ECE 5670/6670 – Lab 8

Torque Curves of Induction Motors

Objectives

The objective of the lab is to measure the torque curves of induction motors. Acceleration experiments are used to reconstruct approximately the torque curves, assuming that the load torque is constant and that a steady-state approximation is valid. The dependency of the torque on the electrical frequency is also investigated.

1. Introduction

The dynamic response of an induction motor is considerably more complex than the response of a brush DC motor or a permanent-magnet synchronous motor. A simple model includes at least five nonlinear differential equations. However, the torque curves obtained under steady-state assumptions provide sufficient information for the design of slip control drives.

Consider a two-phase induction motor with sinusoidal phase voltages

$$\begin{aligned} v_A &= V_s \cos(\omega_e t) \\ v_B &= V_s \sin(\omega_e t) \end{aligned} \tag{1}$$

where V_S is the peak voltage and ω_e is the *electrical frequency*. Assume that the electrical machine is in sinusoidal steady-state and that the speed is constant. Then, the torque is equal to

$$\tau_e(\omega, \omega_e) = \frac{n_p M^2 V_s^2}{R_R} \frac{\omega_e - n_p \omega}{\left(R_s - \sigma L_s \omega_e T_R(\omega_e - n_p \omega)\right)^2 + \left(R_s T_R(\omega_e - n_p \omega) + L_s \omega_e\right)^2} \tag{2}$$

Although equation (2) is only valid under the steady-state assumptions, it is useful in other cases as well, provided that the electrical and mechanical frequencies vary slowly.

Fig. 1 shows a set of torque curves for a typical induction motor. The torque is plotted as a function of the speed ω, expressed in rpm, for a given electrical frequency (the three curves correspond to electrical frequencies of 20 Hz, 40 Hz, and 60 Hz respectively). For the motor under consideration, the number of pole pairs is $n_p = 1$, and a frequency of 60 Hz corresponds to a *synchronous speed* $\omega_e = n_p\omega$ equal to 3,600 rpm. Note that the torque goes to zero when the speed reaches the synchronous speed.

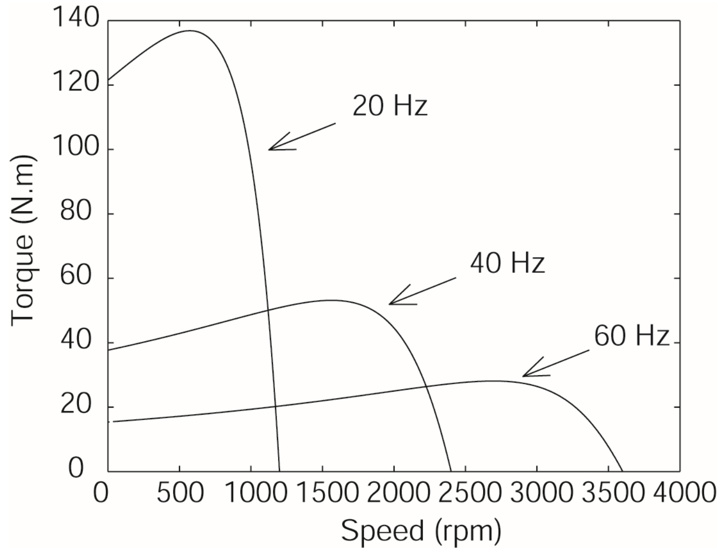

Figure 1: Torque Curves of an Induction Motor

Close to the synchronous speed, the torque is approximately linear in the difference between ω_e and $n_p\omega$. This variable is called the *slip*. Specifically, for small slip, the torque is approximately given by

$$\tau_e(\omega,\omega_e) = \frac{n_p M^2 V_s^2}{R_R}\frac{\omega_e - n_p\omega}{R_s^2 + (L_s\omega_e)^2} \tag{3}$$

Equation (3) as well as Fig. 1 show that the torque decreases rapidly when the electrical frequency increases.

2. Experiments

You will need:

- Induction motor,
- Standalone encoder,
- Dual power amplifier,
- Cable rack,
- Encoder cable
- A metal frame to mount the motors on, with a box of screws and a screwdriver.

2.1 Torque Curves

Download the files *Lab8.mdl* and *Lab8.lax*.

The experiment applies two-phase voltages of the form (1) to the motor. The operator enters values for the peak voltage and for the electrical frequency expressed in terms of the synchronous speed (in rpm). The induction motor has 1 pole pair (n_p = 1), so that a frequency of 60 Hz corresponds to a synchronous speed of 3,600 rpm. 60 Hz is the nominal frequency for the motor under consideration.

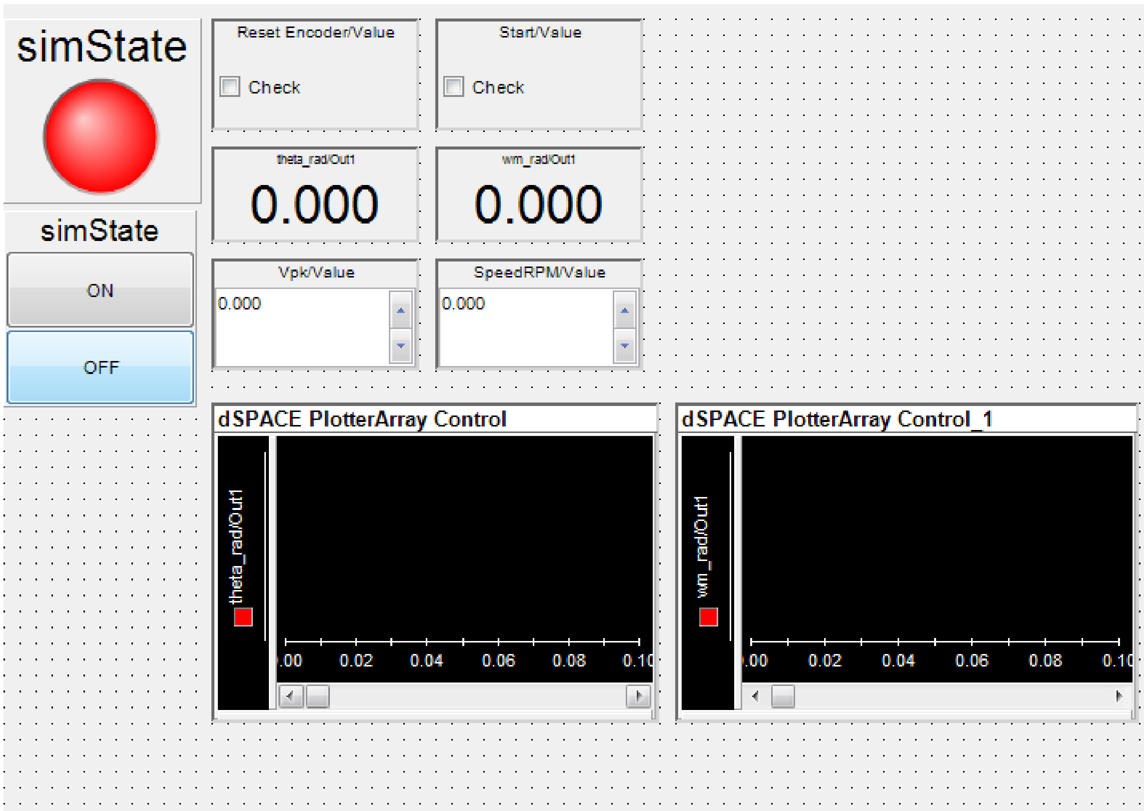

Figure 2: Layout

Important: please note that the two windings of the induction motor are connected together. Although there are four banana plugs on the motor frame, the two middle plugs (colored blue) are connected to the same wire. The two ground plugs of the amplifier are also connected together. Therefore, make sure that the grounds of both amplifiers (black outputs of the amplifiers) are connected to the blue plugs of the motor frame. Also, if the direction of rotation is negative in your initial experiments, swap the phases, so that the direction of rotation becomes positive.

Build the .mdl, create a new project & experiment structure in dSPACE, load the .sdf and layout files. The layout should look something like the one shown in Fig. 2 after you're done mapping the variables to the instruments. Measure the response of the motor for sinusoidal voltages with a peak value of 25V and a synchronous speed of 3,600 rpm. Determine the value of the speed in steady-state (it is useful to average the velocity over some period of time after it has stabilized). Calculate the values of the slip (in rad/s) and of the normalized slip (in %). Repeat for frequencies corresponding to 2,400 rpm and 1,200 rpm, and plot the velocity ω *vs.* time in all three cases.

Measuring the torque curves precisely would require a torque sensor. However, useful results can be obtained from the acceleration of the motor alone. The torque itself is not obtained. Rather, the torque divided by the inertia *J* is obtained. Fortunately, that is all that is needed for control system design. Specifically, one has that

$$\frac{d\omega}{dt} = \frac{\tau_e(\omega, \omega_e)}{J} - \frac{\tau_{LF}}{J} \tag{4}$$

assuming that the electromagnetic torque can be approximated by the steady-state value. Further assuming that the load torque is constant (it consists mostly of Coulomb friction in the set-up), the torque $\tau_e(\omega, \omega_e)$ divided by the inertia *J* is equal to the angular acceleration plus some constant value.

The acceleration of the motor can be reconstructed using numerical differentiation. For this purpose, it is necessary to filter the velocity. Apply a Butterworth filter and differentiate your captured data using, for example, the code below

```
[b,a] =butter(3,0.01);  % 0.01 is the cutoff frequency divided by half the sampling frequency
speed_filt = filter(b,a,inc_vel)
dw = [0 diff(speed_filt)]/Ts;
```

where Ts=1e-3 is the sampling period. Plot the acceleration ($d\omega/dt$) *vs.* slip ($\omega_e - \omega$) for synchronous frequencies corresponding to 3,600 rpm, 2,400 rpm, and 1,200 rpm. You should recognize the shape of the torque curves shown in the Fig. 1, but expect a very approximate resemblance, given the need to reconstruct acceleration.

For small slip and n_p =1, one has that

$$\frac{\tau_e}{J} = k(\omega_e - \omega) \tag{5}$$

where

$$k = \frac{M^2}{J R_R} \frac{V_S^2}{R_S^2 + (L_S \omega_e)^2} \tag{6}$$

Also

$$\frac{\tau_{LF}}{J} = k(\omega_e - \omega_{SS}) \tag{7}$$

where ω_{ss} is the steady-state speed. Therefore

$$\frac{d\omega}{dt} = k\big((\omega_e - \omega) - (\omega_e - \omega_{SS})\big) \tag{8}$$

In other words, k is the slope of the line relating $d\omega/dt$ to $(\omega_e - \omega)$ close to $(\omega_e - \omega_{SS})$.

From the plot of acceleration *vs.* slip, find the value of the constant k such that the best fit is obtained in the linear region. The result can be achieved by manually adjusting the parameters of a line so that it becomes tangent to the plot of the acceleration in Matlab, or by using some other method of your choice. The estimate of k will be very approximate, but only an approximate number is needed for control design.

Report values of k for each of the three speeds. Equation (6) predicts that the constant k is inversely proportional to $(1 + (L_s/R_s)^2.(\omega_e)^2)$. From this fact, and using the values of k at 1,200 rpm and 2,400 rpm, determine a ballpark estimate of the constant L_s/R_s.

<u>Requirements for Full Credit:</u> The list below is a reference for your benefit. Be sure to include comments and explanation for all work performed and results observed/produced.

- Introduction with stated objectives.
- Plot of speed *vs.* time for synchronous speeds of 1,200, 2,400, and 3,600 rpm.
- Tabulated values of the steady-state speed, slip, and normalized slip for each speed.
- Plot of the (filtered) acceleration *vs.* slip, for synchronous speeds of 1,200 rpm, 2,400 rpm, and 3,600 rpm. Include plots of the linear approximation around the steady-state speed.
- Values of the parameter k for each speed.
- Calculation of the estimate of $T_s = L_s/R_s$.
- Conclusion with reference to stated objectives. Describe what worked well and did not work well in this lab, and make suggestions for possible improvements.

***Be sure to LABEL the axes of all your plots and to include UNITS on all of your values. Comments should also always accompany any plot.**