

## ECE 5670/6670 - Lab 5

### Closed-Loop Control of a Stepper Motor

#### Objectives

The objective of this lab is to develop and test a closed-loop control algorithm for a stepper motor. First, field weakening is combined with a DQ transformation to maximize the achievable speed and to guarantee synchronism of the motor. Then, a PID control law is implemented for position control.

#### 1. Introduction

The DQ transformation is given by

$$\begin{aligned}v_d &= \cos(n_p \theta) v_A + \sin(n_p \theta) v_B \\v_q &= -\sin(n_p \theta) v_A + \cos(n_p \theta) v_B\end{aligned}\tag{1}$$

with inverse transformation

$$\begin{aligned}v_A &= \cos(n_p \theta) v_d - \sin(n_p \theta) v_q \\v_B &= \sin(n_p \theta) v_d + \cos(n_p \theta) v_q\end{aligned}\tag{2}$$

In the DQ coordinates, the stepper motor model is

$$\begin{aligned}L \frac{di_d}{dt} &= v_d - R i_d + n_p \omega L i_q \\L \frac{di_q}{dt} &= v_q - R i_q - n_p \omega L i_d - K \omega \\J \frac{d\omega}{dt} &= K i_q - \tau_{LF} \\ \frac{d\theta}{dt} &= \omega\end{aligned}\tag{3}$$

where,  $v_d$  and  $v_q$  (V) are the DQ voltages,  $i_d$  and  $i_q$  (A) are the DQ currents,  $\omega$  (rad/s) is the angular velocity of the motor, and  $\theta$  (rad) is the angular position. The parameters of the motor are:

- $R$  ( $\Omega$ ) the resistance of each of the phase windings,
- $L$  (H) the inductance of each of the phase windings,
- $K$  (N. m/ A or V. s) the torque constant, also called the back-emf constant,
- $J$  (kg. m<sup>2</sup>) the rotor inertia,

- $\tau_{LF}$  (N. m) the load & friction torque,
- $n_p$  the number of the pole pairs

In steady-state operation, one has that

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{1}{R^2 + (n_p \omega L)^2} \begin{bmatrix} R & n_p \omega L \\ -n_p \omega L & R \end{bmatrix} \begin{bmatrix} v_d \\ v_q - K\omega \end{bmatrix} \quad (4)$$

At low speeds, it is reasonable to set  $v_d = 0$ , resulting in  $i_q = v_q/R$ . At high speeds, however, the current  $i_q$  rapidly decreases for a given voltage because of inductive effects and because of the back-emf ( $K\omega$ ). The speed at which inductive effects become significant is approximately a decade below  $\omega = R/(n_p L)$ . It is usually low for stepper motors, because of their large inductance and number of steps. In order to increase the torque available, the current  $i_q$  may be maximized for a bounded amplifier voltage by setting a current  $i_d$  equal to

$$i_d = -\frac{(n_p \omega L)(K\omega)}{R^2 + (n_p \omega L)^2} \quad (5)$$

or, for the voltages

$$v_d = -\frac{n_p \omega L}{R} v_q \quad (6)$$

## 2. Experiments

You will need:

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| <ul style="list-style-type: none"> <li>• Stepper motor,</li> <li>• Standalone encoder,</li> <li>• Dual power amplifier,</li> </ul> | <ul style="list-style-type: none"> <li>• Encoder cable</li> <li>• A metal frame to mount the motors on, with a box of screws and a screwdriver.</li> </ul> |
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### 2.1 Preliminary Testing

In this lab, you will modify the Simulink files of Lab 4. Set the system as for the DQ experiment of Lab 4 and test by applying a voltage  $v_q$  through the layout. Remember that you may have to swap the phases so that a positive direction results from a sequence AB of voltages. Also, the encoder should be reset before every experiment.

## 2.2 Field Weakening

Modify the Simulink file to implement field weakening. Given an input  $v$ , the voltages  $v_d$  and  $v_q$  should now be given by

$$\begin{aligned}v_d &= -\frac{n_p \omega L}{\sqrt{R^2 + (n_p \omega L)^2}} v \\v_q &= \frac{R}{\sqrt{R^2 + (n_p \omega L)^2}} v\end{aligned}\tag{7}$$

instead of  $v_d=0$  and  $v_q=v$ .

If correctly coded and wired, the motor should run very smoothly and should run with 10 volts applied, but may already start with 5V. Once the program is debugged, perform the following experiments:

- Apply steps of voltage  $v = 5V, 10V, 15V, 20V,$  and  $25V$ . Plot velocity vs. time and steady-state velocity vs. voltage. Observe that the response of the system from  $v$  to  $\omega$  is approximately linear. Also observe that the maximum speed reached is much higher than without field weakening. Compare the maximum speed reached to that obtained with open-loop stepping and open-loop DQ control in Lab 4.
- Apply a step of voltage equal to 25V. Considering that the response of the motor is approximately that of a first-order system, estimate its gain and time constant.
- Save a copy of your .mdl under a different name, and include your field weakening code in your lab report.

## 2.3 Closed-loop DQ Control

Modify the program to implement the same PIDF control as you implemented for the DC motor in Lab 3, but this time using the variable  $v$  as a control input. Use the previous procedure to tune the parameters: first look for a good value of  $k_D$ , then adjust  $k_P$  and  $k_I$  using the formulas resulting from the principle of the symmetric optimum. Set  $k_F = 0.5$ . Subsequent fine-tuning of the parameters may be useful but possibly not necessary. After implementing the PIDF control law, add anti-windup for the integrator.

### **Helpful tips:**

- When using the previous procedure to tune the PIDF, set  $k_P = 25$ ,  $k_I = 0$ , and tune by trial and error for a good value of  $k_D$  (should be less than one). You don't need to match the position with the reference input.
- Once you have a good  $k_D$ , look at the velocity response to a voltage step input (using field weakening mode). Similar to Lab 4, you can look at the transient velocity response and obtain a time constant. With this value, a value for the parameter  $a'$  can be deduced.
- Next, look at the *gain* of the system, *i.e.*,  $\frac{\omega_{SS}}{v_{SS}}$ . The gain is equal to  $\frac{k}{a'}$ , and the symmetric optimum method can be applied from Lab 3 to give

$$k_P = \frac{(a')^2}{3k} \quad \text{and} \quad k_I = \frac{(a')^3}{27k}$$

Once you have good gains for the system, capture and plot position *vs.* time for + and - 90 degree steps, and for a 3600 degree step – ensure that the response to the small steps is fast and accurate, and that your anti-windup is working (the response should not overshoot on the 3600 degree step). Then, increase the reference input slowly by 1 degree from 0 to 8 degrees over 10 seconds to see the response to very small changes in position, and plot position data. Note that a full step for a two-phase stepper motor with 12 pole pairs is 7.5 degrees, so that a step of 1 degree is a micro step. In other words, a resolution smaller than the natural stepping angle is achieved.

**Requirements for Full Credit:**

The list below is a reference for your benefit. Be sure to include comments and explanations for all work performed and results observed/produced.

- Introduction with stated objectives.
- Include code/screenshots from the .mdl (every time a change is made throughout the lab).
- Plot of velocity vs. time for multiple steps of voltage.
- Plot of the steady-state velocity for multiple steps of voltage.
- Plot of the speed for a large step (25 V) of voltage and estimates of the gain and of the time constant.
- Value of the derivative gain and calculation of the  $k_P$  and  $k_I$  gain values with explanation.
- Plots of the position for 90 degree moves, 3600 degree moves, and micro-steps.
- Conclusion with reference to stated objectives. Describe what worked well and did not work well in this lab, and make suggestions for possible improvements.

**\*Be sure to LABEL the axes of all your plots and to include UNITS on all of your values. Comments should also always accompany any plot.**