o l} - \rho_o^2\right)}, \tag{1b}$$

$$S_{31} = \frac{\rho_e}{2}\left(1 - \frac{1-\rho_e^2}{e^{2j\beta_e l} - \rho_e^2}\right) - \frac{\rho_o}{2}\left(1 - \frac{1-\rho_o^2}{e^{2j\beta_o l} - \rho_o^2}\right), \tag{1c}$$

$$S_{41} = \frac{\left(1-\rho_e^2\right)e^{j\beta_e l}}{2\left(e^{2j\beta_e l} - \rho_e^2\right)} - \frac{\left(1-\rho_o^2\right)e^{j\beta_o l}}{2\left(e^{2j\beta_o l} - \rho_o^2\right)}, \tag{1d}$$

where

$$\rho_o = -\frac{1 - \frac{Z_{oo}}{Z_o}}{1 + \frac{Z_{oo}}{Z_o}}, \quad \rho_e = -\frac{1 - \frac{Z_{oe}}{Z_o}}{1 + \frac{Z_{oe}}{Z_o}}.$$

These expressions represent a general case where even- and odd-mode impedances Z_{oe} and Z_{oo} do not comply to the familiar $Z_{oe} \cdot Z_{oo} = Z_o^2$ relationship, and the odd- and even-mode velocities (as represented in the above equations by the propagation constants β_e and β_o) are unequal. It can thus be shown that if the usual idealizing assumptions, viz:

$$\beta_e l = \beta_o l = \pi/2,$$
$$Z_{oo} \cdot Z_{oe} = Z_o^2,$$
$$\rho_o^2 = \rho_e^2 = \rho^2,$$

then the above expressions reduce to the somewhat more digestible form

$$S_{11} = 0,$$
$$S_{21} = -j\frac{1-\rho^2}{1+\rho^2},$$
$$S_{31} = \frac{2\rho}{1+\rho^2},$$
$$S_{41} = 0. \quad (2)$$

My interest in the more general equations (1) was based on something of a chance observation that even when couplers are electrically very short, their directional properties are retained. In examining the expression for S_{41} (1d), it can be seen that this term vanishes for any value of the conduction angle, as long as the even- and odd-mode propagation velocities are the same ($\beta_e = \beta_o = \beta$) and the usual odd- and even-mode impedance relationship holds. So even though the expression for the coupling factor S_{31} will always show a strong dependency on electrical length—and, hence, frequency—the coupler will still show directivity, although the precision of the cancellation of the even- and odd-mode terms in (1d) will ultimately limit the useful directional behavior.

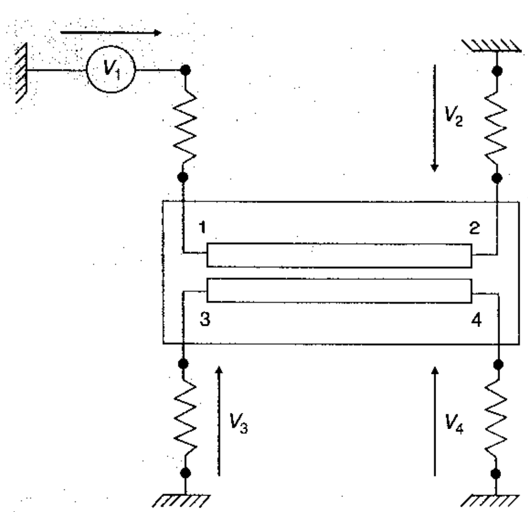

Figure 1. *Port designations for directional coupler analysis.*

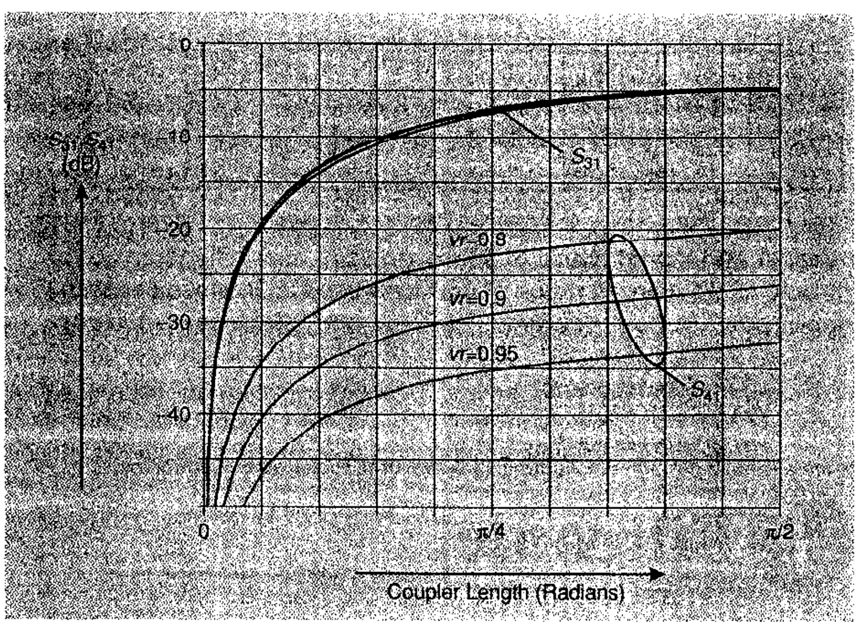

Figure 2. *Coupled and isolated port transmission versus coupler electrical length, for 3 values of velocity ratio* $vr(= \beta_{odd}/\beta_{even})$.

This can be explored more generally by plotting the magnitudes of S_{31} and S_{41} in (1c) and (1d) for various differential velocity factors, as shown in Figure 2. So, for example, if we take a structure that gives a coupling factor of −5 dB when the coupling length is the conventional quarter wavelength, this coupling factor rolls off steadily to a value of −20 dB when the coupling length is just 5°. The corresponding coupling into the isolated port is highly dependent on the degree of mismatch between the odd- and even-mode velocities, and for such a coupler to be useful, it will probably be necessary to employ some equalization. I realize I am probably reinventing a familiar wheel design for many readers; commercial reflectometers for lower VHF and HF applications are often based on such use of "short" lengths of coupled line. I am not so aware, however, of the complemetary use of this technique for RFICs in the microwave region.

Directivity, of course, is the key property of a directional coupler, and as the coupling factor is reduced, the directivity becomes limited by the cancellation of the two terms in (1d), and it becomes more important to come up with a way of equalizing the odd- and even-mode velocities. Two well-known tricks for improving this equalization are the use of a "wiggly gap" and/or the use of compensating capacitors across the coupled lines at the port terminations. The capacitors have the effect of reducing the odd-mode velocity, and it is interesting to note that this form of compensation is generally more effective as the coupling length is reduced, as shown in Figure 3. This can be explained by the fact that as the length decreases, the various reactive elements tend more towards lumped, rather than distributed, properties. Under some circumstances, a narrow band null in directivity can also be obtained near to the quarter wave frequency, as shown also in Figure 3.

It is interesting, and I have always thought somewhat baffling, to note that

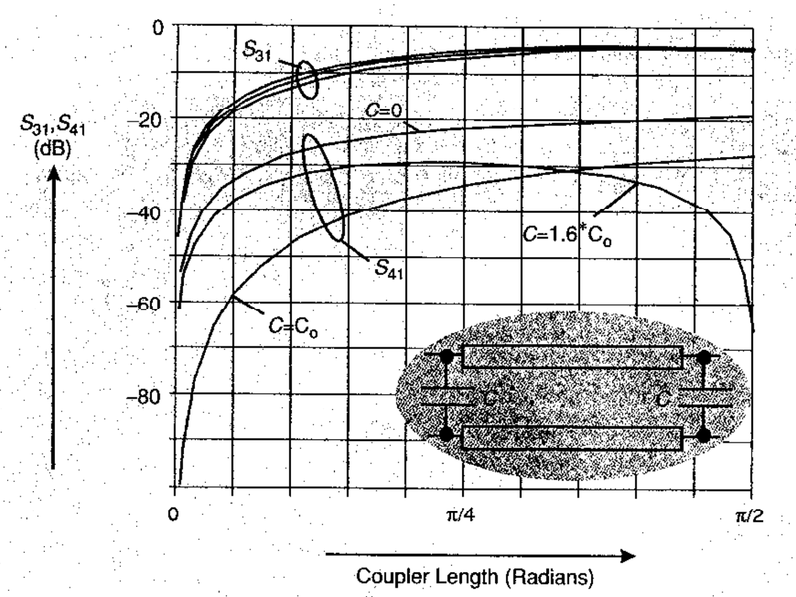

Figure 3. *Use of shunt capacitance to improve directivity of coupled lines.*

the quadrature property of coupled lines is also preserved for short coupling lengths. Taking the expressions for S_{21} and S_{31} (1) and making the usual idealizing assumptions for even and odd mode impedances and velocity factors, we obtain

$$S_{21} = \frac{(1-\rho^2)e^{j\theta}}{e^{2j\theta} - \rho^2},$$

$$S_{31} = -\rho_0 \frac{e^{2j\theta} - 1}{e^{2j\theta} - \rho^2}$$

so the ratio of the coupled outputs, after some further manipulation, is

$$\frac{S_{21}}{S_{31}} = j\frac{(1-\rho^2)}{\rho_0}\frac{1}{\sin\theta},$$

which, remarkably, indicates a quadrature relationship for any value of the conduction angle θ. This result is also preserved very closely when the phase velocity assumption is lifted; even allowing for the typical even and odd mode dispersion in a simple planar microstrip configuration, the phase discrepancy is less than one degree from the ideal quadrature value. I cannot resist the temptation here to quote from Tom Lee's excellent book *Planar Microwave Engineering* [2]. In referring to some of the same results from a general analysis of a directional coupler, he writes:

Sometimes thinking about a result raises more questions than it answers. Depending on your philosophical bent, you might decide that it's therefore best not to think. That might be the case here.

I heartily agree. The standard results, for sure, drop out of the equations, and the applications are well known and widespread. But they are somewhat counterintuitive. The quadrature split, in particular, seems inscrutable. Even in the above analysis, it seems to crawl out of the woodwork at the final line. This result has always defied my own intuition, especially when we are talking about the electrically short coupler. When dealing with a coupled section that is precisely 90° long, it is not unreasonable that there should be a 90° phase difference between two of the outputs. But how and why this is preserved over essentially an infinite bandwidth is a mystery that I have never been able to explain in simple intuitive language.

Some extra light can be shed on this problem by considering the symmetry and reciprocity of the structure. Referring back to Figure 1, if a voltage V is applied to port 1, then the voltage developed across matched loads at ports 3, 4, and 2 are

$$\frac{V}{\alpha}e^{j\phi},\ 0,\ V\left(1-\frac{1}{\alpha^2}\right)^{1/2} e^{j(\phi+\Delta)},$$

respectively, where the coupling factor α is a voltage ratio, the transmission phase is ϕ, and the phase difference between the direct and coupled ports is Δ. If we now apply another equal, cophased voltage V to the isolated port, symmetry and superposition can be applied, giving an equal voltage

$$\frac{V}{\alpha}e^{j\phi} + V\left(1-\frac{1}{\alpha^2}\right)^{1/2} e^{j(\phi+\Delta)}$$

at both ports 2 and 4. If we now apply conservation of power, equating the input powers at ports 1 and 3 to the outputs at ports 2 and 4,

$$2|V|^2 = 2\left|\frac{V}{\alpha}e^{j\phi} + V\left(1-\frac{1}{\alpha^2}\right)^{1/2} e^{j(\phi+\Delta)}\right|^2,$$

which can be expanded to obtain

$$1 = \left\{\frac{1}{\alpha}\cos\phi + \gamma\cos(\phi+\Delta)\right\}^2 + \left\{\frac{1}{\alpha}\sin\phi + \gamma\sin(\phi+\Delta)\right\}^2,$$

where

$$\gamma = \left(1-\frac{1}{\alpha^2}\right)^{1/2},$$

whence

$$\cos\phi\cos(\phi+\Delta) + \sin\phi\sin(\phi+\Delta) = 0,$$

which being the result of a well-known trig identity can be expressed as

$$\cos\{\phi - (\phi+\Delta)\} = 0,$$

so that

$$\cos\Delta = 0,$$

and $\Delta = \pi/2$. Wow!

This is an interesting result, which essentially tells us that the quadrature split is more a property of the symmetry and reciprocity of the structure than having anything much to do with the properties of coupled lines. Once again, the answer seems to drop out rather unexpectedly—just when things seemed to be going nowhere with only a single relationship between the two angles ϕ and Δ. I'm not quite sure, however, that it completely clears the air as far as the counterintuitive behavior of couplers is concerned, but I was once informed with some authority that this is the *real* answer to my intuitive problems.

This analysis leads me into another application, which is the use of a directional coupler as an asymmetrical power combiner. The use of 3-dB hybrids, both as power combiners and to make balanced amplifiers, is familiar enough. The asymmetrical application is less familiar but is encountered, for example, in the output of a feedforward linearization loop, where a correcting or "error" signal is added to the output of a power amplifier. Referring again to Figure 1, we now apply unequal voltages V_1 and $-jV_4$ at ports 1 and 4, the inputs thus having a quadrature relationship. So using superposition again, and now using the simplified equations (2), the terminated port voltages can be written as

$$V_3 = \alpha V_1 - \left(1 - \frac{1}{\alpha^2}\right)^{1/2} V_4,$$

$$V_2 = -j\alpha V_4 - j\left(1 - \frac{1}{\alpha^2}\right)^{1/2} V_1.$$

The interest now focuses on the voltage at the coupled port V_2. The two components are in antiphase, and if $V_1 > V_4$, this voltage will vanish for the particular case given by

$$\left(\frac{V_1}{V_4}\right)^2 = \left(\frac{1}{\alpha^2} - 1\right).$$

So, for example, if we have a 10-dB coupler, and the power ratio of the inputs at ports 1 and 4 is 9 (= 10 − 1), then the power at port 2 will be the sum of the two input powers, with no power "wasted" in port 2. So, if we put 9 W into port 1 and 1 W into port 4, with a correctly polarized quadrature phase difference, we will collect the full 10 W at port 2. This, of course, ignores any copper or dielectric losses in the coupler, but the key point is that the normal transmission to the direct port, which would be measured for a single input at port 1, 0.46 dB for a 10-dB coupler, is not apparent in the power combiner case nor, for that matter, is the 10-dB coupling factor between port 3 and port 4, which would appear in the case of a single input at port 3. Somewhat mysteriously, this will only apply for one specific power ratio represented by $(V_1/V_4)^2$ for a specified coupling factor.

This result is worthy of some further consideration, in particular with respect to feedforward amplifier analysis and efficiency enhancement techniques for power amplifiers. Therefore, I will continue with this theme in my next column.

References

[1] L.S. Napoli and J.J. Hughes, "Characteristics of coupled microstrip lines," *RCA Rev.*, pp. 479–498, Sep. 1970.

[2] T.H. Lee, *Planar Microwave Engineering*. Cambridge, UK: Cambridge Univ. Press, sec. 7.6.4.