

Good

200  
200

11/26/07

Name: Brian Rolfe TA: \_\_\_\_\_  
Random Student # \_\_\_\_\_  
Partner(s): \_\_\_\_\_

| Lab 4 FDTD                                                          |            | Grd        |
|---------------------------------------------------------------------|------------|------------|
| Copy from previous lab or compute R',L',G',C' values                | 10         | 10         |
| Compute alpha,beta, velocity of propagation, wavelength             | 10         | 10         |
| Draw time and space FDTD grid in your lab book and explain notation | 10         | 10         |
| Rewrite telegrapher equations as central difference equations       | 30         | 30         |
| Solve for the future                                                | 20         | 20         |
|                                                                     |            |            |
| Tests and Observations                                              |            |            |
| a) Run and plot simulations                                         | 50         | 50         |
| b) Measure and compare velocity of propagation                      | 20         | 20         |
| c) Summarize Results                                                | 20         | 20         |
| d) Include printout of code                                         | 30         | 30         |
| <b>Total</b>                                                        | <b>200</b> | <b>200</b> |
|                                                                     |            |            |
|                                                                     |            |            |
|                                                                     |            |            |

How much time was spent on this lab?

Comments:

## BRIAN Kolfe

LAB 4 - FDTD PLANE WAVE SIMULATION.  
 PRE-LAB  
 READ THE LAB.

PROCEDURE

## OBJECTIVES

- 1) UNDERSTAND THE TELEGRAPHERS EQUATIONS
- 2) UNDERSTAND HOW TO USE AND IMPLEMENT THE FDTD METHOD.
- 3) CORRECTLY SIMULATE WAVES ON YOUR TRANSMISSION LINES.

I. INTRO TO FDTD METHOD ( TELEGRAPHERS EQS.

$$-\frac{\partial V(z,t)}{\partial z} = R' I(z,t) + L' \frac{\partial I(z,t)}{\partial t} \quad (1)$$

$$-\frac{\partial I(z,t)}{\partial z} = G' V(z,t) + C' \frac{\partial V(z,t)}{\partial t} \quad (2)$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(c) = \frac{f(b) + f(a)}{2}$$

## II FINDING RLGC VALUES.

USING PROGRAM FROM LAB 2

CHECK FOR GIVEN VALUES.

$$f = 1 \text{ GHz}$$

AIR: SAME OR CLOSE

TEFLOW: SAME OR CLOSE

SEA WATER: SAME OR CLOSE.

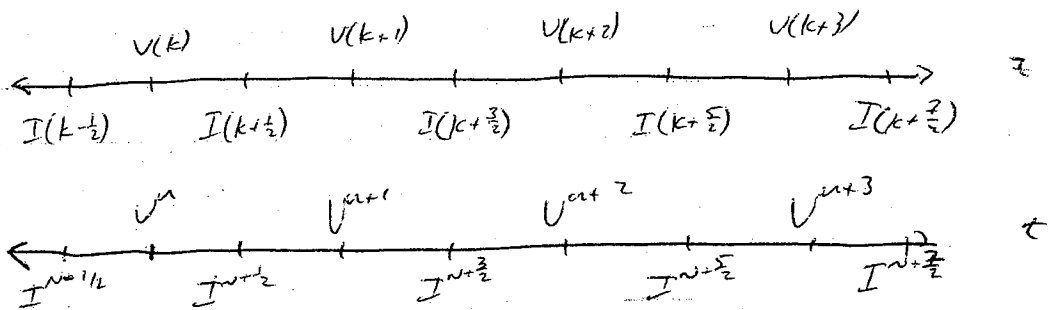
III DIFFERENTIALS VS. DIFFERENCES.

CENTRAL DIFFERENCE EQUATIONS:

$$f''(c) = \frac{f(b) + f(a)}{2}$$

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

IV DISCRETIZING SPACE AND TIME



$V(k)$  = VOLTAGE AT SPACE  $k$

$V(k+1)$  = VOLTAGE AT SPACE  $k+1$

$I(k+\frac{1}{2})$  CURRENT AT SPACE  $k+\frac{1}{2}$

$V^m$  = VOLTAGE AT PRESENT TIME

$V^{m+k}$  = VOLTAGE IN PAST OR FUTURE.

$I^{m+\frac{1}{2}}$  = CURRENT IN PAST OR FUTURE.

V CONVERT FROM DERIVATIVES TO DIFFERENCES.

REWRITE TELEGRAPHERS EQ'S WITH DIFFERENCE NOTATION.

$$\frac{\partial V(z,t)}{\partial z} = -\left(\frac{V_k^N - V_k^N}{dz}\right) = R' \left(\frac{I_{k+\frac{1}{2}}^{m+\frac{1}{2}} + I_{k+\frac{1}{2}}^{m+\frac{1}{2}}}{2}\right) + L' \left(\frac{I_{k+\frac{1}{2}}^{m+\frac{1}{2}} - I_{k+\frac{1}{2}}^{m+\frac{1}{2}}}{dt}\right)$$

$$\frac{\partial I(z,t)}{\partial z} = -\left(\frac{I_{k+\frac{1}{2}}^{m+\frac{1}{2}} - I_{k-\frac{1}{2}}^{m+\frac{1}{2}}}{dz}\right) = G' \left(\frac{V_k^N + V_k^{N+1}}{2}\right) + C' \left(\frac{V_k^{N+1} - V_k^N}{dt}\right)$$

III SOLVE FOR THE FUTURE.

SOLVE EQ. 1 FROM EARLIER FOR  $I_{k+1/2}^{N+1/2}$

$$-\left(\frac{V_{k+1}^N - V_k^N}{dz}\right) = R' \left(\frac{I_{k+1/2}^{N+1/2} + I_{k+1/2}^{N-1/2}}{2}\right) + C' \left(\frac{I_{k+1/2}^{N+1/2} - I_{k+1/2}^{N-1/2}}{dt}\right)$$

MULT THROUGH BY  $R'$  &  $C'$

$$\left(\frac{V_{k+1}^N - V_k^N}{dz}\right) = \frac{R' I_{k+1/2}^{N-1/2} + C' I_{k+1/2}^{N+1/2}}{2} + \frac{C' I_{k+1/2}^{N+1/2} - C' I_{k+1/2}^{N-1/2}}{dt}$$

GROUP LIKE TERMS FOR  $I$  & SIMPLIFY.

$$\left(\frac{V_{k+1}^N - V_k^N}{dz}\right) = I_{k+1/2}^{N-1/2} \left(\frac{R'}{2} - \frac{C'}{dt}\right) + I_{k+1/2}^{N+1/2} \left(\frac{R'}{2} + \frac{C'}{dt}\right)$$

SOLVE FOR  $I_{k+1/2}^{N+1/2}$

$$* I_{k+1/2}^{N+1/2} = \frac{\left(\frac{V_{k+1}^N - V_k^N}{dz}\right) - I_{k+1/2}^{N-1/2} \left(\frac{R'}{2} - \frac{C'}{dt}\right)}{\left(\frac{R'}{2} + \frac{C'}{dt}\right)} \quad (3)$$

SOLVE EQ. 2 FROM EARLIER FOR  $V_{k+1}^{N+1}$

$$\left(\frac{I_{k+1/2}^{N+1/2} - I_{k+1/2}^{N-1/2}}{dz}\right) = G' \left(\frac{V_{k+1}^N + V_k^N}{2}\right) + C' \left(\frac{V_{k+1}^{N+1} - V_k^N}{dt}\right)$$

MULT THROUGH BY  $G'$  &  $C'$

$$\left(\frac{I_{k+1/2}^{N+1/2} - I_{k+1/2}^{N-1/2}}{dz}\right) = \frac{G' V_{k+1}^N + G' V_k^N}{2} + \frac{C' V_{k+1}^{N+1} - C' V_k^N}{dt}$$

GROUP LIKE TERMS & SIMPLIFY.

$$\left(\frac{I_{k+1/2}^{N+1/2} - I_{k+1/2}^{N-1/2}}{dz}\right) = V_k^N \left(\frac{G'}{2} - \frac{C'}{dt}\right) + V_{k+1}^{N+1} \left(\frac{G'}{2} + \frac{C'}{dt}\right)$$

SOLVE FOR  $V_{k+1}^{N+1}$

$$* V_{k+1}^{N+1} = \frac{\left(\frac{I_{k+1/2}^{N+1/2} - I_{k+1/2}^{N-1/2}}{dz}\right) - V_k^N \left(\frac{G'}{2} - \frac{C'}{dt}\right)}{\left(\frac{G'}{2} + \frac{C'}{dt}\right)} \quad (4)$$

TO CLEAN-UP EQUATIONS.

$$A = \frac{-\frac{1}{dz}}{\frac{R'}{2} + \frac{L'}{dt}}$$

$$B = \frac{-\left(\frac{R'}{2} - \frac{L'}{dt}\right)}{\frac{R'}{2} + \frac{L'}{dt}}$$

$$C = \frac{-\frac{1}{dz}}{G'/2 + C'/dt}$$

$$E = \frac{-\left(\frac{G'}{2} - \frac{C'}{dt}\right)}{\left(\frac{G'}{2} + \frac{C'}{dt}\right)}$$

$$VII \quad I(k) = A [V(k+1) - V(k)] + B I(k)$$

$$V(k) = D [I(k) - I(k-1)] + E V(k)$$

PROGRAM THE DIFFERENCE FORM OF THE TELEGRAPHERS EQUATION.

$$VIII \quad f = 1 \text{ GHz}$$

$$dz = \pi / 20 \text{ (CALCULATED } \pi \text{ IN SECTION II)}$$

$$dt = 0.5 dz / \text{VELOCITY OF PROPAGATION (CALC. IN SECTION II)}$$

LINE IS 2000 dz LONG.

START WITH AIR FILLED LINE.

IX PROGRAM A SOURCE.

USE V SRC AT  $k=1$

$$V(1) = (V_0 \sin(\omega t))$$

X TESTS & OBSERVATIONS

(A) ATTACHED PLOTS.

| (B)                | ACTUAL               | CALC.                |
|--------------------|----------------------|----------------------|
| V <sub>0</sub> AIR | 2.99729 <sup>6</sup> | 2.99729 <sup>6</sup> |
| TELEON             | 2.06922 <sup>5</sup> | 2.06922 <sup>5</sup> |
| SEAWATER           | 3.10072 <sup>7</sup> | 3.10072 <sup>7</sup> |

THEY ARE EQUAL.

C) RESULTS. MAIN WRITING  
 THERE IS NO WRITING.

D) MATLAB ATTACHED

E) DISCUSSION & CONCLUSION

FDTD IS VERY EFFECTIVE IN SIMULATING  
 A WAVE PROPAGATING DOWN A T-LINE. AS  
 LONG AS YOU KNOW ALL OF YOUR SOURCES & SOURCES  
 OF ERROR AND ARE ABLE TO ACCOUNT FOR  
 THEM IN THE CODE. OTHERWISE IT IS JUST  
 A GOOD APPROXIMATION FOR WHAT IS GOING TO  
 HAPPEN BUT MAY HAVE LESS VOLTAGE A SLOWER  
 OR FASTER UP... ETC. POSSIBLE SOURCES OF ERROR  
 ARE ERRORS IN THE CODE, IMPROPER VALUES FOR  
 $\epsilon_r$ ,  $\mu_r$  ETC FOR MATERIALS, WAVELENGTH OF  
 LINE EXPECTED, CALCULATION ERRORS WHILE DERIVING  
 EQUATIONS, ~~AND OTHERS~~ AND OTHERS. THEY CAN  
 BE CONTROLLED BY DOUBLE CHECKING AND HAVING  
 SOMEONE ELSE LOOK AT THE CODE FOR  
 COMPLETENESS. THE ERRORS BETWEEN SIMULATED  
 & ACTUAL ARE INCONSISTANT MATERIAL, INTERFERENCE,  
 IMPROPER CALCULATIONS FOR SIMULATION, ~~DIFFERENCES IN~~  
 OTHER ITEMS OF ENVIRONMENT & MATERIAL NOT  
 ACCOUNTED FOR IN THE PROGRAM. THESE ARE  
 HARDER TO CONTROL BUT COULD POSSIBLY BE DONE  
 WITH THE ADDITION OF MORE VARIABLES. ALL IN  
 ALL THIS METHOD WORKS WELL & WILL EVEN  
 SHOW WHAT HAPPENS WITH EACH SUCCESSIVE  
 REFLECTION ON THE LINE. IT IS IMPORTANT TO  
 MAKE SURE ALL EQUATIONS ARE PROPERLY INPUT.  
 IF NOT YOU WILL GET UNDESIRABLE/UNEXPECTED  
 RESULTS LIKE ONE AND. IT IS IMPORTANT TO  
 NOTE THAT IN MOST CASES ACTUAL & CALCULATED DO  
 VARY BUT THE MORE INTEGRATE THE CODE THE LESS

OF A CHANCE OF ERROR. THE CODE  
MAY BE SIMPLIFIED BY LEARNING MORE  
ABOUT MATLAB AND USING SHORTCUTS TO  
REPRESENT THINGS I DID IN LENGTHY WAYS.  
ALSO IMPROVEMENTS CAN BE MADE BY TESTING  
AND IMPROVING & TESTING FOR DESIRED  
FUNCTIONALITY AGAIN UNTIL YOU HAVE  
SIMPLIFIED SEVERAL ITEMS INTO ONE STEP  
WITHOUT SACRIFICING FUNCTIONALITY.

```
clear all
clc
```

```
%This is how many cells are in the transmission line.
```

```
maxZ=200;
```

```
% Maximum number of time steps in 2*maxZ before wave hits the end of the grid...?Why???
```

```
maxT=200;
```

```
% Here is a version of the FDTD program.
```

```
% parameters
```

```
F= 1e9; % Frequency
```

```
w = 2*pi*F;
```

```
Mu_not = 4*pi*(10^-7); %H/m
```

```
Epsilon_not = 8.854*(10^-12);
```

```
% Parameters for copper conductor
```

```
Sigma_c= 5.8e7;
```

```
Mu_c= 0.999991*Mu_not;
```

```
a = 0.445*(10^-3); % outer radius of inner conductor,m, of RG58
```

```
b = 1.765*(10^-3); % inner radius of outer conductor,m, of RG58
```

```
% Comment out parameters not being used in current simulations
```

```
% Parameters for air
```

```
sigma= zeros(1,maxZ);
```

```
Epsilon_i= 1.0005*Epsilon_not*ones(1,maxZ);
```

```
Mu_i= Mu_c*ones(1,maxZ);
```

```
% Parameters for teflon
```

```
% sigma= 10e-15*ones(1,maxZ);
```

```
% Epsilon_i= 2.1*Epsilon_not*ones(1,maxZ);
```

```
% Mu_i= 1*Mu_not*ones(1,maxZ);
```

```
% Parameters for sea water
```

```
% sigma= 5*ones(1,maxZ);
```

```
% Epsilon_i= 72*Epsilon_not*ones(1,maxZ);
```

```
% Mu_i= 1*Mu_not*ones(1,maxZ);
```

```
% Constants computed in previous section
```

```
R_s = sqrt(pi*F.*Mu_c / Sigma_c);
```

```
R = (R_s / (2*pi))*((1/a) + (1/b))
```

```
L = (Mu_i ./ (2*pi)).*log(b/a)
```

```
G = (2*pi.*sigma) ./ (log(b/a))
```

```
C = (2*pi.*Epsilon_i) / (log(b/a))
```

```
Gamma =sqrt((R + j*w*L(1)) .* (G(1) + j*w*C(1)));
```

```
Alpha = real(Gamma)
```

```
Beta = imag(Gamma)
```

```
Lambda = (2*pi) / Beta
```

```
u_p = F * Lambda
```

```
Dz = Lambda/20; % from section II
```

```
Dt = (1/2*Dz)/u_p; % from section II
```

```
%l = 200*Dz; % Line is 200 times dz long
```



Page 53

```
% conversions to make equations simple
A = 1 ./ (-Dz.*(R./2 + L./Dt));
B = (-R./2 + L./Dt) ./ (R./2 + L./Dt);
D = 1 ./ (-Dz.*((G/2) + (C./Dt)));
E = (-G./2 + C./Dt) ./ (G/2 + C./Dt);

% %This is how many cells are in the transmission line.
% maxZ=200;
% % Maximum number of time steps in 2*maxZ before wave hits the end of the grid...?Why???
% maxT= 800;

% Initialize voltage and current values of transmission line.
V=zeros(1,maxZ);
I= zeros(1,maxZ);

%FDTD loops
for N=1:800;
    V(1) = sin(2*pi*F*N*Dt); % sine wave source
    for K=2:maxZ; % find voltage everywhere on the line
        V(K)= D(K)*(I(K)-I(K-1))+E(K)*V(K);
    end

    V(maxZ)=0;
    for K=1:maxZ-1; % find current everywhere on the line
        I(K)= A(K)*(V(K+1)-V(K))+B(K)*I(K);
    end
    plot(V) % plot the voltage all along the line at time N
    axis([0 maxZ -4 4]) % control the axis for uniform pictures
    pause(.001); % give the program time to plot to screen
end

title('Transmission Line with Air as the Insulation Material for n = 200')
xlabel('Time (sec)')
ylabel('Voltage (V)')
```

