<table>
<thead>
<tr>
<th>Task</th>
<th>Grd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copy from previous lab or compute R', L', G', C' values</td>
<td>10</td>
</tr>
<tr>
<td>Compute alpha, beta, velocity of propagation, wavelength</td>
<td>10</td>
</tr>
<tr>
<td>Draw time and space FDTD grid in your lab book and explain notation</td>
<td>10</td>
</tr>
<tr>
<td>Rewrite telegrapher equations as central difference equations</td>
<td>30</td>
</tr>
<tr>
<td>Solve for the future</td>
<td>20</td>
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<table>
<thead>
<tr>
<th>Tests and Observations</th>
</tr>
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<tbody>
<tr>
<td>a) Run and plot simulations</td>
</tr>
<tr>
<td>b) Measure and compare velocity of propagation</td>
</tr>
<tr>
<td>c) Summarize Results</td>
</tr>
<tr>
<td>d) Include printout of code</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

How much time was spent on this lab?

Comments:
LAB 4 - FDTD PLANE WAVE SIMULATION

Pre-Lab
READ THE LAB.

Procedure

Objectives
1) Understand the Telegraphers Equations
2) Understand how to use and implement the FDTD method.
3) Correctly simulate waves on your transmission lines.

I. Introduction to FDTD Method & Telegraphers Eqns.
\[
-\frac{\partial U(\mathbf{z}, t)}{\partial t} = \gamma^2 U(\mathbf{z}, t) + \gamma' \frac{\partial I(\mathbf{z}, t)}{\partial t}
\]
\[
-\frac{\partial I(\mathbf{z}, t)}{\partial t} = \gamma U(\mathbf{z}, t) + \gamma' \frac{\partial U(\mathbf{z}, t)}{\partial t}
\]
\[
\gamma(\mathbf{z}) = \frac{f(x) - f(x)}{b - a}
\]
\[
f(\mathbf{z}) = \frac{f(b) + f(a)}{2}
\]

II. Finding RLCG Values.
Using Program From Lab 2
Check For Given Values.
\( f = 16 \, \text{GHz} \)

Air: Same or Close

Teflon: Same or Close

Sea water: Same or Close.
DIFERENTIALS vs. DIFFERENCES.

Central difference equations:
\[ f'(c) = \frac{f(b + h) - f(b - h)}{2h} \]
\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

IV. DISCRETIZING SPACE AND TIME

\[ U'(k) \quad U'(k+1) \quad U'(k+2) \quad U'(k+3) \]
\[ I(\tau_{k1}) \quad I(\tau_{k1}) \quad I(\tau_{k1}) \quad I(\tau_{k1}) \quad I(\tau_{k1}) \]
\[ V^n \quad V^{n+1} \quad V^{n+2} \quad V^{n+3} \]

\[ V(k) = \text{VOLUME AT SPACE } k \]
\[ V(k+1) = \text{VOLUME AT SPACE } k-1 \]
\[ I(\tau_{k1}) = \text{CURRENT AT SPACE } k-1 \]
\[ V^n = \text{VOLTAGE AT PRESENT TIME} \]
\[ V^{n+1} = \text{VOLTAGE IN PAST OR FUTURE} \]
\[ I^{n+1} = \text{CURRENT IN PAST OR FUTURE} \]

V. CONVERT FROM DERIVATIVES TO DIFFERENCES.

Rewrite the equations with difference notation:
\[ \frac{\partial V(z,t)}{\partial t} = -\left(\frac{V^{n+1}_k - V^n_k}{\Delta t}\right) = R' \left(\frac{I^{n+1}_k + I^{n+1}_{k-1}}{2}\right) + L' \left(\frac{I^{n+1}_k - I^n_{k-1}}{\Delta t}\right) \]

\[ \frac{\partial I(z,t)}{\partial t} = -\left(\frac{I^{n+1}_k - I^n_{k-1}}{\Delta t}\right) = G' \left(\frac{V^n_k + V^{n+1}_k}{2}\right) + C' \left(\frac{V^{n+1}_k - V^n_k}{\Delta t}\right) \]
Solve for the future.

Solve Eq. 1 from earlier for $I_{k+\frac{1}{2}}^{n+\frac{1}{2}}$.

\[ \frac{d}{dz} (U_{k+1}^n - U_k^n) = R' \left( \frac{I_{k+\frac{1}{2}}^n + I_{k+1}^{n+\frac{1}{2}}}{2} \right) + C' \left( \frac{-I_{k+\frac{1}{2}}^n - I_{k+1}^{n+\frac{1}{2}}}{2} \right) \]

Multiply through by $R' \times C'$.

\[ \frac{d}{dz} (U_{k+1}^n - U_k^n) = R' \left( \frac{T_{k+1}^{n-\frac{1}{2}} + \frac{C}{2} \frac{d}{dt} T_{k+1}^{n+\frac{1}{2}}}{2} \right) + C' \left( \frac{-T_{k+\frac{1}{2}}^n - \frac{C}{2} \frac{d}{dt} T_{k+\frac{1}{2}}^{n+\frac{1}{2}}}{2} \right) \]

Group like terms for $T$ and simplify.

\[ \frac{d}{dz} (U_{k+1}^n - U_k^n) = T_{k+\frac{1}{2}}^{n-\frac{1}{2}} \frac{d}{dt} \left( \frac{R' - C'}{2} \right) + \frac{C}{2} \frac{d}{dt} \left( \frac{R' + C'}{2} \right) \]

Solve for $I_{k+\frac{1}{2}}^{n+\frac{1}{2}}$.

\[ I_{k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{-U_{k+1}^n + U_k^n}{2} - \frac{C}{2} \frac{d}{dt} \left( \frac{R' + C'}{2} \right) \]

Solve Eq. 2 from earlier for $V_{k+\frac{1}{2}}^{n+\frac{1}{2}}$.

\[ \frac{d}{dz} (V_{k+1}^n - V_k^n) = G' \left( \frac{V_{k+1}^n + V_k^n}{2} \right) + C' \left( \frac{V_{k+1}^n - V_k^n}{2} \right) \]

Multiply through by $G' \times C'$.

\[ \frac{d}{dz} (V_{k+1}^n - V_k^n) = G' \left( \frac{V_{k+1}^n - V_k^n}{2} + \frac{C}{2} \frac{d}{dt} V_{k+1}^n \right) + C' \left( \frac{V_{k+1}^n - V_k^n}{2} - \frac{C}{2} \frac{d}{dt} V_k^n \right) \]

\[ V_{k+1}^n - V_k^n = \left( \frac{d}{dz} \right) \left( \frac{G' - C}{2} \right) - V_k^n \left( \frac{G' + C}{2} \right) \]

\[ = \left( \frac{G' + C}{2} \right) \frac{d}{dt} \left( \frac{G' - C}{2} \right) \]
To clean up equations,

\[ A = \frac{-\frac{d^2}{dt^2}}{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}} \]

\[ B = -\frac{\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial x}}{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}} \]

\[ C = -\frac{\frac{d}{dt}}{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}} \]

\[ D = -\frac{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}}{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}} \]

\[ E = \frac{-\frac{\partial}{\partial x}}{\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x}} \]

\[ I(k) = A \left[ U(k+1) - U_k \right] + B I(k) \]

\[ V(k) = D \left[ I(k) - I(k-1) \right] + E V(k) \]

Program the difference form of the telegrapher's equation.

\[ f = \frac{1}{6} \text{ cm}^2 \]

\[ C = \frac{\pi}{20} \text{ (calculated } \pi \text{ in Section II)} \]

\[ \frac{d}{dt} = 0.5 \text{ cm}^2 \text{ velocity of propagation (calculated in Section II)} \]

Line is 20 cm long.

Start with air filled line.

Program a source.

Use v = 0.5 at k = 1

\[ U(1) = (0.05 \text{ cm}) \]

Test observations

<table>
<thead>
<tr>
<th>Attacked Plots</th>
<th>Actual</th>
<th>Calc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_p Air</td>
<td>7.9972 e^8</td>
<td>7.9972 e^8</td>
</tr>
<tr>
<td>Metal</td>
<td>7.0684 e^6</td>
<td>2.0684 e^6</td>
</tr>
<tr>
<td>Seawater</td>
<td>3.1047 e^2</td>
<td>3.1047 e^2</td>
</tr>
</tbody>
</table>

They are equal.
(c) RESULTS: As in cur Tiếp
There is no current.

(d) MATLAB ATTACHED

(e) DISCUSSION & CONCLUSION

FORD is very effective in simulating a wave propagation down a 7-line. As long as you know all of your sources & sources of error and are able to account for them in the code, otherwise it is just a good approximation for what is going to happen but may have less voltage, a slower or faster up... etc. Possible sources of error are errors in the code, incorrect values for E_r, E_m, etc for materials, wrong size of line expected, calculation errors while solving equations, and others. They can be controlled by double checking and having someone else look at the code for correctness. The errors between simulated & actual are in consistent material, interfaces, improper calculations for simulation, differences in other items of environment & material not accounted for in the program. These are harder to control but could possibly be done with the addition of more variables. All in all, this method works well if will even show what happens with each successive reflection on the line. It is important to make sure all equations are properly used. If not, you will get undesired unexpected results like we did. It is important to note that in most cases actual & calculated do vary but the more intricate the code the less.
The code may be simplified by learning more about MATLAB and using shortcuts to represent things I did in longer ways. Also, improvements can be made by testing and improving it testing for desired functionality again until you have simplified several items into one step without sacrificing functionality.
clear all
clic

% This is how many cells are in the transmission line.
maxZ=200;
% Maximum number of time steps in 2*maxZ before wave hits the end of the grid...?Why??
maxT=200;

% Here is a version of the FDTD program.
% parameters
F = 1e9; % Frequency
w = 2*pi*F;
Mu_not = 4*pi*(10^-7); % H/m
Epsilon_not = 8.854*(10^-12);

% Parameters for copper conductor
Sigma_c = 5.8e7;
Mu_c = 0.999991*Mu_not;
a = 0.445*(10^-3); % outer radius of inner conductor,m, of RG58
b = 1.765*(10^-3); % inner radius of outer conductor,m, of RG58

% Comment out parameters not being used in current simulations
% Parameters for air
sigma = zeros(1,maxZ);
Epsilon_i = 1.0005*Epsilon_not*ones(1,maxZ);
Mu_i = Mu_c*ones(1,maxZ);

% Parameters for teflon
% sigma = 10e-15*ones(1,maxZ);
% Epsilon_i = 2.1*Epsilon_not*ones(1,maxZ);
% Mu_i = 1*Mu_not*ones(1,maxZ);

% Parameters for sea water
% sigma = 5*ones(1,maxZ);
% Epsilon_i = 72*Epsilon_not*ones(1,maxZ);
% Mu_i = 1*Mu_not*ones(1,maxZ);

% Constants computed in previous section
R_s = sqrt(pi.*F.*Mu_c / Sigma_c);
R = (R_s / (2*pi)).*((1/a) + (1/b))
L = (Mu_i ./ (2*pi)).*log(b/a)
G = (2*pi.*sigma ./ (log(b/a)))
C = (2*pi.*Epsilon_i) ./ (log(b/a))
Gamma = sqrt((R + j*w*L(1)).*(G(1) + j*w*C(1)))
Alpha = real(Gamma)
Beta = imag(Gamma)
Lambda = (2*pi) / Beta
u_p = F * Lambda

Dz = Lambda/20; % from section II
Dt = (1/2*Dz)/u_p; % from section II
l = 200*Dz; % Line is 200 times dz long
% conversions to make equations simple
A = 1 ./ (-Dz.*(R./2 + L./Dt));
B = (-R./2 + L./Dt) ./ (R/2 + L./Dt);
D = 1 ./ (-Dz.*((G/2) + (C./Dt)));
E = (-G./2 + C./Dt) ./ (G/2 + C./Dt);

% This is how many cells are in the transmission line.
% maxZ=200;
% Maximum number of time steps in 2*maxZ before wave hits the end of the grid...?Why???
% maxT= 800;

% Initialize voltage and current values of transmission line.
V = zeros(1,maxZ);
I = zeros(1,maxZ);

% FDTD loops
for N=1:800;
    V(1) = sin(2*pi*F*N*Dt); % sine wave source
    for K=2:maxZ; % find voltage everywhere on the line
        V(K) = D(K) *(I(K)-I(K-1)) + E(K) * V(K);
    end
    V(maxZ)=0;
    for K=1:maxZ-1; % find current everywhere on the line
        I(K) = A(K) *(V(K+1)-V(K)) + B(K) * I(K);
    end
    plot(V) % plot the voltage all along the line at time N
    axis([0 maxZ -4 4]) % control the axis for uniform pictures
    pause(.001); % give the program time to plot to screen
end

title('Transmission Line with Air as the Insulation Material for n = 200')
xlabel('Time (sec)')
ylabel('Voltage (V)')