ECE 5670/6670 - Lab 6

Parameter Estimation of a Brushless DC Motor

Objectives

The objective of the lab is to determine the parameters of a brushless DC motor and to experiment with control strategies using sinusoidal commutation.

1. Introduction

The model of a three-phase permanent magnet brushless DC motor with Y -connected windings is given by

$$L\frac{di_A}{dt} = v_A - Ri_A + K\omega \sin(n_p\theta)$$

$$L\frac{di_B}{dt} = v_B - Ri_B + K\omega \sin(n_p\theta - 120^o)$$

$$L\frac{di_C}{dt} = v_C - Ri_C + K\omega \sin(n_p\theta - 240^o)$$

$$J\frac{d\omega}{dt} = -K[i_A sin(n_p\theta) + i_B \sin(n_p\theta - 120) + i_C \sin(n_p\theta - 240)] - C\mathrm{sgn}(\omega) - B\omega$$

$$\frac{d\theta}{dt} = \omega \qquad (1)$$

If the line voltages satisfy $v_1+v_2+v_3=0$, one has that $v_A=v_1$, $v_B=v_2$, $v_C=v_3$. The parameters of the model are:

- R (Ω) - resistance of each phase winding,
- L (H) - equivalent inductance of each phase winding,
- n_P - number of pole pairs,
- K (N.m/A or V.s) - torque/back-emf constant,
- J (kg.m^2) - rotor inertia,

- C (N.m) - coefficient of Coulomb friction,
- B (N.m.s) - coefficient of viscous friction.

The model predicts that, if the motor is rotated at constant speed with open stator windings, the back-emf voltages on phases A, B and C are $-K\omega\sin(n_p\theta)$, $-K\omega\sin(n_p\theta-120°)$ and $-K\omega\sin(n_p\theta-240°)$, respectively. This observation will be used to determine K. However, since it is typical to not have access to the neutral point, K will be determined through the line-to-line back-emf voltages, such as

$$e_{AB} = -K\omega[\sin(n_p\theta) - \sin(n_p\theta - 120^o)] = -\sqrt{3}\,K\omega\sin(n_p\theta + 30^o) \tag{2}$$

The model (1) can be transformed using the three-phase DQ transformation, resulting in the model

$$\begin{aligned} L\frac{di_d}{dt} &= v_d - Ri_d + n_p\omega L i_q \\ L\frac{di_q}{dt} &= v_q - Ri_q - n_p\omega L i_d - K_{eq}\omega \\ \frac{d\omega}{dt} &= K_{eq}i_q - C\mathrm{sgn}(\omega) - B\omega \end{aligned} \tag{3}$$

where $K_{eq} = \sqrt{\frac{3}{2}}K$. Choosing $v_d = 0$ and considering steady-state operation (all variables of the DQ model are constant), one has

$$\frac{K_{eq}R(v_q - K_{eq}\omega)}{R^2 + (n_p\omega L)^2} = C\mathrm{sgn}(\omega) + B\omega \tag{4}$$

Further, assuming that $n_p\omega L \ll R$, one has

$$v_q = \frac{R}{K_{eq}}C\mathrm{sgn}(\omega) + \left(\frac{R}{K_{eq}}B + K_{eq}\right)\omega \tag{5}$$

This equation can be used to determine B and C by performing a linear fit of the quadrature voltage *vs.* speed, assuming that R and K_{eq} are known.

Once the friction parameters are determined, the inertia can be determined using the transient response of the speed to a step of voltage v_q, while neglecting the transients in i_d and i_q, so that

$$\frac{d\omega}{dt} = -\frac{1}{J}\left(B + \frac{K_{eq}^2}{R}\right)\omega + \frac{K_{eq}}{JR}v_q - \frac{C}{J}\mathrm{sgn}(\omega) \tag{6}$$

This is a first-order system with pole at $-\frac{B+\frac{K_{eq}^2}{R}}{J}$, so that a step response of the system can be used to obtain an estimate of the pole of the system.

2. Equipment Needed

You will need:

- Brushless DC motor,
- Brush DC motor,
- Dual power amplifier,
- Standalone encoder,
- Cable rack,
- Encoder cable.
- Metal frame to mount the motors on, with a box of screws and a screwdriver.

You will also need to buy a resistor with value *0.5 Ω, 3W*, which will also be used in the project.

3. Experiments

3.1 Measuring R

Measure the line-to-line resistance using a multi-meter. The line-to-line resistance is measured across red and yellow, red and black, and black and yellow. Divide the number by two to get the phase resistance. Check this more than once for each combination, and find an average value for the phase resistance.

3.2 Measuring L

Download the files *Lab6.mdl, Lab6.lax* and *Lab6.xml* from the lab web site. Your layout should look like the one shown in figure 1. Use Mode 1 (square wave mode) for this experiment. The time constant is measured by applying a 1V(peak) 10Hz square wave through the linear amplifier. The wave will be applied with the high side of the amplifier connected to winding A (*yellow line*), and the low side to winding B (*red line*) with the *0.5 Ω* resistor connected in series on the low side (black banana plug) of the amplifier. Note that winding C (*black* line) is open. When the signal is applied, the motor should not spin. It may slightly jiggle. In this case, hold the rotor stationary for better results. The current waveform should be measured by connecting ADCH5 to the resistor (with the ground lead connected to the ground of the amplifier) using a BNC to alligator cable. Capture, save, and unpack data to get the time constant from the motor current waveform. The current waveform should be plotted to determine the time constant. Deduce the value of the inductance of a single winding assuming a Y-connection, with $\tau = \frac{L_{tot}}{R_{tot}}$, $L_{tot} = 2L$, $R_{tot} = R_{amp} + R_{sense} + 2R$, R_{sense} is the *0.5 Ω* resistor, and R_{amp} is the internal resistance of the amplifier, equal to about *1Ω*. Solve for L.

Figure 1: Layout

3.3 Back-emf Voltages

Couple the brushless DC motor mechanically to a brush DC motor, and apply a voltage to the brush DC motor to spin both motors. This can be performed by using Mode 2 of the experiment. Also connect the DC motor's encoder and measure the velocity in dSPACE. The sensing resistor is not needed for this section. Capture the motor position and the line-to-line voltages v_{AC} and v_{BC}. Using BNC to alligator clips, connect ADCH6 to phase A of the motor, with the ground connected to C, and similarly for ADCH7 to phase B. Repeat for 5 different speeds. In Matlab, observe the line-to-line back-emf voltages and determine the number of pole pairs by comparing the electrical frequency to the mechanical speed of rotation. Also estimate the peak value of the sinusoidal back-emf for each speed and plot the peak value as a function of speed. Deduce an estimate of the constant K.

3.4 Open-loop Control

Disconnect the A/D channels of the dSPACE breakout box, and instead connect channel one from the amplifier to terminals A and C (red to A, black to C), and channel two to terminals B and C (red to B, black to C). Connect the BNC inputs of amplifier channels 1 and 2 to DACH1 and DACH2, respectively. Also connect a standalone encoder mechanically and electrically to measure the velocity in dSPACE. Use Mode 3 of the experiment, which applies three-phase sinusoidal voltages v_A, v_B, v_C to the motor windings

$$v_A = K\omega_{REF}\sin(\theta_e)$$
$$v_B = K\omega_{REF}\sin(\theta_e - 120^o)$$
$$v_C = K\omega_{REF}\sin(\theta_e - 240^o) \quad (7)$$

where ω_{REF} is a desired velocity variable to be updated through the layout and θ_e is obtained by taking the time integral of $\omega_e = n_p\omega_{REF}$. For K and n_P, you need to insert in the code the estimates found in previous section. The Simulink application will apply a voltage v_A-v_C to channel 1 and v_B-v_C to channel 2.

As for the stepper motor, it is necessary to align the encoder. The program will apply 2V through channel 1 momentarily to align the motor when the encoder reset box is checked. *Make sure to align before starting each experiment, or your results will vary from expected.* Capture the position and velocity using the dSPACE system. Unpack the data to produce plots that show the responses.

Determine the maximum speed that can be reached in open-loop, starting abruptly from zero speed. Determine whether or not a greater speed can be reached by increasing the voltage slowly.

3.5 Open-loop Quadrature Voltage Command

In this section, you will apply constant voltages v_d and v_q in the DQ coordinate frame, and use the results to estimate the remaining motor parameters. You should use Mode 3 of the Simulink model, but modify the appropriate blocks so that:

- Based on an input v_q of the layout, two-phase variables $v_{\bar{A}}, v_{\bar{B}}$ are computed using an inverse DQ transformation assuming $v_d=0$. The encoder position is needed to perform the DQ transformation. Remember that the output of the th_rad gain block is in radians.
- Based on $v_{\bar{A}}, v_{\bar{B}}$, voltages v_A, v_B, v_C are computed using an equal power 2-3 transformation assuming $v_h=0$.

Once the program has been tested, apply a sequence of five positive values for v_q over 10 seconds. Capture the motor position and velocity using the dSPACE system. From the (intermediate) steady-state values of the speed, determine *B* and *C*. From the time constant of the responses, determine *J.* Create a plot of v_q *vs.* steady-state speed. Use the function polyfit to determine the equation for the line (first order). Use the results to solve for *B* and *C* in the equations below*:*

$$\frac{R}{Keq}B + Keq = slope\ (first\ element\ returned\ from\ polyfit) \tag{12}$$

$$\frac{R}{Keq}C = intercept\ (second\ element\ returned) \tag{13}$$

In order to obtain *J*, enlarge the parts of the data showing the step response. Obtain an average time constant and deduce *J* using

$$J = \left(B + \frac{(Keq)^2}{2R}\right)\tau_c \tag{14}$$

<u>Requirements for Full Credit:</u> The list below is a reference for your benefit. Be sure to include comments and explanations for all work performed and results observed/produced.

- INTRODUCTION WITH STATED OBJECTIVES
- 3.1: List or table of resistance values
- 3.2: Captured current waveform, derivation of time constant and inductance value
- 3.3:
  List or table of speeds and voltages (5 total)
  Plot of position and voltages V_{ab} and V_{ac} *vs*. time (show only analysis of one speed)
  Derivation and/or explanation of values for number of pole pairs
  Derivation and value for *K* based on computed coefficients
- 3.4:
  Capture and plot of position vs. time and velocity vs. time (show a minimum of one speed)
  Discuss and compare maximum speeds observed
- 3.5:
  Screenshot of updated Simulink model
  Plot the captured sequence of speeds for five different voltages. Then, plot of steady-state values and use of a linear fit (such as *polyfit*) to produce values of slope and intercept
  Obtain an average time constant of speed responses
  Show derivations of *B*, *C*, and *J*
- Conclusion with reference to stated objectives. Describe what worked well and did not work well in this lab, and make suggestions for possible improvements.

***Be sure to LABEL the axes of all your plots and to include UNITS on all of your values. Comments should also always accompany any plot.**