Speed Sensorless Identification of the Rotor Time Constant in Induction Machines

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Abstract

A method is proposed to estimate the rotor time constant $T_R$ of an induction motor without measurements of the rotor speed/position. The method consists of solving for the roots of a polynomial equation in $T_R$ whose coefficients depend only on the stator currents, stator voltages, and their derivatives. Experimental results are presented.

1 Introduction

Induction motors are very attractive in many applications owing to their simple structure, low cost, and robust construction. Field-oriented control is now used to obtain high performance drive of the induction motor because it gives control characteristics similar to separately excited DC motors. Implementation of a (rotor-flux) field-oriented controller requires knowledge of the rotor speed and the rotor time constant $T_R$ to estimate the rotor flux linkages. There has been considerable work done in the last several years to implement a field-oriented controller without the use of a speed sensor [1][2][3][4][5][6]. However, many of these methods still require the value of $T_R$, which can change with time due to ohmic heating. That is, to be able to update the value of $T_R$ to the controller as it changes is valuable. The work presented here uses an algebraic approach to identify the rotor time constant $T_R$ without the motor speed information. It is is most closely related to the ideas described in [7][8][9][10][11][12]. Specifically, it is shown that $T_R$ satisfies a polynomial equation whose coefficients are functions of the stator currents, the stator voltages, and their derivatives. A zero of this polynomial is the value of $T_R$. It is further shown that $T_R$ is not identifiable under steady-state operation because it is not sufficiently excited.

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The paper is organized as follows. Section 2 introduces a space vector model of the induction motor. Section 3 uses this model to develop an algebraic equation that $T_R$ must satisfy. Section 4 shows that in steady state, $T_R$ is not identifiable by either the proposed algebraic method nor a standard linear least-squares method. Section 5 presents the experimental results, while Section 6 gives the conclusions and future work.

## 2 Mathematical Model of Induction Motor

The starting point of the analysis is a space vector model of the induction motor given by (see, e.g., pp. 568 of [13])

\[
\frac{d}{dt}i_S = \frac{\beta}{T_R} (1 - jn_P \omega T_R) \psi_R - \gamma i_S + \frac{1}{\sigma L_S} u_S \tag{1}
\]

\[
\frac{d}{dt} \psi_R = -\frac{1}{T_R} (1 - jn_P \omega T_R) \psi_R + M T_R i_S \tag{2}
\]

\[
\frac{d\omega}{dt} = \frac{n_p M}{J L_R} \text{Im} \{ i_S \psi_R^* \} - \frac{\tau_L}{J} \tag{3}
\]

where $i_S \triangleq i_{Sa} + j i_{Sb}$, $\psi_R \triangleq \psi_{Ra} + j \psi_{Rb}$, and $u_S \triangleq u_{Sa} + j u_{Sb}$. Here, $\theta$ is the position of the rotor, $\omega = d\theta/dt$ is the rotor speed, $n_p$ is the number of pole pairs, $i_{Sa}, i_{Sb}$ are the (two-phase equivalent) stator currents, $\psi_{Ra}, \psi_{Rb}$ are the (two-phase equivalent) rotor flux linkages, $R_S, R_R$ are the stator and rotor resistances, respectively, $M$ is the mutual inductance, $L_S$ and $L_R$ are the stator and rotor inductances, respectively, $J$ is the moment of inertia of the rotor, and $\tau_L$ is the load torque. The symbols $T_R = \frac{L_R}{R_R}, \sigma = 1 - \frac{M^2}{L_S L_R}, \beta = \frac{M}{\sigma L_S L_R}, \gamma = \frac{R_S}{\sigma L_S} + \frac{\beta M}{T_R}$ have been used to simplify the expressions. $T_R$ is referred to as the rotor time constant, while $\sigma$ is called the total leakage factor.

## 3 Algebraic Approach to $T_R$ Estimation

The idea of the approach is to solve (1) and (2) for $T_R$. However, equations (1) and (2) are only four equations while there are six unknowns, namely $\psi_{Ra}, \psi_{Rb}, d\psi_{Ra}/dt, d\psi_{Rb}/dt, \omega$, and $T_R$. Equation (3) is not used because it introduces the additional unknown $\tau_L$. To find two more independent equations, equation (1) is differentiated to obtain

\[
\frac{d^2}{dt^2} i_S = \frac{\beta}{T_R} (1 - jn_P \omega T_R) \frac{d}{dt} \psi_R - jn_P \beta \psi_R \frac{d\omega}{dt} - \gamma \frac{d}{dt} i_S + \frac{1}{\sigma L_S} \frac{d}{dt} u_S. \tag{4}
\]

Using the (complex-valued) equations (1) and (2), one can solve for $\psi_R$ and $\frac{d}{dt} \psi_R$ in terms of $\omega, i_S$, and $u_S$ and substitute the resulting expressions into (4) to obtain
\[
\frac{d^2 i_s}{dt^2} = - \frac{1}{T_R} (1 - jn_p \omega T_R) \left( \frac{d}{dt} i_s + \frac{\gamma i_s - 1}{\sigma L_s} u_s \right) + \frac{\beta M}{T_R^2} (1 - jn_p \omega T_R) i_s - \frac{\gamma}{\sigma L_s} \frac{d}{dt} i_s + \frac{1}{\sigma L_s} \frac{d}{dt} u_s
\]

Solving (5) for \( \frac{d \omega}{dt} \) gives

\[
\frac{d \omega}{dt} = - \frac{(1 - jn_p \omega T_R)^2}{jn_p T_R^2} + \frac{\beta M}{jn_p T_R^2} \frac{(1 - jn_p \omega T_R) i_s - \gamma}{\sigma L_s} \frac{d}{dt} i_s + \frac{1}{\sigma L_s} \frac{d}{dt} u_s - \frac{d^2 i_s}{dt^2}
\]

The left-hand side of (6) is real, so the right-hand side must also be real. Note by (1) that \( \frac{d i_s}{dt} + \gamma i_s - u_s / (\sigma L_s) = \frac{\beta}{T_R} (1 - jn_p \omega T_R) \psi \), so that the right-hand side of (6) is singular if and only if \( \left| \frac{i_s}{i_s} \right| = 0 \). Other than at startup, \( \left| \frac{i_s}{i_s} \right| \neq 0 \) in normal operation of the motor. Separating the right-hand side of (6) into its real and imaginary parts, the real part has the form

\[
\frac{d \omega}{dt} = a_2 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \omega^2 + a_1 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \omega + a_0 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})
\]

The expressions for \( a_2 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \), \( a_1 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \), and \( a_0 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \) are lengthy in terms of \( u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb} \), and their derivatives as well as of the machine parameters including \( T_R \). As a consequence, they are not explicitly presented here. Appendix 7.2 gives their steady-state expressions.

On the other hand, the imaginary part of the right-hand side of (6) must be zero. In fact, the imaginary part of (6) is a second degree polynomial equation in \( \omega \) of the form

\[
q(\omega) = q_2 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \omega^2 + q_1 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \omega + q_0 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})
\]

and, if \( \omega \) is the speed of the motor, then \( q(\omega) = 0 \). The \( q_i \) are functions of \( u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb} \), and their derivatives as well as of the machine parameters including \( T_R \). The expressions for \( q_2 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \), \( q_1 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \), and \( q_0 (u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \) are also lengthy and not explicitly presented here. (Their steady-state expressions are given in Appendix 7.1.) If the speed was measured, then (8) would be equal to zero and could then be solved for \( T_R \). However, in the problem being considered, \( \omega \) is not known. To eliminate \( \omega \), \( q(\omega) \) in (8) is differentiated to obtain

\[
\frac{d}{dt} q(\omega) = (2q_2 + q_1) \frac{d \omega}{dt} + q_2 \omega^2 + q_1 \omega + q_0
\]

where \( dq(\omega)/dt \equiv 0 \) if \( \omega \) is equal to the motor speed. Next, \( d\omega/dt \) in (9) is replaced by the right-hand
side of (7) so that (9) may be written as

\[ \frac{dq(\omega)}{dt} = g(\omega) = 2q_2a_2\omega^3 + (2q_2a_1 + q_1a_2 + \dot{q}_2)\omega^2 + \left(2q_2a_0 + q_1a_1 + \dot{q}_1\right)\omega + q_1a_0 + \dot{q}_0. \]  

(10)

g(\omega) is a third-order polynomial equation in \( \omega \) (with time-varying coefficients) for which the speed of the motor is one of its roots. Dividing\(^1\) \( g(\omega) \) in (10) by \( q(\omega) \) in (8), \( g(\omega) \) may be rewritten as (\( q_2 \neq 0 \) if \( \omega \) and the stator electrical frequency \( \omega_S \) are nonzero. See [6][14])

\[ g(\omega) = \frac{1}{q_2}\left((2q_2a_2\omega + 2q_2a_1 - q_1a_2 + \dot{q}_2)q(\omega) + r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\right) \]

(11)

\[ r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \equiv 2q_2^2a_0 - q_2q_1a_1 + q_2\dot{q}_1 - 2q_2q_0a_2 + q_1^2a_2 - q_1\dot{q}_2 \]  

(12)

\[ r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \equiv q_2q_1a_0 + q_2\dot{q}_0 - 2q_2q_0a_1 + q_0q_1a_2 - q_0\dot{q}_2. \]  

(13)

If \( \omega \) is equal to the speed of the motor, then both \( g(\omega) = 0 \) and \( q(\omega) = 0 \), and one obtains

\[ r(\omega) \equiv r_1(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb})\omega + r_0(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) = 0. \]  

(14)

This is now a first-order polynomial equation in \( \omega \) which uniquely determines the motor speed \( \omega \) as long as \( r_1 \) (the coefficient of \( \omega \)) is nonzero. (It is shown in Appendix 7.3 that \( r_1 \neq 0 \) in steady state if \( q_2 \neq 0 \).) Solving for the motor speed \( \omega \) using (14), one obtains

\[ \omega = -r_0/r_1. \]  

(15)

Next, replace \( \omega \) in (8) by the expression in (15) to obtain

\[ q_2r_0^2 - q_1r_0r_1 + g_0r_1^2 \equiv 0. \]  

(16)

The expressions for \( q_i, r_i \) are in terms of motor parameters (including \( T_R \)) as well as the stator currents, voltages, and their derivatives. Expanding the expressions for \( q_0, q_1, q_2, r_0, \) and \( r_1 \), one obtains a twelfth-order polynomial equation in \( T_R \), which can be written as

\[ \sum_{i=0}^{12} C_i(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) T_R^i = 0. \]  

(17)

Solving equation (17) gives \( T_R \). The coefficients \( C_i(u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}) \) of (17) contain third-order derivatives of the stator currents and second-order derivatives of the stator voltages making noise

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\(^1\)Given the polynomials \( g(\omega), q(\omega) \) in \( \omega \) with \( \deg\{g(\omega)\} = n_q, \deg\{q(\omega)\} = n_q \), the Euclidean division algorithm ensures that there are polynomials \( \gamma(\omega), r(\omega) \) such that \( g(\omega) = \gamma(\omega)q(\omega) + r(\omega) \) and \( \deg\{r(\omega)\} \leq \deg\{q(\omega)\} - 1 = n_q - 1 \). Consequently if, for example, \( \omega_0 \) is a zero of both \( g(\omega) \) and \( q(\omega) \), then it must also be a zero of \( r(\omega) \).
a concern. For short time intervals in which $T_R$ does not vary, (17) must hold identically with $T_R$ constant. In order to average out the effect of noise on the $C_i$, (17) is integrated over a time interval $[t_1, t_2]$ to obtain

$$
\sum_{i=0}^{12} \left( \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} C_i \left( u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb} \right) dt \right) T_R^i = 0.
$$

The measured variables appear into the coefficients of (17) in a nonlinear manner, so that it would be difficult to quantify exactly how much noise is filtered out. However, assuming a sufficient frequency separation between the noise and the signal, one would expect that such filtering would help and the experimental results presented below bare this out.

There are 12 solutions satisfying (18). However, simulation results have always given 10 conjugate solutions. The remaining two solutions include the correct value of $T_R$ while the other one was either negative or close to zero. The method is to compute the coefficients $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} C_i dt$ and then compute the roots of (18). Among the positive real roots is the correct value of $T_R$. Experimental results using this method are presented in Section 5.

Remark The expression (14) was used by the authors in [6] (assuming $T_R$ is known) as a technique to estimate the speed of an induction motor for speed sensorless field-oriented control.

4 Identifiability of $T_R$ in Steady State

The goal of this section is to show that $T_R$ is not identifiable with the machine in steady-state because it is not sufficiently excited. We show this explicitly for the method proposed here and then show it explicitly for a linear least-squares formulation. The terminology “steady state” means the machine is running at constant speed and the voltages/currents are in steady state.

4.1 Algebraic approach

The polynomial (18) is now considered with the machine in steady-state so that, in particular, the speed is constant. That is, $u_{Sa} + ju_{Sb} = U_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = I_S e^{j\omega_S t}$ are substituted into (8) and (14) where $\omega_S$ is the electrical frequency. In steady state, the motor speed in (15) becomes (see Appendix 7.3 and [14])

$$
\omega = -\frac{r_0}{r_1} = \frac{\omega_S (1 - S)}{n_p} \tag{19}
$$

where $S \triangleq (\omega_S - n_p \omega)/\omega_S$ is the normalized slip. Substituting the steady-state expressions for $q_2$, $q_1$, and $q_0$ from Appendix 7.1 as well as the expression (19) for $\omega$ into (8), one obtains $q_2 \omega^2 + q_1 \omega + q_0 = 0$,

$$
\frac{n^2 p^2 T_R^2 |L_S|^4 \omega_S^2 L_S (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_S^2 T_R^2)} \left( \frac{\omega_S (1 - S)}{n_p} \right)^2 + \frac{n_p \omega_S |L_S|^4 L_S (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_S^2 T_R^2)} = 0.
$$
That is, in steady state (8) and (14) hold independent of the value of $T_R$ and thus so does (17) making $T_R$ unidentifiable in steady state by this method.

### 4.2 Linear least-squares approach

Vélez-Reyes et al [3][4] have used least-squares methods for simultaneous parameter and speed identification in induction machines. In the approach used herein, $d\omega/dt$ is taken to be zero so that a linear (in the parameters) regressor model can be obtained. Specifically, consider the mathematical model of the induction motor in (5). With $d\omega/dt = 0$ this equation reduces to

$$\frac{d^2 i_s}{dt^2} = -\frac{1}{T_R} (1 - jn_p\omega T_R) \left( \frac{di_s}{dt} + \frac{1}{\sigma L_s} u_s - \frac{1}{\sigma L_s} \frac{d^2 i_s}{dt^2} \right) + \frac{\beta M}{T_R} (1 - jn_p\omega T_R) i_s - \frac{\gamma d}{dt} i_s + \frac{1}{\sigma L_s} \frac{d u_s}{dt}$$

(20)

where $i_s = i_{sa} + j i_{sb}$ and $u_s = u_{sa} + j u_{sb}$. Decomposing equation (20) into its real and imaginary parts gives

$$\frac{d^2 i_{sa}}{dt^2} = \frac{1}{T_R} \left( -\frac{di_{sa}}{dt} - \frac{R_s}{\sigma L_s} i_{sa} + \frac{1}{\sigma L_s} u_{sa} \right) + n_p\omega \left( -\frac{di_{sb}}{dt} - \frac{R_s}{\sigma L_s} i_{sb} + \frac{1}{\sigma L_s} u_{sa} \right) - \frac{\gamma d}{dt} i_{sa} + \frac{1}{\sigma L_s} \frac{d u_{sa}}{dt}$$

(21)

and

$$\frac{d^2 i_{sb}}{dt^2} = \frac{1}{T_R} \left( -\frac{di_{sb}}{dt} - \frac{R_s}{\sigma L_s} i_{sb} + \frac{1}{\sigma L_s} u_{sb} \right) - n_p\omega \left( -\frac{di_{sa}}{dt} - \frac{R_s}{\sigma L_s} i_{sa} + \frac{1}{\sigma L_s} u_{sa} \right) - \frac{\gamma d}{dt} i_{sb} + \frac{1}{\sigma L_s} \frac{d u_{sb}}{dt}$$

(22)

The goal here is to estimate $T_R$ without knowledge of $\omega$. So, it is now assumed the motor parameters are all known except for $T_R$. The set of equations (21) and (22) may then be rewritten in regressor form as

$$y(t) = W(t) K$$

(23)

where $K \triangleq [1/T_R \ n_p\omega]^T \in \mathbb{R}^2$, and $y \in \mathbb{R}^2$, $W \in \mathbb{R}^{2 \times 2}$ are given by

$$y(t) \triangleq \begin{bmatrix} \frac{du_{sa}}{dt} - \sigma L_s \frac{d^2 i_{sa}}{dt^2} - R_s \frac{di_{sa}}{dt} \\ \frac{du_{sb}}{dt} - \sigma L_s \frac{d^2 i_{sb}}{dt^2} - R_s \frac{di_{sb}}{dt} \end{bmatrix}, \quad W(t) \triangleq \begin{bmatrix} L_s \frac{di_{sa}}{dt} - u_{sa} + R_s i_{sa} & \sigma L_s \frac{di_{sb}}{dt} - u_{sb} + R_s i_{sb} \\ L_s \frac{di_{sb}}{dt} - u_{sb} + R_s i_{sb} & -\sigma L_s \frac{di_{sa}}{dt} + u_{sa} - R_s i_{sa} \end{bmatrix}$$

The regressor system (23) is linear in the parameters. The standard linear least-squares approach is to let (i.e., collect data at) $t = 0, T, 2T, \cdots, NT$, multiply (23) on the left by $W^T(nT)$, sum $W^T(nT)y(nT) = W^T(nT)W(nT)K$ from $t = 0$ to $t = NT$, and finally compute the solution to

$$R_W K = R_{YW}$$

(24)
where
\[ R_W \triangleq \sum_{n=0}^{N} W^T(nT)W(nT), \quad R_{YW} \triangleq \sum_{n=0}^{N} W^T(nT)y(nT). \]

A unique solution to (24) exists if and only if \( R_W \) is invertible. However, \( R_W \) is never invertible in steady state as is now shown. To proceed, define
\[
D(t) = \begin{bmatrix} i_{Sa}(t) & -i_{Sa}(t) \\ i_{Sa}(t) & i_{sb}(t) \end{bmatrix}.
\]

In steady state where \( u_{Sa} + ju_{sb} = U_se^{j\omega s t} \) and \( i_{Sa} + ji_{sb} = I_se^{j\omega s t} \), det\((D(t)) = i_{Sa}^2(t) + i_{sb}^2(t) = |L_s|^2 \), \( D(t)^T D(t) = |L_s|^2 I_{2 \times 2} \). Multiply both sides of (23) on the left by \( D(t) \) to obtain
\[
D(t) y(t) = D(t) W(t) K
\]
or
\[
\begin{bmatrix} R_s \omega |L_s|^2 - \omega_s P \\ \sigma L_s \omega_s^2 |L_s|^2 - \omega_s Q \end{bmatrix} = \begin{bmatrix} -\omega_s L_s |L_s|^2 + Q & R_s |L_s|^2 - P \\ R_s |L_s|^2 - P & \sigma L_s \omega_s |L_s|^2 - Q \end{bmatrix} K
\]
where \( P \triangleq u_{Sa}i_{Sa} + u_{sb}i_{sb} \) and \( Q \triangleq u_{sb}i_{Sa} - u_{Sa}i_{sb} \) are the real and reactive powers, respectively, whose steady-state expressions are given by (30) and (31) in the Appendix. Using (30) and (31) to replace \( P \) and \( Q \) in (25), one obtains
\[
\begin{align*}
\bar{D} & \triangleq D(t) W(t) = -\frac{|L_s|^2 (1 - \sigma) \omega_s L_s}{1 + S^2 \omega_s^2 T_R^2} \begin{bmatrix} S^2 \omega_s^2 T_R^2 & S \omega_s T_R \\ S \omega_s T_R & 1 \end{bmatrix} \\
\bar{Y} & \triangleq D(t) y(t) = -\omega_s \frac{|L_s|^2 (1 - \sigma) \omega_s L_s}{1 + S^2 \omega_s^2 T_R^2} \begin{bmatrix} S \omega_s T_R \\ 1 \end{bmatrix}.
\end{align*}
\]

That is, in steady state, \( \bar{D} \triangleq D(t) W(t) \in \mathbb{R}^{2 \times 2} \) and \( \bar{Y} \triangleq D(t) y(t) \in \mathbb{R}^2 \) are constant matrices. Further, it is easily seen that the determinant of \( \bar{D} \triangleq D(t) W(t) \) is zero. Also,
\[
R_{DW} \triangleq \sum_{n=1}^{N} (D(nT)W(nT))^T (D(nT)W(nT)) = |L_s|^2 \sum_{n=1}^{N} W^T(nT)W(nT) = |L_s|^2 R_W.
\]

\( R_{DW} \) is singular as \( D(t) W(t) \) is constant and singular. It then follows that \( R_W \) is also singular using steady-state data. Further,
\[
R_{DWY} \triangleq \sum_{n=1}^{N} (D(nT)W(nT))^T (D(nT)y(nT)) = |L_s|^2 \sum_{n=1}^{N} W^T(nT)y(nT) = |L_s|^2 R_{YW}.
\]
Thus $R_W$ and $R_{YW}$ are given by

$$R_W = \frac{R_{DW}}{|I_S|^2} = \frac{N |I_S|^2 (1-\sigma)^2 \omega_S^2 L_S^2}{1 + S^2 \omega_S^2 T_R^2} \left[ \begin{array}{cc} S^2 \omega_S^2 T_R^2 & S \omega_S T_R \\ S \omega_S T_R & 1 \end{array} \right]$$

$$R_{YW} = \frac{R_{DWY}}{|I_S|^2} = \frac{N |I_S|^2 (1-\sigma)^2 \omega_S^2 L_S^2}{1 + S^2 \omega_S^2 T_R^2} \left[ \begin{array}{c} S \omega_S T_R \\ 1 \end{array} \right],$$

where again $\bar{D}$ and $\bar{Y}$ are from (26) and (27), respectively.

By inspection of (28) and (29), $K = [0 \omega_S]^T$ is one solution to (24). The null space of $R_W$ is generated by $[-1/T_R S \omega_S]^T$ so that all possible solutions are given by $[0 \omega_S]^T + \alpha [-1/T_R S \omega_S]^T$ for some $\alpha \in \mathbb{R}$. In summary, solving (24) using steady-state data leads to an infinite set of solutions so that $T_R$ is not identifiable using the linear regressor (23) with steady-state data.

**Remarks** There are a few ways to deal with the problem in a real-time control application. For example, a small perturbation could be added to the speed reference. This type of technique has often been used for the adaptive control of insufficiently excited systems. A more interesting approach, however, would be to vary the flux reference while keeping the torque reference constant. The speed of the motor would not vary, but the voltages and currents would no longer be in sinusoidal steady-state, so that the speed and the rotor time constant would be identifiable. In the work [4], a linear regressor was obtained by assuming constant speed, but the data collected in [4] was not in sinusoidal steady state (see Figures 7.1a and 7.1b in [4]). In the identification method given in [15], the speed is assumed constant, but it requires the flux magnitude be perturbed by a small amplitude sinusoidal signal so it is also not in sinusoidal steady state.

### 5 Experimental Results

To demonstrate the viability of the speed sensorless estimator (18) for $T_R$, experiments were performed. A three-phase, 0.5 hp, 1735 rpm ($n_p = 2$ pole-pair) induction motor was driven by an ALLEN-BRADLEY PWM inverter to obtain the data. Given a speed command to the inverter, it produces PWM voltages to drive the induction motor to the commanded speed. Here a step speed command was chosen to bring the motor from standstill up to the rated speed of 188 rad/s. The stator currents and voltages were sampled at 10 kHz. The real-time computing system RTLAB from OPAL-RT with a fully integrated hardware and software system was used to collect data [16]. Filtered differentiation (using digital filters) was used for the derivatives of the voltages and currents. Specifically, the signals were filtered with a third-order Butterworth filter whose cutoff frequency was 100 Hz. Further, if one uses a low-pass Butterworth filter, the derivatives of the filtered input signal (i.e., stator currents/voltages) are state variables in the state-space implementation of the filter, i.e., no differentiation is needed [15]. See also [7] for another approach to estimating derivatives without numerical differentiation. The voltages and currents were put through a $3 - 2$
transformation to obtain their two-phase equivalent values (see Figure 1).

The data was collected just after the machine is turned on to obtain the “cold” value of $T_R$ for comparison with the results in [17]. Using the data \{$u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}$\} collected between 0.84 sec to 0.91 sec, which includes the time the motor accelerates, the quantities $du_{Sa}/dt$, $du_{Sa}/dt$, $di_{Sa}/dt$, $di_{Sb}/dt$, $d^2i_{Sa}/dt^2$, $d^2i_{Sb}/dt^2$, $d^3i_{Sa}/dt^3$, $d^3i_{Sb}/dt^3$ are calculated and used to evaluate the coefficients $C_i, i = 1, 2, \cdots, 12$ in equation (18).

![Figure 1: Left: Sampled two-phase equivalent voltages $u_{Sa}, u_{Sb}$. Right: Sampled phase b current $i_{Sb}$ and its simulated response $i_{Sb-sim}$.](image)

Solving (18), one obtains the 12 solutions

$$T_{R1} = +0.1064$$
$$T_{R3} = -0.0576 + j0.0593$$
$$T_{R5} = -0.0037 + j0.0166$$
$$T_{R7} = -0.0072 + j0.0103$$
$$T_{R9} = +0.0125 + j0.0077$$
$$T_{R11} = +0.0065 + j0.0018$$
$$T_{R2} = -0.0186$$
$$T_{R4} = -0.0576 - j0.0593$$
$$T_{R6} = -0.0037 - j0.0166$$
$$T_{R8} = -0.0072 - j0.0103$$
$$T_{R10} = +0.0125 - j0.0077$$
$$T_{R12} = +0.0065 - j0.0018$$

$T_R$ must be a real positive number, so $T_R = 0.1064$ is the only possible choice. This value compares favorably with the “cold” value of $T_R = 0.11$ obtained using the method of Wang et al [18], which required a speed sensor.

To illustrate the identified $T_R$, a simulation of the induction motor model was carried out using the measured voltages as input. The simulation’s output [stator currents computed according to (1) and (2)] are used to compare with the measured (stator currents) outputs. The right side of Figure 1 shows the sampled two-phase equivalent current $i_{Sb}$ and its simulated response $i_{Sb-sim}$. (The phase a current $i_{Sa}$ is similar, but shifted by $\pi/(2n_p)$.) The resulting phase b current $i_{Sb-sim}$
from the simulation corresponds well with the actual measured current $i_{sb}$. Note that in equation (1) the parameter $\gamma = \frac{R_S}{\sigma L_S} + \frac{\beta M}{T_R}$ also depends on $T_R$.

6 Conclusions and Future Work

This paper presented an algebraic approach to the estimation of the rotor time constant of an induction motor without using a speed sensor. The experimental results demonstrated the practical viability of this method. Though the method is not applicable in steady state, neither is a standard linear least-squares approach. Future work includes studying an on-line implementation of the estimation algorithm and using such an online estimate in a speed sensorless field-oriented controller.

7 Appendix: Steady-State Expressions

In the following, $\omega_S$ denotes the stator frequency and $S$ denotes the normalized slip defined by $S \triangleq (\omega_S - n_p \omega) / \omega_S$. With $u_{st} + j u_{sb} = U_s e^{j \omega st}$ and $i_{sa} + j i_{sb} = I_s e^{j \omega st}$, it is shown in [19] that under steady-state conditions, the complex phasors $U_s$ and $I_s$ are related by ($S_p \triangleq \frac{R_R}{\sigma \omega S L_R} = \frac{1}{\sigma \omega S T_R}$)

$$I_s = \frac{U_s}{R_S + j \omega_S L_S \left( \frac{1 + j \frac{s}{\sigma_p}}{1 + j \frac{s}{\sigma_p}} \right)} = \frac{U_s}{R_S + \left( \frac{(1 - \sigma) S \omega_S^2 L_S T_R}{1 + S^2 \omega_S^2 T_R^2} \right) + j \omega_S L_S \left( 1 + \sigma S^2 \omega_S^2 T_R^2 \right)}$$

and straightforward calculations (see [6]) give

$$P \triangleq u_{sa} i_{sa} + u_{sb} i_{sb} = R_e (U_s I_s^*) = |I_s|^2 \left( R_S + \frac{(1 - \sigma) S \omega_S^2 L_S T_R}{1 + S^2 \omega_S^2 T_R^2} \right)$$

$$Q \triangleq u_{sb} i_{sa} - u_{sa} i_{sb} = I_m (U_s I_s^*) = |I_s|^2 \frac{\omega_S L_S (1 + \sigma S^2 \omega_S^2 T_R^2)}{1 + S^2 \omega_S^2 T_R^2}.$$  

7.1 Steady-State Expressions for $q_2$, $q_1$, and $q_0$

The steady-state expressions for $q_2$, $q_1$, and $q_0$ are from [6] and given by

$$q_2 = n_p^2 T_R^2 |I_s|^4 \frac{\frac{\omega_S^2}{\sigma^2} L_S (1 - \sigma)^2 (1 - S)}{(1 + S^2 \omega_S^2 T_R^2)}$$

$$q_1 = n_p \omega_S |I_s|^4 \frac{L_S (1 - \sigma)^2 (1 - \omega_S^2 T_R^2 (1 - S)^2)}{\sigma (1 + S^2 \omega_S^2 T_R^2)}$$

$$q_0 = -|I_s|^4 \frac{\frac{\omega_S^2}{\sigma^2} L_S (1 - \sigma)^2 (1 - S)}{(1 + S^2 \omega_S^2 T_R^2)}.$$  

With $\omega \neq 0$ (equivalent to $S \neq 1$), it is seen that $q_2 \neq 0$. Conversely, $q_2 = 0$ if and only if $S = 1$ (i.e., $\omega = 0$). Also, if $\omega = 0$, then $S = 1$ and $q_1 \neq 0$. 

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7.2 Steady-State Expressions for $a_2, a_1, a_0$

The steady-state expressions for $a_2, a_1, and a_0$ are (see [6])

\[
a_2 = -n_p^2 |l_s|^4 \frac{\omega_S (1 - \sigma)^2}{\sigma^2 (1 + S^2 \omega_S^2 T_R^2)} \frac{1}{\text{den}} \tag{35}
\]

\[
a_1 = n_p |l_s|^4 \frac{2 \omega_S^2 (1 - \sigma)^2 (1 - S)}{\sigma^2 (1 + S^2 \omega_S^2 T_R^2)} \frac{1}{\text{den}} \tag{36}
\]

\[
a_0 = -|l_s|^4 \frac{\omega_S^3 (1 - \sigma)^2 (1 - S)^2}{\sigma^2 (1 + S^2 \omega_S^2 T_R^2)} \frac{1}{\text{den}} \tag{37}
\]

\[
den = n_p T_R |l_s|^4 \left( \frac{(1 - \sigma) 1 + S^2 \omega_S^2 T_R^2 - S \omega_S^2 T_R^2}{\sigma T_R} + \frac{(1 - \sigma) \omega_S}{1 + S^2 \omega_S^2 T_R^2} \right)^2. \tag{38}
\]

Recall from Section 3 [following (6)] that $\text{den} = 0$ if and only if $|\bar{\psi}_R| = 0$.

7.3 Steady-State Expression for $r_1$ and $r_0$

It is now shown that the steady-state value of $r_1$ in (12) is nonzero. Substituting the steady-state values of $q_2, q_1, q_0, a_2, a_1, and a_0$ (noting that $\dot{q}_1 \equiv 0$ and $\dot{q}_2 \equiv 0$ in steady state) into (12) gives

\[
r_1 = -|l_s|^6 \left( \frac{1}{1 + S^2 \omega_S^2 T_R^2} \right)^3 n_p^4 (1 - \sigma)^6 L_S^2 \omega_S^3 \left( 1 + T_R^2 \omega_S^2 (1 - S)^2 \right)^2 / \text{den}
\]

\[
r_0 = |l_s|^6 \left( \frac{1}{1 + S^2 \omega_S^2 T_R^2} \right)^3 n_p^4 (1 - \sigma)^6 L_S^2 \omega_S^3 (1 - S) \left( 1 + \omega_S^2 T_R^2 \times (1 - S)^2 \right)^2 / \text{den}
\]

where $\text{den}$ is given by (38). It is then seen that $r_1 \neq 0$ in steady state.

References


