**Problem 1.2:** the problem is designed to develop familiarity with *Simulink*. A tutorial is available on the web site showing how the system can be built and simulated. The objective is to simulate the response of the RL circuit of Fig. 1.25.

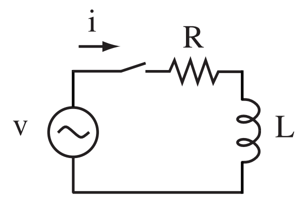


Figure 1.25: RL circuit for Problem 1.2

(a) Simulate the response of the RL circuit with a voltage

$$v(t) = V_{pk} \cos(2\pi f\ t), \tag{1.47}$$

applied at $t = 0$ s. Let $V_{pk} = 2$ V and $f = 60$ Hz. Fig. 1.26 shows an implementation using blocks from the file *generator_blocks.mdl* available on the web site. Blocks can be dragged to the simulation file *blank.mdl*, which can be renamed as desired. Adjust the RL circuit parameters so that $R = 1\ \Omega$ and $L = 30$ mH. Run the simulation for 0.3 s and plot the voltage and the current as functions of time.

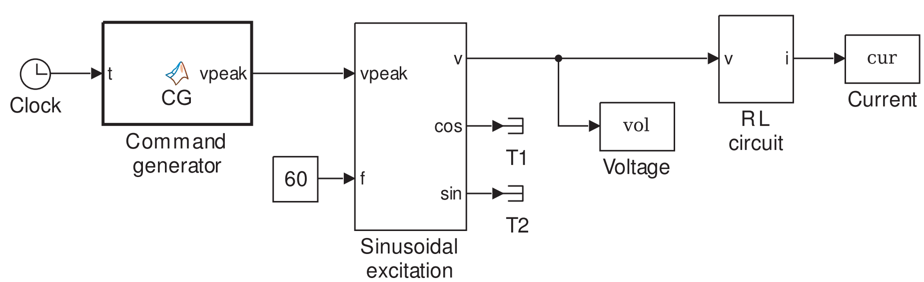


Figure 1.26: Simulation of an RL circuit with a sinusoidal voltage

(b) In the *Command generator* block, let $V_{pk} = 0$ for $t < t_0$ and $V_{pk} = 2$ V for $t \geqslant t_0$, where $t_0 = 3/4\ T$ and $T$ is the period of the sinusoidal voltage. Note that this adjustment is equivalent to replacing the cos function applied previously by a sin function. Plot the voltage and the current as functions of time, and observe that the transient current is much larger than in part (a).

Estimate the percentage overshoot of the current compared to the steady-state value.

**Problem 1.3:** (a) Consider the analytic expression for the coefficient of power of a wind turbine proposed in [18], or

$$C_P(\lambda) = \left(\frac{c_1}{\lambda_1} - c_2\right) e^{-\frac{c_3}{\lambda_1}}, \ \ \lambda_1 = \frac{\lambda}{1 - c_4\lambda}. \tag{1.48}$$

Find analytic expressions for $\lambda_{OPT}$, the TSR for which $C_P(\lambda)$ is maximum, and for $\lambda_{MAX}$, the TSR for which $C_P(\lambda)$ is 0.
(b) Using the results of part (a), compute $\lambda_{OPT}$, $C_{OPT}$, and $\lambda_{MAX}$ for $c_1 = 58$, $c_2 = 2.5$, $c_3 = 21$, and $c_4 = 0.035$. Plot $C_P(\lambda)$ for $\lambda$ ranging from 1 to 14 and check that the values that you computed are consistent with the graph.
(c) Assuming a turbine with $R = 9$ m and a wind speed $v_W = 10$ m/s, plot $P_T$, the mechanical power available from the wind turbine as a function of the turbine speed. Label the axes in kW and rpm. What speed would the turbine reach if rotating freely (i.e., if $P_F = P_G = 0$)?
(d) Consider a grid-tied squirrel-cage induction generator (SCIG) connected to the turbine through a gear, as shown in Fig. 1.22. The generating torque is given by

$$\tau_G = -\frac{k_1 S}{1 + (k_2 S)^2}, \tag{1.49}$$

where $S = \omega_S - n_P\omega$ is the so-called slip frequency in rad/s, $\omega_S = 120\pi$ rad/s is the angular electrical frequency of the grid voltages (at 60 Hz), $n_P = 2$ is the number of pole pairs, and $\omega$ is the speed of the generator (in rad/s). The model represents the SCIG using a steady-state approximation. Assume that there are no friction losses, so that $\tau_{PM} = \tau_G$ and $P_{PM} = P_G$ in (1.22)).

Let $k_1 = 50$, $k_2 = 0.032$, and assume that the generator is connected to the wind turbine of parts (a) to (c) using a gear with ratio $G = 29$. Plot on the same graph $P_T$ and $P_G = \tau_G\omega$ (in kW) as functions of the turbine speed $\omega_T$ in rpm. The curves should be similar to Fig. 1.8, but with $\omega_T$ on the x-axis. The values of $\omega_T^*$ where $P_T = P_G$ are the possible operating points of the turbine/generator set. Using the graph, estimate the speed and the power generated at this condition.

**Problem 1.4:** the simulation results of this problem illustrate the computations of Problem 1.3.
(a) Download the *Simulink* file *wind_turbine.mdl* and the data file *wind.mat* from the web site, and place the files in the working directory of *Matlab*. Fig. 1.27

shows the block diagram of the simulation. There is a switch that directs the simulation to use either a constant wind speed of 10 m/s or the wind data in the file *wind.mat*. You must load the wind data in the workspace by typing *load wind* for the simulation to work. Run the simulation for a constant wind speed of 10 m/s and plot the turbine speed as a function of time over 150 s. Compare the steady-state value of the turbine speed to the value computed in part (c) of Problem 1.3.

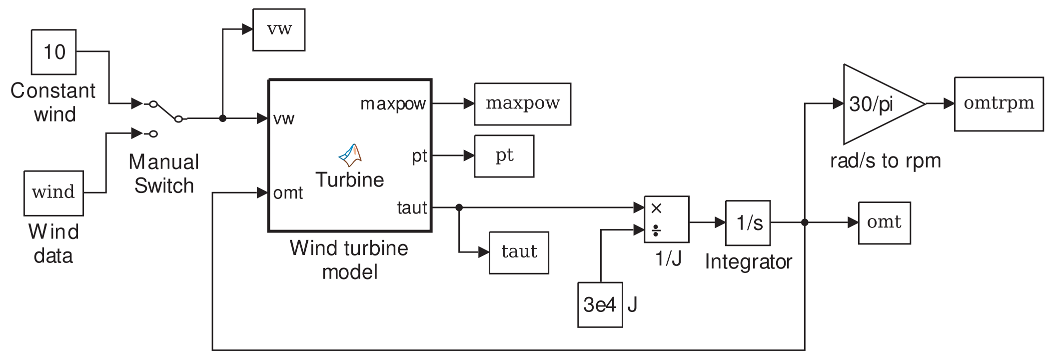


Figure 1.27: Simulation of a wind turbine

(b) Insert the model of the generator of Problem 1.3, part (d), in the *Simulink* model. Specifically, the torque given in (1.49) should be implemented in a block connecting the turbine speed $\omega_T$ to the generator torque $\tau_{GT}$ on the turbine side, and the generator torque should be subtracted from the turbine torque. The variables on the turbine side are related to the variables on the generator side by (1.41), or

$$\omega = G\ \omega_T, \quad \tau_{GT} = G\ \tau_G. \tag{1.50}$$

Fig. 1.28 shows the block diagram of a possible implementation. The *Generator model* block should implement (1.49) with (1.50).

Run the simulation with the constant wind speed of 10 m/s and plot the turbine speed in rpm as a function of time over 100 s (reduce from 150 s to match the length of the wind data set). Also plot the generator power in kW as a function of time. Give your interpretation of what happens during the first few seconds of the simulation, and compare the steady-state value of the speed to the value computed in part (d) of Problem 1.3. Finally, compute the efficiency of the system at the end of the simulation as the generated power $P_G$ over the maximum available power $P_W$ (available as the variable *maxpow*).

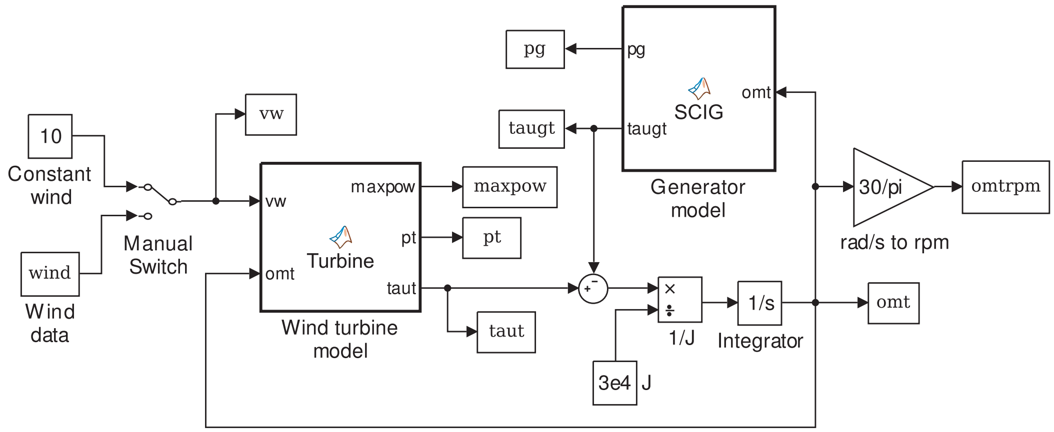


Figure 1.28: Simulation of a wind turbine with a grid-tied SCIG

(c) Run the simulation with the wind data instead of the constant wind speed. Plot the turbine speed (in rpm) and the generated power (in kW) as functions of time over 100 s. Observe that the turbine speed is nearly constant, even with variable wind. This property is due to the steep nature of the torque curve of the generator. Compute the average of $P_G$ and $P_W$. Deduce a measure of the average efficiency of the turbine/generator set. Note that, as the turbine speed is much lower than the value corresponding to $\lambda_{OPT}$ at the prevailing wind conditions, the efficiency is relatively low. A grid-tied squirrel-cage induction generator can only capture the maximum power at a specific wind speed. Finally, compute the total electrical energy produced (in kWh) during the period of the simulation.

(d) For the simulation of part (c), plot the wind speed as a function of time and compute the average wind speed. Compute the average turbine speed, $\omega_T$, and the corresponding values of $\lambda$ and $C_P$. Interpret the results in relation to the efficiency computed in part (c).

$\omega$ is related to the torque $\tau_{PM}$ of the prime mover through the relationship

$$\omega = \omega_U - g_1 \tau_{PM}, \tag{2.48}$$

where $\omega_U$ and $g_1$ are positive constants. Assuming that mechanical losses due to friction are negligible, derive an expression for the droop curve of the generator, i.e., for the function $v(i_L)$.

(b) Repeat part (a) for a prime mover that is controlled so that

$$\omega = \omega_U - g_2 P_{PM}, \tag{2.49}$$

where $P_{PM}$ is the power delivered by the prime mover and $\omega_U$, $g_2$ are positive constants.

(c) Assuming that $g_2 = g_1/\omega_U$, which of the generators of parts (a) and (b) would deliver the highest current if the two generators were placed in parallel?

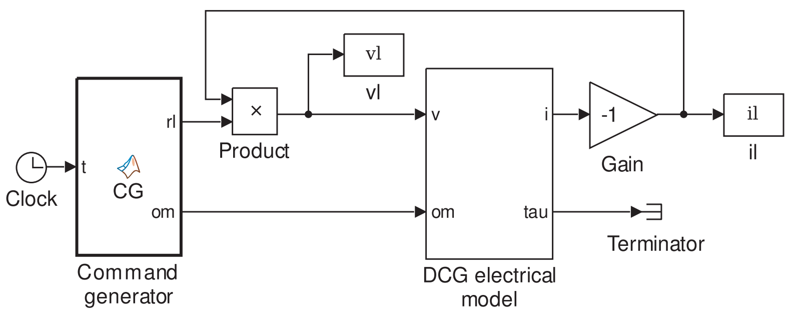


Figure 2.21: Simulation of a DC generator

**Problem 2.4:** the objective of this problem is to measure the droop characteristics of DC generators in simulations. Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 2.21. The *DCG electrical model* block contains the equations of the generator. The *Command generator* block is programmed to vary the load resistance while setting the speed to a constant. Set *Max step size* in *Simulation/Model Configuration Parameters/Solver* to 1e-3, and *Absolute tolerance* to 1e-9.

(a) Set $\omega = 1800$ rpm (convert to rad/s) and $R_L = 120\ \Omega$. After 1 s, vary $R_L$ from 120 $\Omega$ to 10 $\Omega$ over a period of 5 s. Plot $v_L(i_L)$ after removing the first second of data. Relate the droop rate to the parameter $R$ in the *DCG electrical model*.

*Note:* although $R_L$ varies as a function of time, the variation is sufficiently slow that one can plot the steady-state characteristic approximately using the time-varying simulation data.

(b) Modify the simulation to have two identical DC generators in parallel. Fig. 2.22 shows a possible implementation. Let $\omega_1 = \omega_2$ as in part (a). Plot $v_L(i_L)$, $v_L(i_{L1})$, and $v_L(i_{L2})$ on the same graph. Then, repeat the simulation after multiplying the parameter $R$ in the second generator model by a factor of 2. Discuss the results.

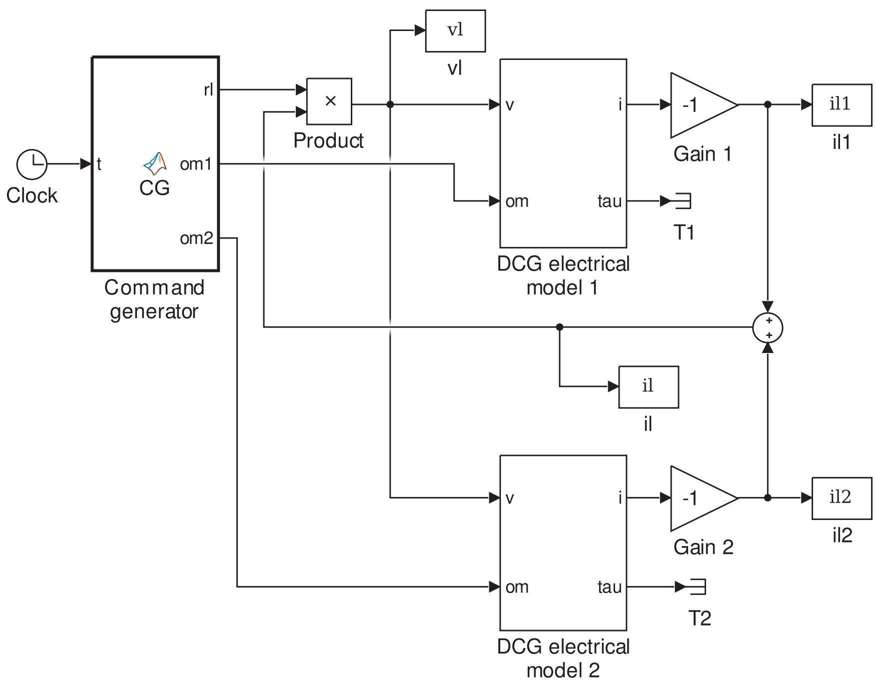


Figure 2.22: Simulation of two DC generators connected in parallel

**Problem 2.5:** (a) A shunt DC generator is a wound-field DC generator where the armature and the field windings are placed in parallel, resulting in the model

$$\begin{aligned} L\frac{di}{dt} &= v - Ri - K_F i_F \omega \\ L_F\frac{di_F}{dt} &= v - R_F i_F. \end{aligned} \tag{2.50}$$

Assuming that the generator is connected to a purely resistive load, one has the additional equation

$$v = -R_L(i + i_F). \tag{2.51}$$

Assume that the speed $\omega$ is constant and that, at $t = 0$, the currents are $i(0) = 0$, $i_F(0) = i_{F0}$. Denoting $I(s) = \mathcal{L}(i)$, the Laplace transform of the current $i(t)$, compute $I(s)$ as a function of $i_{F0}$. Recall the formula

$$\mathcal{L}(\frac{di}{dt}) = s\mathcal{L}(i) - i(0). \tag{2.52}$$

The result should be a transfer function from $i_{F0}$ to $I(s)$ with a denominator of degree 2.

(b) From the transfer function of part (a), compute the speed above which the transfer function is unstable, i.e., the minimum speed required for self-excitation. Determine the number and nature (real or complex) of the unstable pole(s).

**Problem 2.6:** (a) Consider the self-excited series DC generator with the model

$$L_T \frac{di_L}{dt} = -\left(R_T + R_L\right)\ i_L + K_M\ \omega + K_F(i_L)\ i_L\ \omega. \tag{2.53}$$

The term $K_M\ \omega$ was added to (2.40) to represent the effect of residual magnetism in the machine. Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 2.23. The *SEDC electrical model* block contains the equations of the generator. The *Command generator* block is to be programmed to provide the speed profile. Simulate the response of the system when the speed increases linearly from 0 to 3200 rpm in 10 seconds. Plot the voltage on the load $v_L = R_L i_L$ as a function of the speed in rpm and observe the rapid rise of voltage once a certain speed is reached.

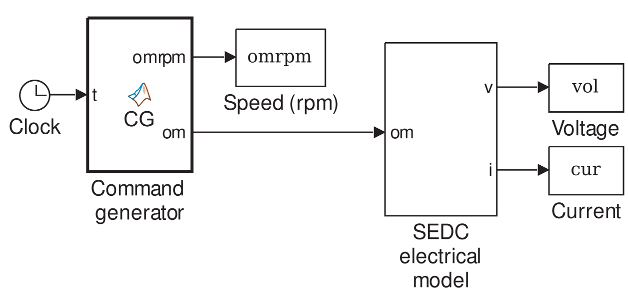


Figure 2.23: Simulation of self-excited DC generator

(b) Compare the value of the speed where the voltage rises rapidly to the value that can be predicted from (2.42) using the parameters of the simulation model (neglecting $K_M$).

(c) Switch the sign of $K_M$ in the simulation and discuss the results.

**Problem 2.7:** consider the system shown on Fig. 2.24 where two identical DC machines are connected in parallel electrically, and the first machine (generator) is driven by a prime mover while the second machine (motor) rotates freely.

In other words, the three-phase reactive power of (3.39) is the two-phase reactive power of (3.25) scaled by 3/2.

(3.106) and (3.107) provide instantaneous estimates of active and reactive power. (3.106) is the true instantaneous active power. However, (3.107) is an extension of the value defined for balanced, steady-state conditions.

### 3.6.3 Relationship to Fortescue's transformation

While Fortescue's transformation was applied to phasors that are constant complex numbers, the 3−2 transformation is applied to real signals that are functions of time, Fortescue's transformation assumes sinusoidal steady-state operation, but the $3-2$ transformation does not.

In sinusoidal steady-state, $v_a$, $v_b$, and $v_h$ are described by phasors $\overline{V}_a$, $\overline{V}_b$, and $\overline{V}_h$. Using (3.96), (3.97), followed by (3.86),

$$\begin{aligned}
\overline{V}_a &= \frac{2}{3}\left(\overline{V}_A - \frac{1}{2}\overline{V}_B - \frac{1}{2}\overline{V}_C\right) = \overline{V}_1 + \overline{V}_2 \\
\overline{V}_b &= \frac{2}{3}\left(\frac{\sqrt{3}}{2}\,\overline{V}_B - \frac{\sqrt{3}}{2}\,\overline{V}_C\right) = j\left(\overline{V}_2 - \overline{V}_1\right) \\
\overline{V}_h &= \frac{\overline{V}_A + \overline{V}_B + \overline{V}_C}{3} = \overline{V}_0.
\end{aligned} \tag{3.108}$$

In other words, the variables computed through the $3-2$ and Fortescue transformations are closely related.

When applied to systems, the $3-2$ transformation decouples the homopolar variable from the other variables, as Fortescue's transformation. However, both two-phase variables generally remain different from zero, even if only the positive sequence component is present

Fortescue's transformation is typically used for the analysis of faulted power systems and the design of protection circuits assuming sinusoidal steady-state [1]. On the other hand, the 3−2 transformation is used to analyze transient dynamics and to implement real-time control systems. Several examples of application will be presented in the following chapters.

## 3.7 Problems

**Problem 3.1:** the objective of this problem is to explore concepts of single-phase power.

(a) Create a *Simulink* diagram to simulate the response of a series RL circuit with a voltage $v(t)$ as the input and a current $i(t)$ as the output. Fig. 3.22 shows an implementation using blocks from *generator_blocks.mdl* available on the web site. The block *Sinusoidal excitation* produces a signal

$$v(t) = V_{pk}\cos(\omega_S t), \tag{3.109}$$

where $\omega_S = 2\pi f_S$. Simulate the response of the system for $V_{pk} = 2$ V, $f_S = 60$ Hz, $R = 1\ \Omega$, and $L = 3$ mH. Let the simulation time be 0.1 s.

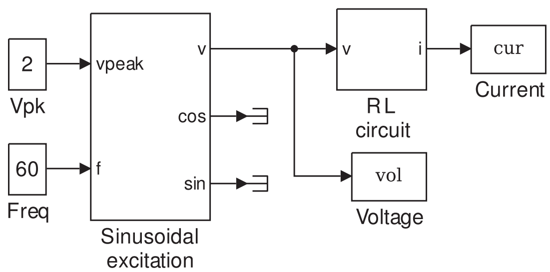


Figure 3.22: Simulation of an RL circuit with sinusoidal excitation

(b) Add a block to compute the instantaneous power absorbed by the RL circuit, i.e.,

$$p(t) = v(t)i(t). \tag{3.110}$$

The average power can be estimated in discrete-time by computing

$$P(n) = \frac{1}{N}\sum_{k=n+1-N}^{n} v_d(k)i_d(k), \tag{3.111}$$

where $v_d(k)$ and $i_d(k)$ are sampled values of $v(t)$ and $i(t)$, and $N$ is the number of samples over which averaging takes place ($N$ should be a multiple of the period of the signal $p(t)$).

The averaging formula can be implemented as a recursive algorithm using

$$P(n) = P(n-1) + \frac{v_d(n)i_d(n) - v_d(n-N)i_d(n-N)}{N}. \tag{3.112}$$

Fig. 3.23 shows a *Simulink* diagram (from the file *generator_blocks.mdl* available on the web site) of an implementation of the recursive filter (3.112) with $N = 167$. The averaging window of 167 samples corresponds to one period of a 60 Hz signal (or two periods of the instantaneous power signal) at a sampling frequency of 10 kHz. The sampling period $T_S = 10^{-4}$ s should be set in the delay blocks, or as a parameter in *Matlab*'s workspace.

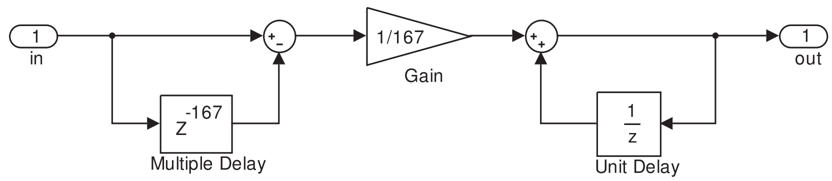


Figure 3.23: Averaging filter

Add the averaging filter to the *Simulink* diagram of the RL circuit to compute the average power. Fig. 3.24 shows a *Simulink* block diagram that includes the averaging filter, together with a solution for parts (b)-(d).

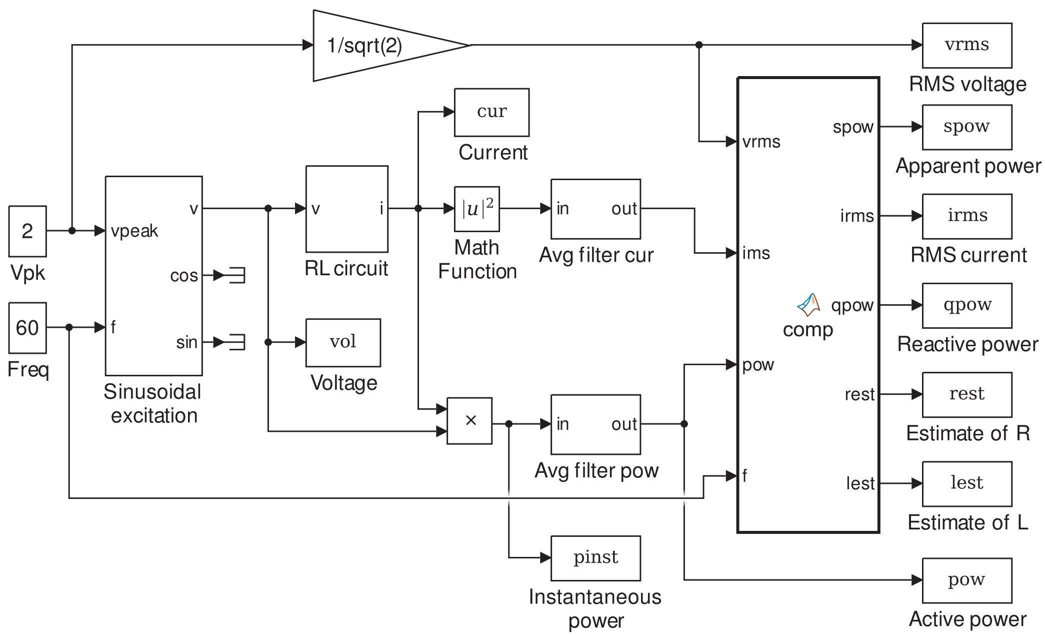


Figure 3.24: *Simulink* diagram for Problem 3.1

(c) Add blocks to compute $V_{rms} = V_{pk}/\sqrt{2}$ and an estimate of $I_{rms}$ using

$$I_{rms} = \sqrt{AVG(i^2(t))}, \tag{3.113}$$

implementing the same averaging filter as for the power. Then, compute the values of the apparent power

$$S = V_{rms} I_{rms} \tag{3.114}$$

and of the reactive power

$$Q = \sqrt{S^2 - P^2}. \tag{3.115}$$

(d) In theory, one should have

$$P = RI_{rms}^2, \quad Q = 2\pi f_S \, LI_{rms}^2. \tag{3.116}$$

In the *Simulink* diagram, use these formulas to compute estimates of $R$ and $L$ based on the estimates of $P$ and $Q$. To avoid division by zero at $t = 0$, replace the estimate of $I_{rms}$ by 0.1 when the estimate of $I_{rms}$ is smaller than 0.1.

(e) Plot $v(t), i(t)$, and $p(t)$, as well as the estimates of $R$ and $L$ as functions of time. Compare the values observed in steady-state to those used in the simulation.

**Problem 3.2:** show that in a balanced three-phase system with voltages (3.30) and currents (3.36), the instantaneous reactive power given by (3.39) is constant and satisfies (3.40).

*Hint:* the following equalities may be helpful

$$\begin{aligned} \cos(a)\cos(b) &= \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b) \\ \cos(a) - \cos(b) &= 2\sin(\frac{a+b}{2})\sin(\frac{b-a}{2}). \end{aligned} \tag{3.117}$$

**Problem 3.3:** Fig. 3.25 shows the representation of an ideal transformer with voltages $v_1$ and $v_2$ on the primary and secondary sides. The notation $1 \mid \alpha$, means that there are $\alpha$ turns in the secondary for every turn in the primary. In other words, $\alpha$ is the transformer ratio and, ideally, $v_2 = \alpha v_1$.

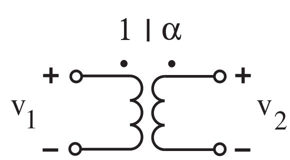


Figure 3.25: Ideal transformer

(a) The circuit shown in Fig. 3.26 is used in the U.S. to provide dual single-phase service to residential customers based on a three-phase distribution line. The voltages $v_A$, $v_B$, and $v_C$ are line-to-ground voltages given by (3.30). The ground (zero voltage) is connected to the center tap of the secondary of the transformer. Compute the line-to-ground voltages $v_{A1}$ and $v_{A2}$. Find the value of $\alpha$ such that the peak voltage of $v_{A1}$ is the same as the peak voltage of $v_A$.

(b) In the circuit shown in Fig. 3.27, the three-phase voltages are the same as in part (a). The transformer on the left is connected to the center tap of the transformer on the right. The transformer ratios are $\alpha$ and $\beta$ and are taken to be

(c) Is it possible to vary the angle $\varphi$ as a function of the power $P$ so that $V$ is nonzero and independent of $P$? In that case, what is the maximum power $P$ that can be drawn by the load?

**Problem 4.3:** the objective of this problem is to simulate a PMSG at constant speed with a variable load. Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 4.17. The *Power Voltage Current* block provides estimates of the peak voltage, peak current, active power, and reactive power using (3.43), (3.44), (3.35), and (3.39), with signs of power changed to give the generated powers.

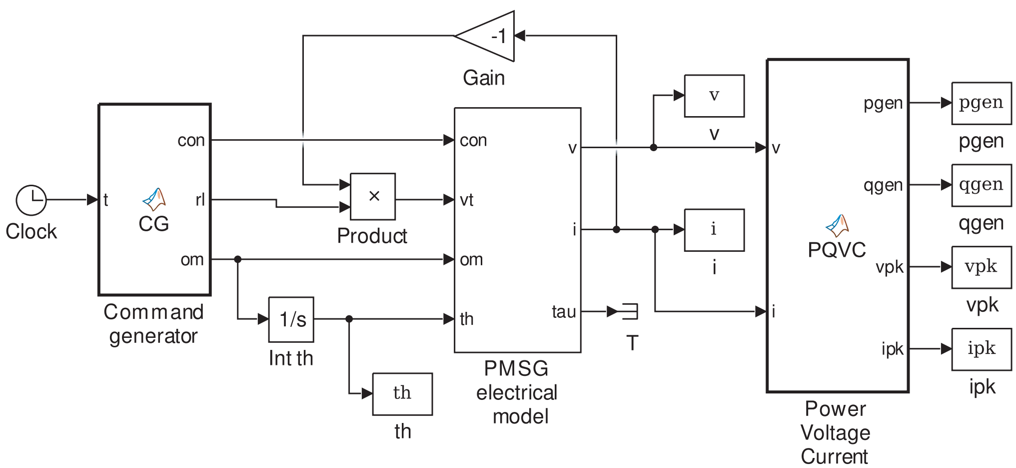


Figure 4.17: Simulation of a PMSG with resistive load

(a) Let $\omega = 22.5$ rpm (convert to rad/s), leaving the generator disconnected from the resistive load (set the connection variable *con* to 0). Plot the three-phase voltages over a 0.5 s segment. Relate the frequency of the voltages to the speed of rotation and the number of pole pairs found in the simulation model. Estimate $V_{pk}$, the peak magnitude of the back-emf voltage, and relate $V_{pk}$ to the back-emf constant found in the simulation model. Plot $v_A(t)$ together with $-V_{pk}\ \sin(n_P\theta)$ (use a dashed line for the second variable), and observe that the signals are the same.

(b) Let $R_L = 2.5$ ohms and connect the generator to the resistive load by setting $con = 1$ at $t = 1$ s. After another second, vary the load resistance $R_L$ from 2.5 to 0.25 ohms over a period of 5 s. Plot $V_{pk}(I_{pk})$ after removing the first two seconds of data. For the final value of $R_L$, give the value of the line-to-line rms voltage (in V), the generated power (in MW), and the power factor.

(c) Replace the resistive load by an *RC* load (resistor in parallel with a capacitor). A block is available in *generator_blocks.mdl* and a *Simulink* diagram is shown on Fig. 4.18. Vary $R_L$ as in part (b) and let $C$ (in F) $= 0.009\ /R_L$ (in ohms). Show that the choice of $C$ corresponds to setting a constant power factor and give its value. Plot $V_{pk}(I_{pk})$ as in part (b). For the final value of $R_L$, give the values of the line-to-line rms voltage (in V), the generated power (in MW), the power factor, and the capacitor (in mF). The values should be similar to those of the example on p. 86. Comment on the results, in particular the impact of the additional capacitor on the voltage and power generated on the side of the load.

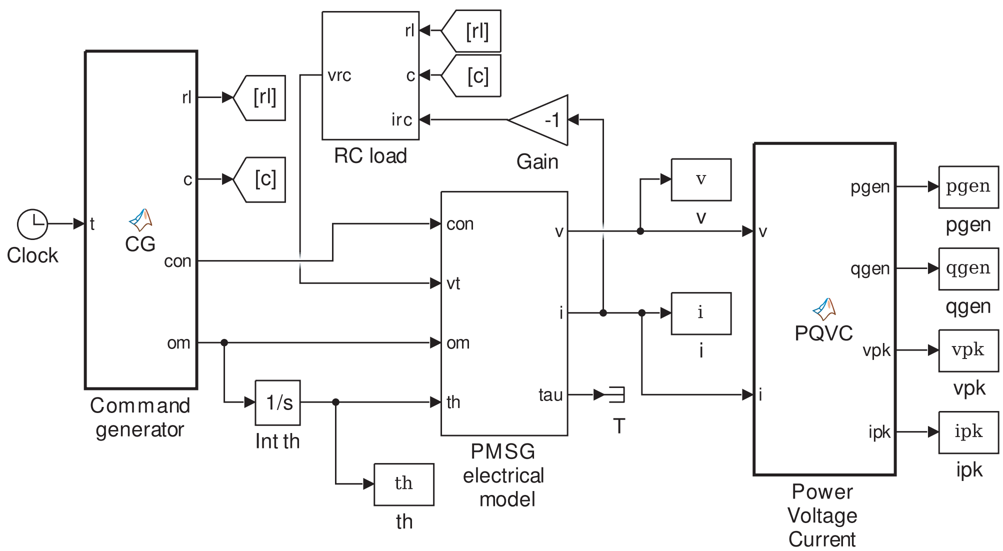


Figure 4.18: Simulation of a PMSG with $RC$ load.

**Problem 5.2:** (a) Determine the value of the capacitance that is needed in Fig. 5.10 so that the capacitor provides 90% of the reactive current for $R = 0.3\ \Omega$, $L = 0.014$ H, and $f_S = 60$ Hz. Compare the value to the value of $C$ such that 60 Hz is the resonant frequency of the circuit.

(b) Repeat part (a) at 120 Hz.

**Problem 5.3:** (a) Consider the circuit shown in Fig. 5.26, representing a simplified model of a self-excited induction generator in steady-state. Use the loop impedance method to obtain conditions describing the onset of self-excitation. Compute the impedance for the loop shown on the figure, where $M$ is in parallel with the two resistances on the right. Derive equations relating the system variables when the impedance is zero. From these equations, deduce expressions for the electrical frequency $\omega_S$, the speed $\omega$, and the range of capacitance $C$ for self-excitation.

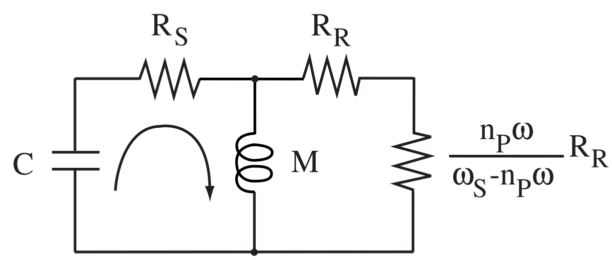


Figure 5.26: Simplified steady-state model of an SEIG

(b) Apply the results of part (a) to an induction machine with $R_S = 0.3\ \Omega$, $R_R = 0.7\ \Omega$, $M = 14$ mH, and $n_P = 2$. For $C = 100\ \mu$F, compute the electrical frequency (in Hz), the speed in rpm, the normalized slip (in %) at the onset of self-excitation, and the range of capacitance for which self-excitation is possible.

**Problem 5.4:** the objective of this problem is to simulate a grid-tied SCIG in conditions similar to the example of p. 108. The active and reactive powers are plotted as functions of speed, similarly to Figs. 5.5 and 5.6.

Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 5.27. The machine is rated for 380 V (line-to-line rms), 50 Hz operation, and has two pole pairs. The simulation should start with the generator connected to the grid ($con = 1$) and run for 6 s.

(a) Find a speed such that the machine acts as a motor and produces a mechanical power equal to approximately to 2.2 kW in steady-state. Limit the search to speeds that are integer numbers in rpm. Give the values of the speed (in rpm),

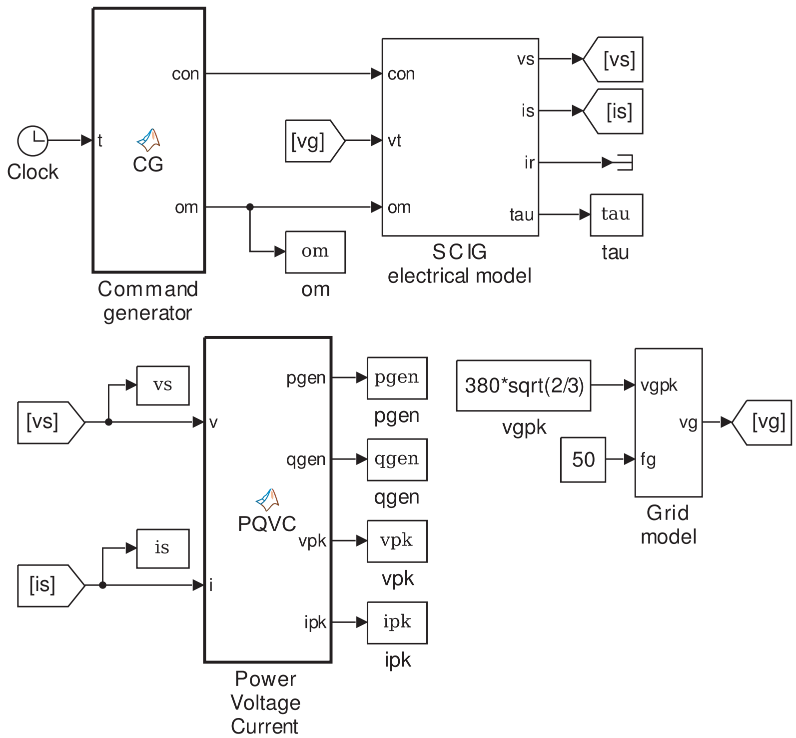


Figure 5.27: Simulation of a grid-tied SCIG

the slip (in %), the mechanical power produced, the electrical power absorbed, the reactive power absorbed, and the rms current at the end of the simulation.

(b) Find a speed such that the machine acts as a generator and produces an electrical power equal to approximately to 2.2 kW in steady-state. Give the values of the speed (in rpm), the slip (in %), the mechanical power absorbed, the electrical power generated, the reactive power generated, and the rms current at the end of the simulation.

(c) Start the simulation at a speed equal to 85% of the synchronous speed. A second into the simulation, raise the speed from 85% to 115% of the synchronous speed over a period of 5 s. Plot $P_{GEN}(\omega)$ and $Q_{GEN}(\omega)$ with the speed labelled in rpm, and removing the initial second of the simulation so that the electrical transients do not affect the plots. Use the peak value of $Q_{GEN}$ to find an estimate of $L_S$ using (5.25).

**Problem 5.5:** the objective of this problem is to observe self-excitation in a squirrel-cage induction generator. Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 5.28. A *Power Voltage Current* block with storage of the variables should also be included, but is not shown. Set $R_L = 65\ \Omega$ and $C = 55\ \mu F$.

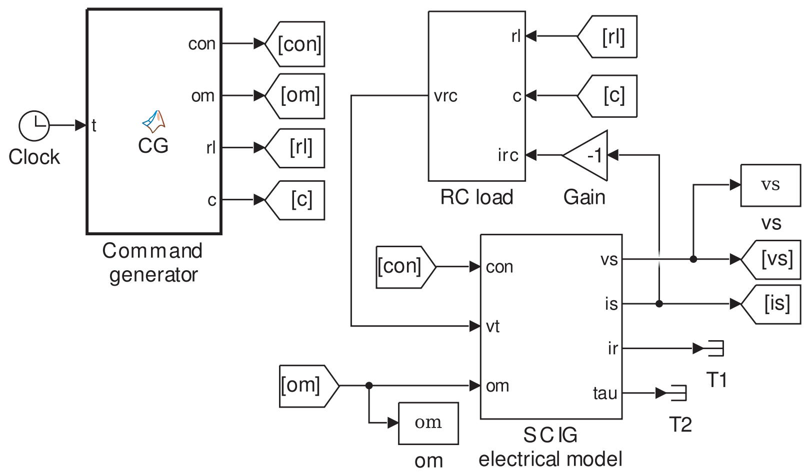


Figure 5.28: Simulation of an SEIG

(a) Vary the speed from 1400 rpm to 1600 rpm over 10 s. Plot $V_{pk}$ as a function of $\omega$ in rpm and observe the rapid growth of the voltage once a certain speed is reached.

(b) Set the speed at 1500 rpm and plot the voltage of phase $A$ of the generator as a function of time. Then, plot the voltage over about 10 periods of the signal after 9 s. Observe that the voltage appears sinusoidal, despite the operation in magnetic saturation. Estimate the frequency (in Hz) and the rms line-to-line voltage.

(c) Self-excitation begins at a lower speed (around 1200 rpm), but the build-up of the voltages takes a much longer time. Still, the stability or instability of the system at a given speed can be determined by zooming on the plot of $V_{pk}$ as a function of time. By trial-and-error, determine within 1 rpm the speed at which self-excitation begins. Plot $V_{pk}$ as a function of time for the two speeds bordering the unstable behavior.

**Problem 5.6:** (a) A $Y-$connected load consists of three identical resistors placed in parallel with three identical capacitors. What value of resistance corresponds to a total absorbed power of 2.2 kW for a line-to-line rms voltage of 380 V?

(b) Let the load of part (a) be connected to a squirrel-cage induction generator. Consider the simplified per-phase model of Fig. 5.15 with the parameters $M = 0.2$ H, $R_R = 1.87\ \Omega$, and $n_P = 2$. Assume that the generator reaches steady-state self-excitation for a speed of 1500 rpm. What is the normalized slip $S_n$ of line rms back-emf voltage). The range of power in the plot should span from 0 to 900 MW.

*Note:* be careful with the fact that equations of the book assume that voltages are peak line-to-neutral voltages and currents are peak line currents.

**Problem 6.2:** the objective of this problem is to compute the edges of the reactive power capability curve of Fig. 6.20 for $P \geqslant 0$ and $Q \geqslant 0$. The limits $L1$ and $L2$ are given by (6.43) and (6.44), where $V$ and $E_{MAX}$ are peak line-to-neutral voltages and $I_{MAX}$ is a peak line current. Consider a generator with a grid voltage equal to 26 kV (line-to-line rms), 60 Hz grid frequency, $L = 0.0025$ H, and maximum stator current of 22 kA (line rms).

(a) Give the value of the back-emf voltage $E$ (peak line-to-neutral) needed to connect the generator to the grid with zero power output (origin of Fig. 6.20).

(b) Compute the maximum active power that can be generated and the values of $Q$ and $E$ needed to obtain that power (point $P3$).

(c) Assume that $E$ is adjusted so that the curves $L1$ and $L2$ intersect at an operating point with PF $= 0.9$. Find the values of $P$, $Q$, and $E$ at that condition (point $P2$)

(d) Find the maximum value of $Q$ that can be produced for the value of $E = E_{MAX}$ obtained in part (c), and give the corresponding power $P$ (point $P1$).

**Problem 6.3:** consider the droop control scheme shown on Fig. 6.25, letting $P_G = 0$ and $P_F = 0$. Assume that the mechanical dynamics are modeled by a transfer function $H(s) = N(s)/D(s)$, where $N(s)$ and $D(s)$ are polynomials in $s$.

(a) Compute $\omega(s)$ as a function of $\omega_{REF}(s)$ and $P_{REF}(s)$.

(b) Give the polynomial equation that specifies the closed-loop poles of the system

(c) Assuming that $R = 0$ and that $H(s)$ is approximated by the DC gain $H(0)$, show that the system has a single pole and find the value of $k_I$ such that the pole is placed at $s = -a_D$.

(d) Under the same approximation for $H(s)$ but $R \neq 0$, compute the pole of the system for $k_I$ computed in part (c).

(e) For the system $H(s) = 20/(s + 10)$, find the $k_I$ of part (c) for $R = 0$ and $a_D = 1$ rad/s. Compute the approximate pole of part (d) for the same $k_I$ and $R = 0.1$. Using part (b), compute the exact poles for the same $k_I$ and $R = 0.1$.

**Problem 6.4:** the objective of this problem is to simulate the operation of a WFSG including synchronization, the effect of varying excitation, and the edges of the operating limits. Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 6.44. The PQVC block is not shown, but should also be included. The number of pole pairs of the generator is $n_P = 1$, so that the synchronous speed associated with 60 Hz is 3600 rpm. The prime mover model is such that the speed is 3600 rpm for $P_{PM} = 0$, and increases by 0.1 pu in steady-state for every 1 pu increase in $P_{PM}$. The model of damping included with the prime mover ensures a stable system, but is not meant to be accurate.

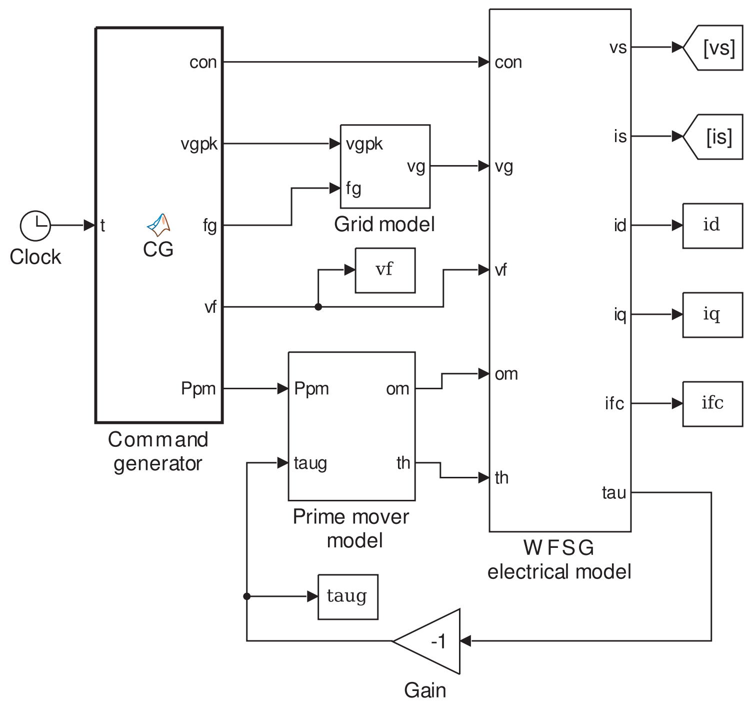


Figure 6.44: Simulation of a WFSG

(a) With the generator disconnected from the grid (variable *con*=0), set the peak value of the grid voltage (*vgpk*) so that the line-to-line grid voltage is 26 kV. Let the frequency (*fg*) be 60 Hz. Set the voltage $v_F = 291$ V and the prime mover power $P_{PM} = 0.02$ pu. Run the simulation for 3 s and verify that the generator reaches a speed 0.2% ($0.1 \times 0.02$) higher than the synchronous speed with this power setting. Observe the three-phase voltages of the generator and of the grid, and note that $v_F$ was chosen so that the voltage magnitudes are approximately the same. Find a time where the voltages are the closest (around 2 s) and connect the generator to the grid by setting the variable *con* to 1 at that time. For this purpose, a practical method consists in plotting $v_{SA} - v_{GA}$ and identifying a time where the waveform is closest to zero. Run the simulation again for 15 seconds and plot $\omega$ in rpm as a function of time. Observe that

the speed drops to the synchronous speed after the connection, although the response is quite oscillatory. Plot $P_{GEN}$ and $Q_{GEN}$ as functions of time (in MW and MVAR) and observe that the generated power is positive in steady-state. Verify that the steady-state power is 2% of the base power (equal to 991 MW).
(b) Extend the simulation to 75 s. From $t = 15$ s to $t = 75$ s, raise the voltage $v_F$ from 291 V to 554 V. Using the new 60 s of data, plot the generated active and reactive powers (in MW and MVAR) as functions of the field current (in kA). The field current is labelled *ifc* in the simulation, Then, plot the rms stator current (in kA) as a function of the field current. Note that this plot represents one half of a V-curve. Comment on the results.
(c) The last experiment is designed to follow the reactive power capability curve, reaching $(P, Q)$ coordinates at $(0.02 \text{ pu}, 0)$, $(0.02 \text{ pu}, Q_{MAX})$, $(P_{rated}, Q_{rated})$, and $(P_{MAX}, 0)$. The values are close to those of Problem 6.2. Extend the total simulation time to 210 s. The first two coordinates were obtained in parts (a) and (b). For the third coordinate, keep $v_F$ at 554 V and raise the power of the prime mover from 0.02 pu to 0.9 pu between $t = 80$ s and 140 s. Then, between $t = 145$ s and $t = 205$ s, bring $v_F$ down from 554 $V$ to 464 V and $P_{PM}$ from 0.9 to 1 pu. The fourth coordinate is then reached. Using data from 15 s to 210 s, plot the generated active and reactive powers as functions of the prime mover power. Also plot the reactive power as a function of the active power. The plot should draw approximately the reactive power capability curve in the first quadrant of Fig. 6.20. Comment on the results.

The control problem corresponds to the diagram shown on Fig. 7.12. The dynamics of the system to be controlled are those of an integrator with a negative gain. For control design, the gain of the system can be made positive by switching the sign of $P_{REF}$ before sending it to the PQ control law. The input power $P_{IN}$ is handled as a disturbance to be rejected.

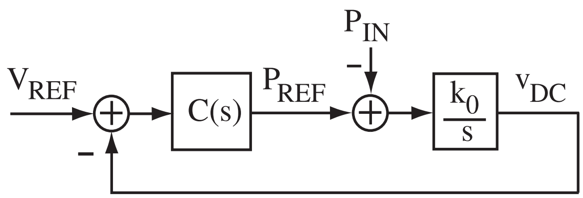


Figure 7.12: Control system for DC bus voltage regulation

## 7.5 Problems

**Problem 7.1:** the objective of the problem is to simulate the responses of a three-phase converter with DQ control. The converter is connected to a 575 V (line-to-line rms), 60 Hz grid. A *Simulink* implementation is shown in Fig. 7.13. Most of the blocks are available from the file *generator_blocks.mdl*, specifically the *DQ*, *(DQ)-1*, *Grid model*, and *Grid sync* blocks. The *Three-phase RL circuit* block is an extension of the single-phase RL circuit, which is obtained by replacing the scalar variables by 3-dimensional vectors. For this purpose, the initial condition of the integrator is replaced by the vector $[0; 0; 0]$. The variables *edq* and *idq* are two-dimensional vectors containing the d and q variables. The variables *vg*, *vrl*, and *irl* are three-dimensional vectors representing $v_{ABC}$, $v_{ABC} - e_{ABC}$ (the voltage applied to the RL filter), and $i_{ABC}$.

The resistances and the inductances of the RL elements should be changed to $R = 0.001\ \Omega$ and $L = 0.0001$ H. The $P_{REF}$ and $Q_{REF}$ blocks are step functions from the *Simulink* library. The values should be set to zero initially, with steps of $P_{REF} = 250$ kW and $Q_{REF} = 120$ kVAR applied at $t = 0.5$ s and $t = 1.5$ s, respectively. The main task is then to design the *DQ control* block, which includes the power control and current control algorithms of Sections 7.3.4 and 7.3.5.

Implement the current control scheme, setting $k_{IC}$ to zero and $k_{PC}$ to a value such that the closed-loop pole of the control loop is located at $s = -100$ rad/s. Add the feedforward power control system. Compute $P_{GEN}$, $Q_{GEN}$ using (7.42)

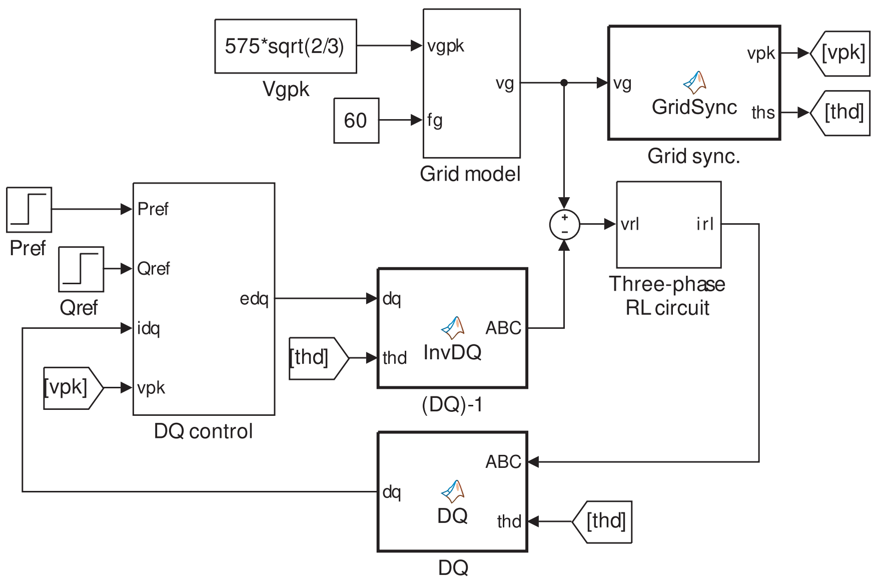


Figure 7.13: *Simulink* diagram of a three-phase inverter and associated controller for active and reactive power

or a *Power Voltage Current* block (PQVC). Run the simulation for a period of 2.5 s and plot $P_{GEN}$, $P_{REF}$, $Q_{GEN}$, and $Q_{REF}$ as functions of time (in kW and kVAR). The outputs should track the references with a first-order time constant corresponding to the choice of closed-loop pole.

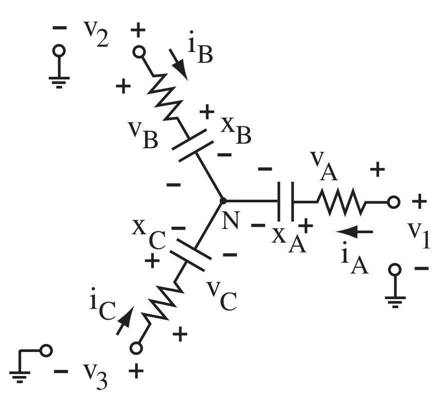

Figure 7.14: Three-phase RC network

**Problem 7.2:** (a) Consider the circuit of Fig. 7.14. Assume that the three resistors have the same value $R$, and that the three capacitors have the same value $C$. Let $x_A$, $x_B$, and $x_C$ be the voltages on the capacitors, $v_A$, $v_B$, and $v_C$

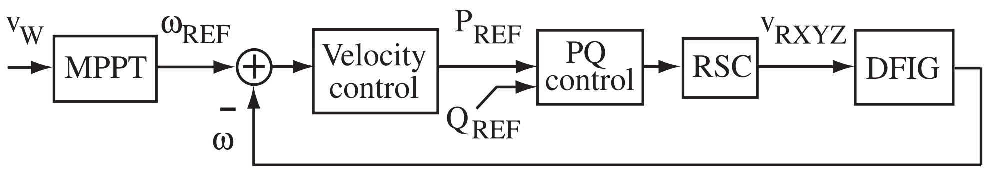


Figure 8.13: Speed control of a DFIG for maximum power point tracking

## 8.5 Problems

**Problem 8.1:** the objective of this problem is to simulate the responses of a DFIG with a basic control scheme and to demonstrate the independent control of the active and reactive powers. The properties of the DFIG are observed in idealized conditions, with responses matching the example of p. 194.

Using the file *generator_blocks.mdl* available on the web site, build the *Simulink* diagram shown on Fig. 8.14. *Power Voltage Current* (PQVC) blocks should also be added for the stator and for the rotor variables, but are not shown. The *DFIG PQ control* block implements the power control law (8.86) together with the rotor current control law (8.82) where $R_S$ is neglected, $k_{PC} = 100$, $k_{IC} = 0$, and $\overline{u}_R$ is simplified to the first terms of (8.81), i.e.,

$$\overline{u}_R = R_R\overline{i}_R + j\omega_R(L_R\overline{i}_R + M\overline{i}_S). \tag{8.92}$$

The control parameters are such that the poles are placed at $-100$ rad/s.

The stepwise commands $P_{COM}$, $Q_{COM}$, produced by the *Command generator* are filtered by transfer functions $20/(s + 20)$ to provide smoother references $P_{S,REF}$, $Q_{S,REF}$, to the PQ controller. Set the variable *vgpk* such that the line-to-line rms voltage of the grid is 575 V. Let the speed of the generator be 120% of the synchronous speed for a 60 Hz grid and for a generator with three pole pairs.

(a) Start with the system disconnected (variable *con*=0) and with $P_{COM} = 0$, $Q_{COM} = 0$. Run the simulation for 2 s and plot the stator voltage (converted from peak line-to-neutral to line-to-line rms). Observe that the stator voltage magnitude converges to the grid value.

(b) Connect the generator to the grid at 2 s and run the simulation for 5 s. The transient at the time of connection should be small, as the parameters of the control law are the same as those in the DFIG model. Apply commands $P_{COM} = 1.3$ MW at 2.5 s and $Q_{COM}$ corresponding to a power factor PF=0.9 at 3.5 s. Bring $P_{COM}$ and $Q_{COM}$ back to zero at 4 s. Plot the generated active

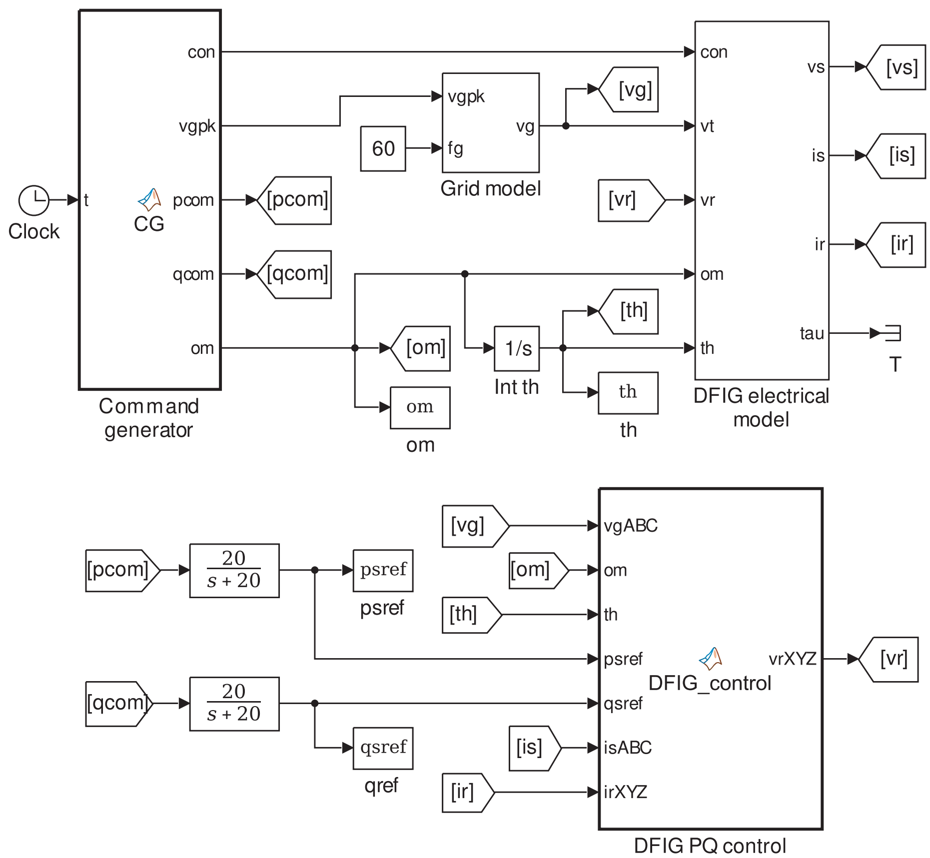


Figure 8.14: Simulation of a DFIG with DQ control of active and reactive power

and reactive powers (in kW and kVAR) together with their commands over a period of 3 s after connection of the generator. Next, plot the active and reactive powers for the rotor. Then, plot the active powers generated by the stator, by the rotor, and by the sum of the stator and rotor (in kW). Observe that the machine generates slightly over 1.5 MW (total power). Finally, plot the stator and rotor currents (converted to Arms) over the same period. Notice that the rotor current is significantly smaller than the stator current, which means that a reduced size converter can be used.

(c) Starting at $t = 5$ s, bring the speed of the generator to the synchronous speed linearly over 1 s. Keep the speed synchronous for one second. From $t = 7$ s to 8 s, bring the speed from 100% to 80% of the synchronous speed over 1 s, and leave the speed at 80% afterwards. Run the simulation for 8.5 s. Plot the stator and rotor voltages from 4.5 s to 8.5 s (converted from peak line-to-neutral to line-to-line rms). Observe that the magnitude of the stator voltage is at the grid value. The rotor voltage is approximately proportional to the absolute value of the slip, and is smaller than the grid voltage with the limited range of slip of the simulation. Finally, plot the first two phases (X and Y) of the rotor voltages and relate the forward/backward sequence of the voltages to the sign of the slip. Comment on the results.

In the second step, the feedback loop is closed with a small gain $g$ and the gain is progressively increased. Hopefully, the feedback improves or at least does not degrade the response of the local mode. Tuning is stopped when oscillations appear in the response. Then, the gain is reduced by a factor of 3 and the PSS is considered tuned. More details on the procedure can be found in [36]. It is interesting to note that this design procedure could be applied to many other systems with needs for active damping.

## 9.5 Problems

**Problem 9.1:** the objective of this problem is to observe the short-circuit current of a wound-field synchronous generator. Build the simulation model of Problem 6.4 and, having connected the generator to the grid with $v_F$ at 291 V and $P_{PM}$ at 0.02 pu in part (a), drop $P_{PM}$ to 0 at $t = 15$ s. The stator currents should converge to zero after a few seconds. Apply a short-circuit at $t = 30$ s by setting *vgpk* to zero in the simulation. Plot the currents $-i_d(t)$ and $-i_q(t)$ (in kA) for 3 s after the short-circuit. Estimate the ratio $i_d(0)/i_d(\infty)$ and the time constant of the response. Compare the results to values computed using (9.38) with the simulation parameters. Also plot $i_A(t)$ together with $i_d(t)$ and $-i_d(t)$ (in the same plot but with a different color than $i_A$), and confirm that $\pm i_d(t)$ gives the envelope of the stator current.

**Problem 9.2:** the objective of this problem is to estimate the critical clearing time of a wound-field synchronous generator. Build the simulation model of Problem 6.4 and, having connected the generator to the grid with $v_F$ at 291 V and $P_{PM}$ at 0.02 pu in part (a), raise $v_F$ to 464 V and $P_{PM}$ to 1 pu at $t = 15$ s. At $t = 30$ s, disconnect the generator from the grid and reconnect it after a predetermined period of time (switching the *con* variable to 0 and then back to 1). Through trial and error looking at the speed response, find an estimate of the critical clearing time as a multiple of the period of the grid voltages. The number of cycles should be between 18 and 20. Plot the active and reactive powers as functions of time (in MW and MVAR) for the highest number of cycles such that the generator resynchronizes, and for a number of cycles greater by one. Also plot the speed in rpm and the angle $\delta$ in degrees for both cases (where $\delta$ is computed using (9.7) and $\theta_S$ is the grid angle *thg* found inside the grid model). Deduce an estimate of the critical clearing time in seconds. Let the time scale for the plots span 10 s from the time of disconnection, but limit the scale to 1 s for the second plot of $\delta$ to better view the response. Comment on the results.

*Note:* power transients are very large and the clearing time observed in the simulation is longer than the time predicted by the theory in (9.99). A major difference is that the analytical formula assumes a constant current $i_F$, while the simulation implements a constant voltage $v_F$. In practice, various elements can affect the clearing time. Nevertheless, the analysis and the simulation make it possible to observe the basic features of the disconnection and reconnection of a synchronous generator.

**Problem 9.3:** assume that an AC voltage source $v_S$ is connected to a WFSG so that $v_A = v_S$, $v_B = v_C = 0$. The source current is $i_S = i_A = -i_B - i_C$. The machine is at standstill and the field winding is short-circuited.

(a) Using the definition of the DQ transformation (9.2), show that

$$v_d + jv_q = \frac{2}{3} v_S e^{-jn_P\theta}, \; i_d + ji_q = \left( i_S + j\frac{i_B - i_C}{\sqrt{3}} \right) e^{-jn_P\theta}. \tag{9.121}$$

(b) Show that $v_S = 3/2 \; v_d$ and $i_S = i_d$ when $\theta = 0$. In other words, the impedance corresponding to $v_S/i_S$ is $3/2 \; Z_d$ for this alignment of the rotor.

(c) Show that $v_S = 3/2 \; v_q$ and $i_S = i_q$ when $\theta = -\pi/(2n_P)$. In other words, the impedance corresponding to $v_S/i_S$ is then $3/2 \; Z_q$.