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FREQUENCY-DEPENDENT FINITE-DIFFERENCE TIME-DOMAIN)TD) METHOD FOR INDUCED CURRENT AND SAR CALCULATIONS R A HETEROGENEOUS MODEL OF THE HUMAN BODY

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This paper describes a new frequency-dependent FDTD [(FD)^{2}TD] method for ultrawide-band applications such as irradiation by extremely short pulses, where the effects of dispersive tissue properties must be considered. Based on a differential-equation approach for the time-domain representation of **D**(t), this method may be used for general dispersive media where $\varepsilon^*(\omega)$ and $\mu^*(\omega)$ can be described by rational functions in the frequency domain, or for human tissues where $\varepsilon^*(\omega)$ is described by a Debye equation with several relaxation constants. Based on a single run with a broadband pulse excitation, the (FD)^{2}TD method is used to calculate SARs and induced currents in a heterogeneous anatomically based man model over an ultra-wide band.

A weakness of the conventional FDTD algorithm is that the dispersion of the tissue's dielectric properties is ignored and frequency-independent properties are assumed. Although this approach may be permissible for CW or narrow-band irradiation, it yields substantial errors for short pulses with ultra-wide bandwidths. We have developed a frequency-dependent finite-difference time-domain [(FD)^{2}TD] method by adding a differential equation relating to **D** and **E** to the traditional FDTD algorithm. Previously used for homogeneous models,[1] the (FD)^{2}TD approach is now extended to the heterogeneous anatomically based model of the human body, where the tissue properties $\varepsilon^*(\omega)$ are described by second-order Debye equations. Using a single run with a broadband pulse excitation, we obtain layer-averaged SARs and induced currents at various frequencies by taking the Fourier components of the induced **E**(t) fields.

THE DIFFERENTIAL-EQUATION-BASED (FD)^{2}TD METHOD

The procedure is similar to that used for the conventional FDTD method,[2] except that an additional differential equation relating **D**(t) and **E**(t) is used. This equation is developed from the assumption that the dispersion of the various tissues can be fitted to a Debye equation with two relaxation constants, as shown in Fig. 1 for muscle and fat:

$$\mathbf{D}(\omega) = \varepsilon^*(\omega)\,\mathbf{E}(\omega) = \varepsilon_0\left[\varepsilon_\infty + \frac{\varepsilon_{s1}-\varepsilon_\infty}{1+j\omega\tau_1} + \frac{\varepsilon_{s2}-\varepsilon_\infty}{1+j\omega\tau_2}\right]\mathbf{E}(\omega) \tag{1}$$

which may be converted to a differential equation in the time domain by assuming $e^{j\omega t}$ time dependence:

$$\tau_1\tau_2\frac{\partial^2\mathbf{D}}{\partial t^2} + (\tau_1+\tau_2)\frac{\partial\mathbf{D}}{\partial t} + \mathbf{D} = \varepsilon_0\left[(\varepsilon_{s1}+\varepsilon_{s2}-\varepsilon_\infty)\mathbf{E} + (\varepsilon_{s1}\tau_1+\varepsilon_{s2}\tau_2)\frac{\partial\mathbf{E}}{\partial t} + \varepsilon_\infty\tau_1\tau_2\frac{\partial^2\mathbf{E}}{\partial t^2}\right] \tag{2}$$

For the (FD)^{2}TD method, we solve Ampere's law to find **D** and **E**, and Faraday's law to find **H** at each cell location. This **D** → **E** → **H** loop is then repeated for each time step.

APPLICATION OF THE (FD)^{2}TD METHOD TO THE ANATOMICALLY BASED HETEROGENEOUS MAN MODEL

The (FD)^{2}TD method is applied to an anatomically based heterogeneous man model previously used in Ref. 2. This model is made up of $24\times45\times135$ 1.31cm (approximately 0.5in.) cells imbedded in a total region of $42\times63\times153$ cells; 7.67 MWords of memory and approximately 165 cpuminutes are needed to run

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this test case on an IBM-3090. By comparison, the traditional FDTD takes 6.37 MW words and 155 cpuminutes, but must be rerun for each frequency. Tissue properties in each cell are obtained by volume-averaging properties for all tissues in each cell. The incident plane wave is a raised cosine pulse with 3 GHz maximum frequency and E_z polarized (parallel to the long axis of the body), and is frontally incident on the body.

Layer-averaged current and SAR distributions computed by both FDTD and $(FD)^2TD$ methods are shown in Figs. 2 and 3, respectively, for frequencies from 20 to 915 MHz. These values differ slightly because of the slight discrepancies between measured tissue properties and their fit to the Debye equation as shown in Fig. 1.

This test case illustrates the power of the $(FD)^2TD$ method. The new method provides accurate broad-band and time-domain data for a highly dispersive model from a single run, whereas the traditional FDTD method would require a run at every frequency of interest and could not provide accurate time-domain data for this dispersive model.

CONCLUSION

This paper describes a new frequency-dependent FDTD $[(FD)^2TD]$ method for general dispersive media for which permittivity and/or permeability can be described by rational functions in the frequency domain. This approach was applied to a *heterogeneous* anatomically based model of the human body where the tissue properties $\varepsilon^*(\omega)$ are described by multiterm Debye equations. A single run with a broadband pulse excitation provides SARs and induced currents in either the time or frequency domain for this highly dispersive model.[4]

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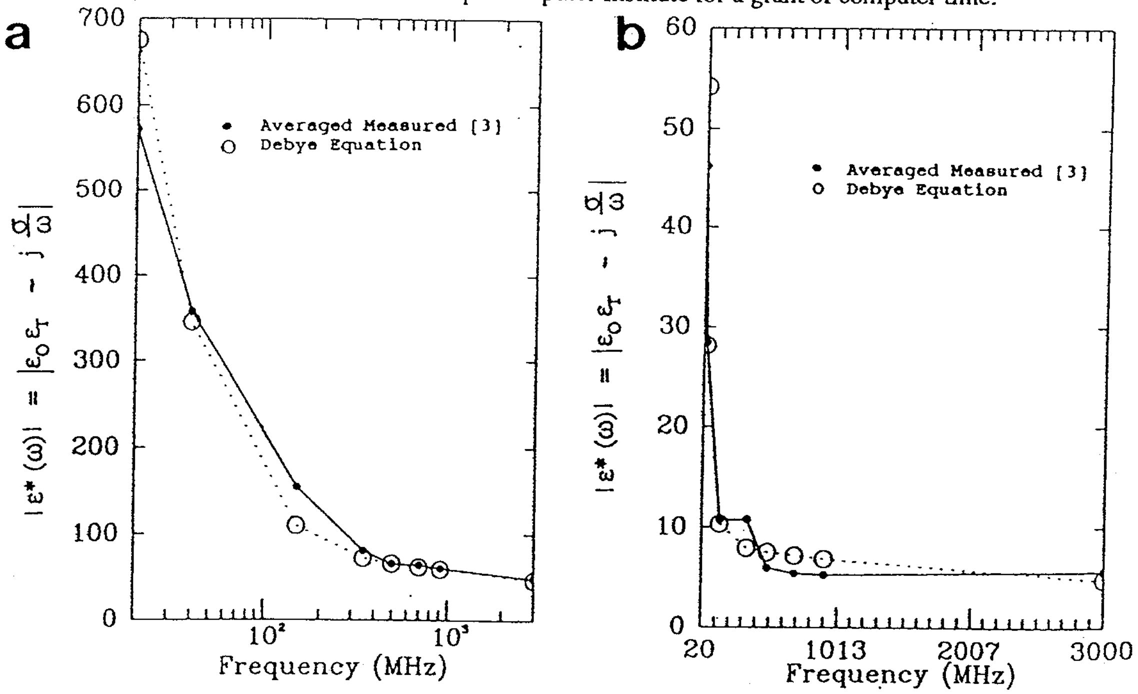

FIG. 1.—Fit of Debye equation with two relaxation constants to measured tissue properties of (a) muscle, (b) fat.

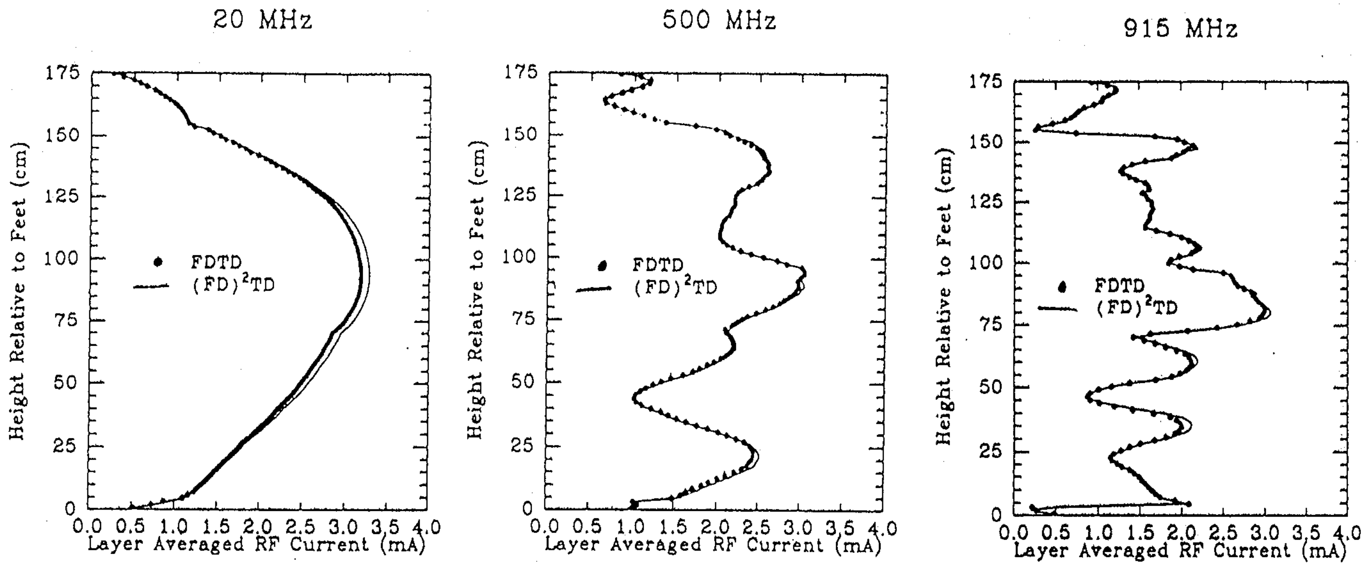

FIG. 2.—Layer-averaged RF current.

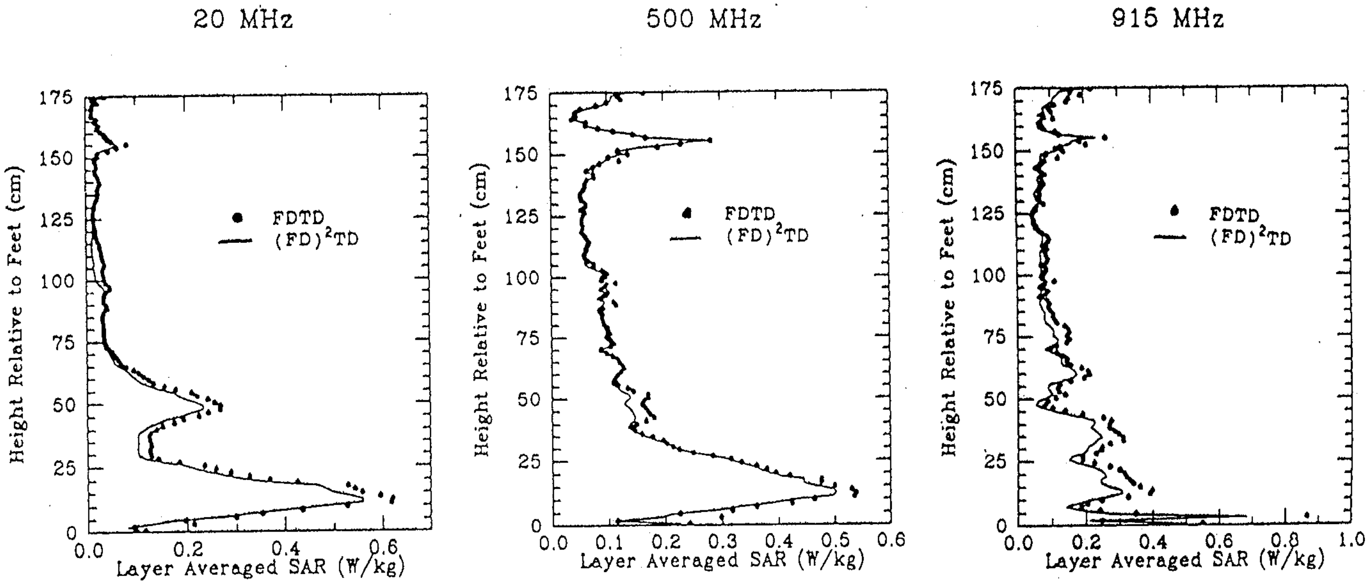

FIG. 3.—Layer-averaged SAR.