An Adaptive Algorithm for the Tuning of Two Input Shaping Methods

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Abstract

The paper presents an algorithm for the tuning of two input shaping methods. These methods are designed to prevent the excitation of oscillatory modes in resonant systems. The first input shaping method produces a control signal that is the linear combination of delayed versions of the reference input. The resulting control system is linear time-invariant but infinite-dimensional. Its transfer function has an infinite number of complex zeros, with some of them placed exactly at the locations of the resonant modes of the plant. In contrast, the second input shaping method is based on a pole/zero cancellation of the resonant modes using a finite-dimensional controller. For both input shaping methods, tuning is useful to optimize performance and an algorithm is developed for the automatic adjustment of the controller parameters. Experimental results are presented for a system in which a motor is used to control the position of a flexible arm. The step response of the plant is poorly damped, but is much improved with input shaping. The control performance is found to be comparable for both methods, and the tuning method is found to be simple and effective.

Keywords: adaptive control, feedforward control, inverse dynamics control, flexible arms, input shaping, vibration control.

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1 Introduction

The problem of controlling systems with poorly damped modes, *i.e.,* resonant systems, occurs in many applications. Space structures, flexible aircraft wings, and robotic manipulators are important examples. Another area of interest is in disk drives, where read/write heads mounted at the end of small but flexible assemblies must be moved rapidly to distant tracks while being subjected to minimum residual vibrations ([3]).

There are often strong limitations to the use of feedback to control resonant modes, due to sensor noise and unmodelled dynamics. Whenever resonant modes remain after feedback loops are closed, the problem is to design feedforward control algorithms so that the resonant modes are not excited by the input signals applied to the system. Such methods have recently received attention under the name of *input shaping* algorithms.

Several groups of researchers have studied input shaping methods where the control input is a linear combination of delayed versions of the reference signal. The control signal can be viewed as the convolution of the reference signal with a sequence of impulses whose magnitudes and time separations are appropriately selected. The idea was proposed by [6], who referred to it as *posicast control.* Nonadaptive and adaptive extensions of the algorithm were presented by [5] and [8] (among others). Recently, such methods were tested in the Space Shuttle as part of the *Middeck Active Control Experiment* (see [7]). The input shaping methods using delayed inputs produce infinite-dimensional feedforward control systems with an infinite number of complex zeros, some of them turning out to be coincident with the plant poles (*cf.* [2]). A natural question to investigate is how such methods compare to finite-dimensional control laws based on the same principle of pole/zero cancellation. In this paper, we perform such a comparison. The first method is referred to as the *delayed input method*, and the second method as the *pole/zero cancellation method.*

Experimental results indicate that both methods yield comparable performance. The simpler case of second-order compensation is considered, although the plant under testing is far more complex than a second-order system. Because both methods rely on pole/zero cancellation for a poorly damped system, tuning of the control laws is necessary, an operation which is difficult to achieve by trial and error. However, we show that a simple procedure can be applied to perform tuning and that it is effective in practice.

2 Input Shaping Using Delayed Inputs

We consider the problem of feedforward control system design, or *input shaping*, so that the system is described, in the Laplace domain, by

$$\begin{aligned} y(s) &= P(s)u(s), \\ u(s) &= C(s)r(s), \end{aligned} \tag{1}$$

where $r(s)$ is the reference input, $u(s)$ is the control input, and $y(s)$ is the plant output. $C(s)$ is the (feedforward) compensator transfer function, and $P(s)$ is the plant transfer function. Although not a restriction of the methods discussed in this paper, the plant is assumed to be a second-order system with transfer function

$$P(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}. \tag{2}$$

For the systems under consideration, the damping factor ζ is small (much less than 1), leading to poles close to the $j\omega$-axis and to highly oscillatory step responses.

A controller $C(s)$ using delayed inputs is

$$C(s) = K_1 + K_2 e^{-sT}. \tag{3}$$

Because a second-order system is considered, only one delay is necessary. The reference input $r(t)$ is thus delayed by a time interval T and summed with the original input using scaling factors K_1 and K_2 to produce the control signal $u(t)$. To calculate the controller parameters, we note that the response of system (2) to a double step input

$$u(t) = K_1 \text{ step}(t) + K_2 \text{ step}(t - T) \tag{4}$$

is given by

$$\begin{aligned} y(t) &= \left(K_1 + \frac{K_1\omega_n}{b} e^{-at} \sin(bt - \phi)\right) \text{step}(t) \\ &+ \left(K_2 + \frac{K_2\omega_n}{b} e^{-a(t-T)} \sin(b(t - T) - \phi)\right) \text{step}(t - T). \end{aligned} \tag{5}$$

In (5), $-a$ and b are the real and imaginary parts of the poles of the plant, equal to

$$a = \zeta\omega_n, \qquad b = \omega_n\sqrt{1 - \zeta^2}. \tag{6}$$

The phase $\phi = \tan^{-1}(\frac{b}{-a})$. By properly adjusting the magnitude and the timing of the two steps, it is possible to produce a response of the system to the second step which cancels exactly the response to the first step. The result is a finite-time response precisely to the desired value. The condition for this occurrence is that $y(t) = 1$ for all $t \geq T$, and is satisfied if

$$K_1 + K_2 = 1, \qquad K_1 = K_2 e^{aT}, \qquad bT = \pi. \tag{7}$$

The transfer function of the control system (3) has an infinite number of zeros located at

$$s = -\frac{1}{T}\ln\left(\frac{K_1}{K_2}\right) \pm jn\frac{\pi}{T}, \tag{8}$$

where $n = 1, 3, 5, ...$ For the values of K_1, K_2, and T that solve (7), the zeros are given by

$$s = -a \pm jnb. \tag{9}$$

Note that, for $n = 1$, the locations of the complex zeros match exactly those of the plant poles. The fact that the plant poles are cancelled by the controller zeros ensures that the method works for arbitrary inputs, and not only for step inputs. However, the control law introduces far more zeros than are necessary for that purpose.

3 Input Shaping Using Pole/Zero Cancellation

Since the principle behind the delayed input design is pole/zero cancellation, one may wonder what success would be achieved through explicit cancellation of the two plant poles with a finite-dimensional controller. To address this issue, we first define a desired transfer function, called the *reference model*

$$M(s) = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}. \tag{10}$$

Typically, ζ_m will be chosen greater than or equal to 0.707 to yield adequate damping. The controller transfer function is set to

$$C(s) = M(s)P^{-1}(s) = \frac{\omega_m^2}{\omega_n^2} . \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}, \tag{11}$$

so that the combined transfer function $C(s)P(s)$ is equal to $M(s)$. The poorly damped modes of the plant are cancelled and replaced by those of the reference model in the cascade transfer function. Generally, methods based on pole/zero cancellation are viewed unfavorably by control engineers, because of sensitivity considerations. However, they can be made to work effectively if combined with adaptive methods to provide the necessary tuning. Adaptive feedforward control methods are attractive in cases where sensor accuracy and bandwidth constraints severely limit the capabilities of feedback control. Such methods are sometimes referred to as *adaptive inverse control* methods (see [9]). As observed in equation (11), the principle of the control law is indeed that of plant inversion.

The step response with the controller (11) is not a finite-time response, but rather converges exponentially to the steady-state. The distinction is not significant from a practical point of view. On the other hand, the design allows to select the speed of response of the controlled response, in contrast to the previous method where the speed of response was determined by the natural frequency of the original system.

4 Adaptation

Both methods require fine parameter tuning for optimal performance. In experiments, this tuning was found to be difficult to perform by trial and error. [8] proposed a method for the delayed input design, based on the calculation of the Fourier transform of the plant response. The method was shown to be effective, although computationally demanding. We present an alternative method which is simple and effective, based on a parameterization of the pole/zero cancellation controller. First, the compensator transfer function (11) is rewritten as

$$C(s) = M(s) + A(1 - M(s)) + BsM(s), \tag{12}$$

where

$$A = \frac{\omega_m^2}{\omega_n^2}, \qquad B = 2\,\frac{\zeta\omega_n - \zeta_m\omega_m}{\omega_n^2}. \tag{13}$$

The motivation for (12) is that the expression is linear in the parameters A and B, so that standard adaptive laws can be applied. We define the signals

$$\begin{aligned} x_1(s) &= (1 - M(s))y(s), \\ x_2(t) &= sM(s)y(s), \\ x_3(s) &= M(s)u(s), \\ y_m(s) &= M(s)y(s). \end{aligned} \tag{14}$$

Note that the four signals can be reconstructed from the known signals $u(t)$ and $y(t)$. Because the transfer function $M(s)$ has relative degree two, $sM(s)$ is strictly proper and can be implemented without differentiation. Using (11) and (12), one finds that

$$(A(1 - M(s)) + BsM(s))\,P(s) = M(s) - M(s)P(s). \tag{15}$$

Applying both sides of (15) to the signal u, it follows that, in the time domain,

$$Ax_1(t) + Bx_2(t) = x_3(t) - y_m(t). \tag{16}$$

The last equation can be used as the basis for the identification of the parameters A and B, with a number of adaptive algorithms (see [4]). For identifiability, the signals $x_1(t)$ and

$x_2(t)$ need to be linearly independent functions of time, a condition which will be satisfied for most non-constant reference inputs (explicit persistency of excitation and frequency-domain conditions for convergence can be derived using the techniques discussed in [4]).

The adaptive method may also be applied to the delayed input control method. Specifically, one estimates A and B using the same method, and then calculates

$$\omega_n = \frac{\omega_m}{\sqrt{A}}, \qquad \zeta\omega_n = \frac{B{\omega_n}^2}{2} + \zeta_m\omega_m. \tag{17}$$

From those parameters,

$$\begin{aligned} T &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \\ K_1 &= \frac{e^{\zeta\omega_n T}}{1+e^{\zeta\omega_n T}}, \\ K_2 &= 1 - K_1. \end{aligned} \tag{18}$$

There is a difficulty with the implementation if the estimated parameters are such that $A < 0$ or $|\zeta| \geq 1$. A simple fix to the problem consists in performing the calculations with the estimate of A replaced by $A_{min} > 0$ if the estimate is less than A_{min}, and the estimate of ζ replaced by $\zeta_{max}.\text{sign}\zeta$, with $0 < \zeta_{\max} < 1$, if the magnitude of the estimate exceeds ζ_{max}.

5 Experimental Results

Experiments were carried out using a testbed represented on Fig. 1. In the testbed, a flexible beam composed of a thin metallic plate of approximately 40 cm long is attached to a brush DC motor, model 1050-01 from *Aerotech*. An encoder provides position information about the shaft of the motor and its pulses are decoded by a board from the *Dynamics Research Corporation*, yielding a resolution of 2000 counts/rev. An accelerometer from *Kistler* is attached at the end of the beam. However, measurements of the tip acceleration are not used by the adaptive algorithm, which observes bending motions of the beam only through the induced motions of the shaft. Accelerometer measurements are used later in this section to demonstrate that damping of the shaft and of the tip motions are achieved simultaneously.

A linear current amplifier, model 4020-LS from *Aerotech*, ensures direct torque control. A *Pentium*-based computer, programmed in C, is used for the computation of the control algorithm. The real-time clock of the PC is re-programmed for a 500 Hz sampling rate. Analog interfaces are provided by a *Data Translation* board DT-2801, with a D/A output connected to the power amplifier, an A/D input used to read a potentiometer position, and another A/D input used to read the accelerometer measurement. The potentiometer is

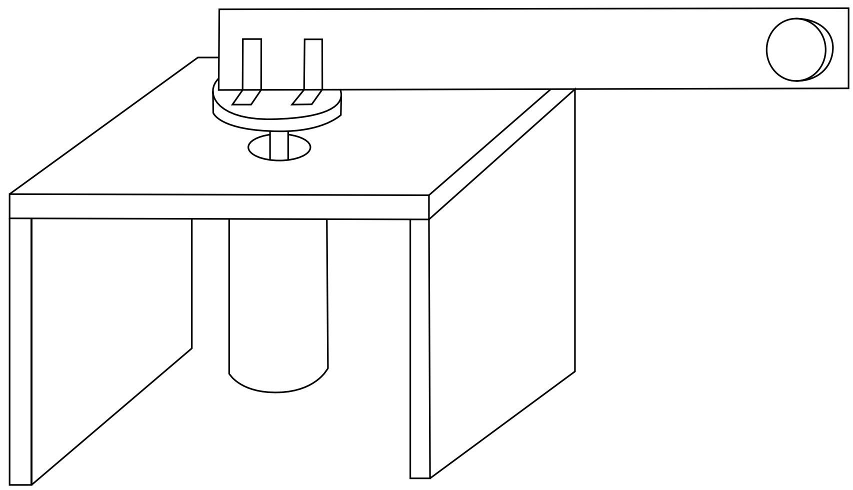

Figure 1: Schematic of the Experimental Testbed

adjusted by the operator to specify a reference input. An inner control loop is provided in the form of a proportional derivative control law, which is tuned for the DC motor without the flexible beam.

Fig. 2 shows the step response of the system with the beam. The dashed curve is the reference input and the solid curve is the angular response of the system, which exhibits overshoot because of the inertia and flexibility of the beam.

The performance of the adaptive input shaping methods was evaluated with the operator applying step inputs to the system through the potentiometer. Figs. 3 and 4 show the results with the pole/zero cancellation method. The reference model was chosen to be $M(s) = \frac{144}{(s+12)^2}$. The computation of the estimates was performed using a stabilized recursive least-squares algorithm with forgetting factor discussed in [1]. The initial values of the parameters were $A = 1$ and $B = 0$. Those parameters are such that the transfer function of the compensator is equal to the identity (*i.e.,* there is no compensation when the system starts). Fig. 3 shows the response of the system, with the solid line giving the angular position of the shaft, the dashed line giving the reference input provided by the operator, and the dot-dashed line being the control input (that is, the filtered reference input). The responses show that the overshoot is reduced within the first step of the reference input, and eliminated in subsequent steps, demonstrating the effective tuning of the control law. Fig. 4 shows the responses of the two adaptive parameters, which are found to converge rapidly. The solid line is the parameter estimate for A, and the dashed line is the parameter estimate

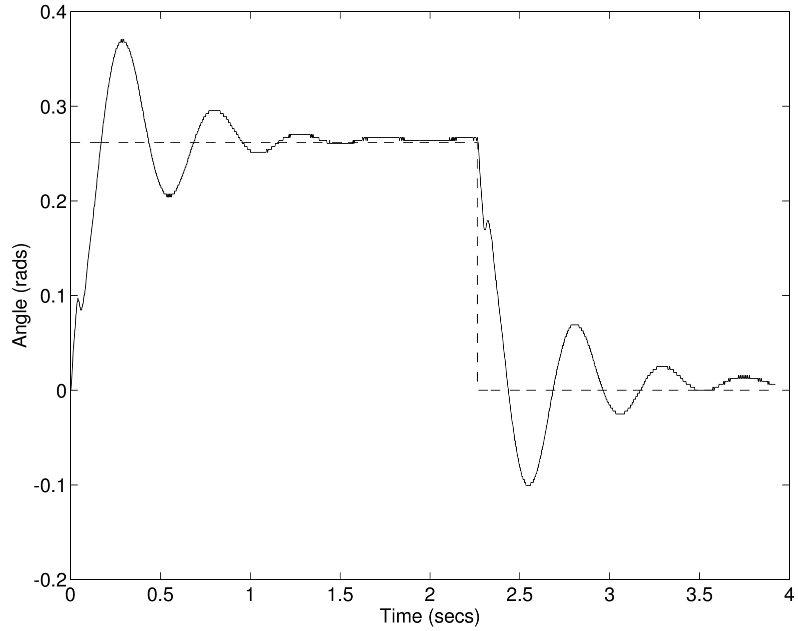

Figure 2: Step Response without Input Shaping

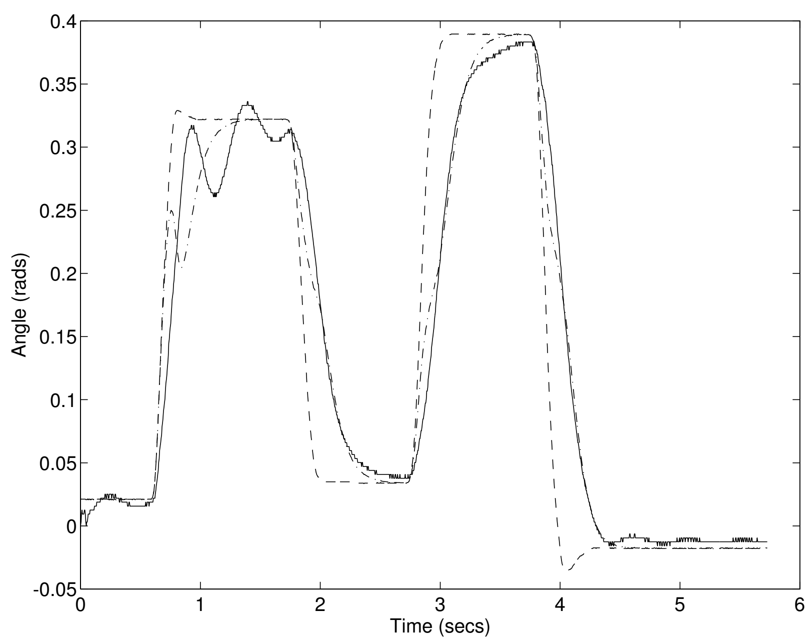

Figure 3: Responses with Pole/Zero Cancellation

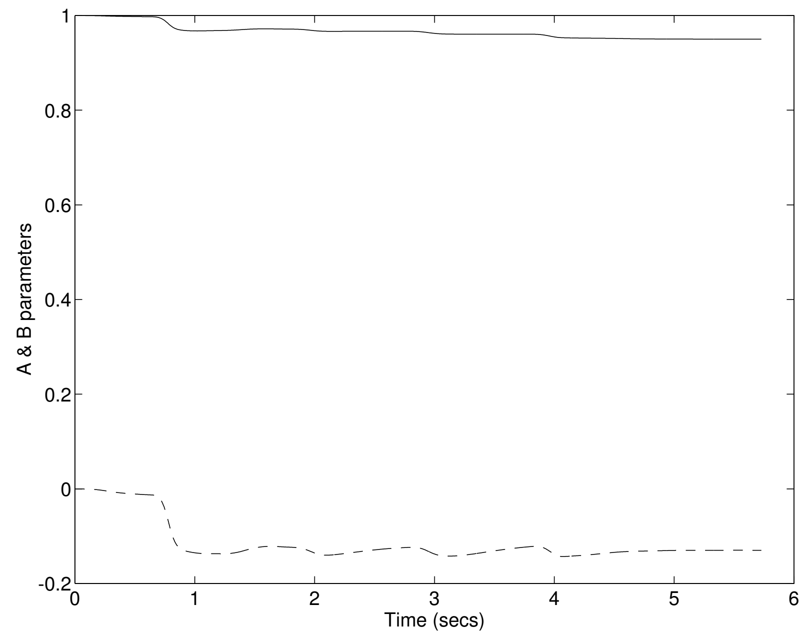

Figure 4: Parameters with Pole/Zero Cancellation

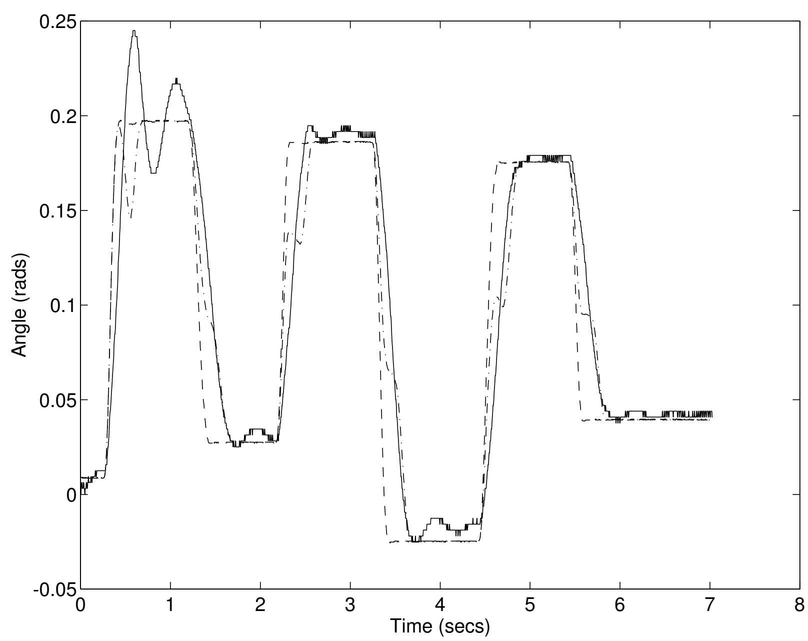

Figure 5: Responses with Delayed Input

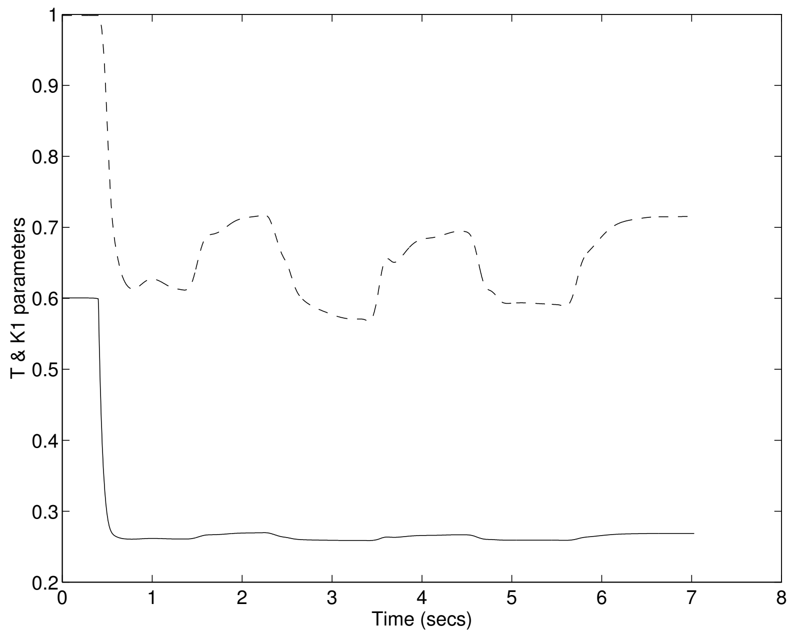

Figure 6: Parameters with Delayed Input

for B.

Fig. 5 shows the results for the delayed input method. The overshoot in the first step is again smaller than for the uncompensated system, and subsequent step responses are much improved, as for the pole/zero cancellation method. The parameters, shown in Fig. 6, are obtained through the transformation of the A, B adaptive parameters. The solid line is the estimated time delay T, in seconds, which converges from 0.6s to 0.25s. The dashed line is the K_1 parameter. There is a visible fluctuation in this parameter, although the variation is not detrimental to performance.

Although the adaptive algorithm does not require the use of the accelerometer measurement, the signal was used to evaluate the amount of oscillation at the end of the beam. Because of the large noise affecting the measurement, the sensor data was filtered in *Matlab* using the function *filtfilt* and a Butterworth filter of order 5 and bandwidth of 50 Hz. On Fig. 7, the solid line is the filtered acceleration, in g's. The dashed line is the desired acceleration at the tip of the beam. For the pole/zero cancellation method used in the experiment, the desired acceleration is the one that would be obtained with the reference model $M(s)$ and a rigid beam. It is equal, in the Laplace domain, to

$$a_d(s) = (\frac{L}{g})s^2 M(s) r(s) \tag{19}$$

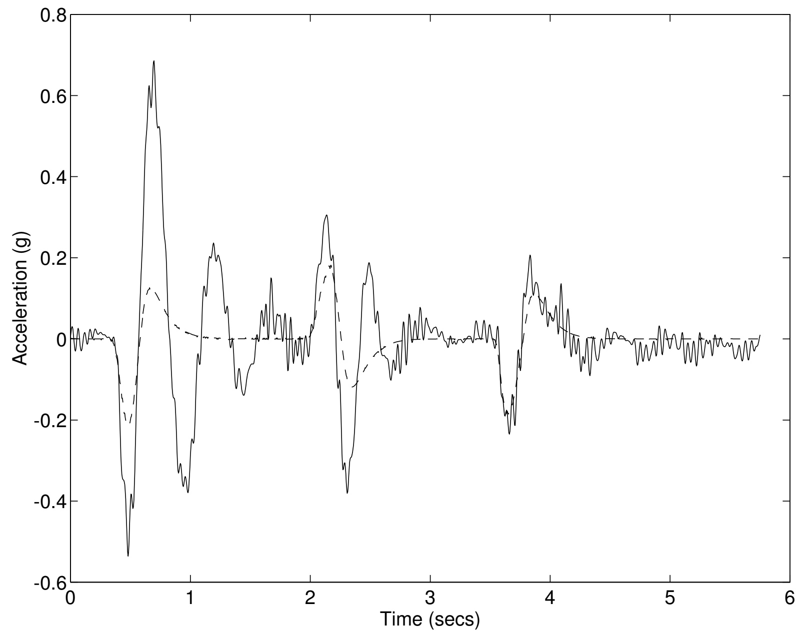

Figure 7: Acceleration of the Tip of the Beam

where $r(s)$ is the reference position of the motor (in rad), L is the distance between the axis of the motor and the position of the accelerometer (38 cm in the testbed), and g is the acceleration of gravity. Note that the transfer function $s^2M(s)$ is proper. For step inputs provided by the operator, the desired acceleration is seen on Fig. 7 to have the form of doublets, as expected. Contrary to previous experiments, the adaptation was postponed for two seconds to show the behavior of the uncompensated system. During that time, one finds that there is a considerable excess of acceleration in the response and a large residual oscillation. After two seconds, adaptation of the parameters produces a significant decrease in the transient oscillation. The response to the last step input does not exhibit any undesired oscillation and is very close to the desired response. Overall, the adaptive algorithm is successful in reducing the oscillation of the shaft as well as of the beam. This situation is due in part to the fact that the motor is connected to the beam without a gear box and with little friction, so that any beam oscillation is observed on the shaft of the motor.

6 Conclusions

Two methods were discussed for the control of resonant systems: the first was based on delayed inputs, and the second was based on explicit pole/zero cancellation. Both methods actually rely on pole/zero cancellation, so that a more intrinsic distinction was the structure of the control law, in particular the infinite *vs.* finite dimensionality.

A method for the automatic tuning of the control laws was presented and was proved to be effective experimentally. An interest of the adaptation method is its simplicity, which makes it easily implementable in real-time. With adaptation, both control methods were found to perform comparably. Although the delayed input method has received considerable attention in the literature, it would seem that the approach based on straighforward pole/zero cancellation is as effective. Both methods were found to be easy to code. The delayed input method required slightly less computations and somewhat more storage (300 memory locations for a 0.6 secs delay at a 500 Hz sampling rate).

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References

[1] Bodson, M. (1995), "An Adaptive Algorithm with Information-Dependent Data Forgetting," *Proc. of the American Control Conference,* Seattle, WA, pp. 3485-3489.

[2] Hillsley, K.L., and S. Yurkovich (1992), "Vibration Control of a Two-Link Flexible Robot Arm," *Dynamics and Control*, Vol. 3, pp. 261-280, 1993.

[3] Miu, D. (1993). *Mechatronics,* Springer-Verlag, New York.

[4] Sastry, S. and M. Bodson (1989). *Adaptive Control: Stability, Convergence, and Robustness*, Prentice-Hall, Englewood Cliffs, NJ.

[5] Singer, N.C. and W.P. Seering (1990). "Preshaping Command Inputs to Reduce System Vibration," *Journal of Dynamic Systems, Measurement, and Control*, vol. 112, pp. 76-82.

[6] Smith, O.J. (1958), *Feedback Control Systems,* Mc Graw-Hill, New York.

[7] Tuttle, T.D. and W.P. Seering (1995). "Vibration Reduction in 0-g Using Input Shaping on the MIT Middeck Active Control Experiment," *Proc. of the IEEE Conference on Decision and Control*, Seattle, WA, pp. 919-923.

[8] Tzes, A. and S. Yurkovich (1993). "An Adaptive Input Shaping Control Scheme for Vibration Suppression in Slewing Flexible Structures," *IEEE Trans. on Control Systems Technology*, vol. 1, no. 2, pp. 114-121.

[9] Widrow, B. and S.D. Stearns (1985). *Adaptive Signal Processing,* Prentice-Hall, Englewood Cliffs, NJ.