ECE 5670/6670 - Lab 4

Modeling and Identification of a Stepper Motor

Objectives

The objective of this lab is to estimate the parameters of a permanent magnet stepper motor using electrical measurements, measurements of the position under open-loop stepping, and measurements of the position under open-loop DQ control. The performance of the stepper motor is compared to the performance of the DC motor measured in a previous lab.

1. Introduction

The model of a two-phase stepper motor with n_p pole pairs is given by

$$L\frac{di_A}{dt} = v_A - Ri_A + K\omega\sin(n_p\theta)$$
$$L\frac{di_B}{dt} = v_B - Ri_B - K\omega\cos(n_p\theta)$$
$$J\frac{d\omega}{dt} = -Ki_A\sin(n_p\theta) + Ki_B\cos(n_p\theta) - \tau_{LF}$$
$$\frac{d\theta}{dt} = \omega \tag{1}$$

where v_A and v_B (V) are the voltages applied to the two phases of the motor, i_A and i_B (A) are the phase currents, ω (rad/s) is the angular velocity of the motor, and θ (rad) is the angular position. The parameters are:

- R (Ω) the resistance of each of the phase windings,
- L (H) the inductance of each of the phase windings,
- K (N. m/ A or V. s) the torque constant, also called the back-emf constant,
- J (kg. m^2) the rotor inertia,
- τ_{LF} (N. m) the load/ friction torque,
- n_p the number of the pole pairs

The model predicts that if the motor is rotated at constant speed ω with open stator windings, the back-emf voltages measured on phases A and B are $-K\omega\sin(n_P\theta)$ and

$K\omega\cos(n_P\theta)$, respectively. This observation will be used to determine *K*. Also, the dynamics of the system linearized around $n_P\theta$ = 90 degrees, ω = 0, i_A = 0, and i_B = I satisfy

$$\frac{d}{dt}\begin{bmatrix}\delta i_A\\ \delta i_B\\ \delta\omega\\ \delta\theta\end{bmatrix} = \begin{bmatrix}-\frac{R}{L} & 0 & \frac{K}{L} & 0\\ 0 & -\frac{R}{L} & 0 & 0\\ -\frac{K}{L} & 0 & 0 & -\frac{KIn_p}{J}\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\delta i_A\\ \delta i_B\\ \delta\omega\\ \delta\theta\end{bmatrix} + \begin{bmatrix}\frac{1}{L} & 0\\ 0 & \frac{1}{L}\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta v_A\\ \delta v_B\end{bmatrix} \tag{2}$$

where the load torque is assumed to be zero. Two eigenvalues of the system matrix are usually lightly damped for practical values of motor parameters. If one neglects δi_A

$$\frac{d^2}{dt^2}(\delta\theta) = -\frac{KIn_p}{J}\,\delta\theta \tag{3}$$

so that an estimate of the resonant frequency (in rad/s) is

$$\omega_n = \sqrt{\frac{KIn_p}{J}} \tag{4}$$

For control design, a more useful representation of the stepper motor is obtained through the DQ transformation

$$\begin{aligned} v_d &= \cos(n_p\theta)v_A + \sin(n_p\theta)v_B\\ v_q &= -\sin(n_p\theta)v_A + \cos(n_p\theta)v_B \end{aligned} \tag{5}$$

whose inverse is given by

$$\begin{aligned} v_A &= \cos(n_p\theta)v_d - \sin(n_p\theta)v_q\\ v_B &= \sin(n_p\theta)v_d + \cos(n_p\theta)v_q \end{aligned} \tag{6}$$

In the DQ coordinates, the stepper motor model becomes

$$\begin{aligned} L\frac{di_d}{dt} &= v_d - Ri_d + n_p\omega Li_q\\ L\frac{di_q}{dt} &= v_q - Ri_q - n_p\omega Li_d - K\omega\\ J\frac{d\omega}{dt} &= Ki_q - \tau_{LF}\\ \frac{d\theta}{dt} &= \omega \end{aligned} \tag{7}$$

An interesting property of this model is that the response from i_q to ω is the same as that of a current-controlled brush DC motor. The same PID control laws can therefore be used for position or velocity control in the new reference frame. At low speeds, the inductance L can be neglected, so that

$$i_q = \frac{v_q - K\omega}{R} \tag{8}$$

The response from v_q to ω is then the same as that of a voltage-controlled brush DC motor with negligible inductance, namely

$$J\frac{d\omega}{dt} = -\frac{K^2}{R}\omega + \frac{K}{R}\, v_q - \tau_{LF} \tag{9}$$

It is also useful to consider the steady-state operation, *i.e.,* operation such that the speed and DQ variables are constant. For constant DQ variables and $v_d = 0$,

$$Ri_d = n_p\omega L i_q, \qquad v_q = K\omega + Ri_q + n_p\omega L i_d \tag{10}$$

And it follows that

$$v_q = Ri_q + K\omega + \frac{(n_P L)^2}{R} i_d \omega^2 \tag{11}$$

With $i_q = \frac{\tau_{LF}}{K}$,

$$v_q - K\omega = \tau_{LF}\left(\frac{R}{K} + \frac{\left(n_p L\right)^2}{RK}\,\omega^2\right) \tag{12}$$

2. Experiments

You will need:

- Stepper motor,
- Brush DC motor,
- Standalone encoder,
- Dual power amplifier,
- An encoder cable,
- A metal frame to mount the motors on, with a box of screws and a screwdriver.
- Multimeter
- For the measurements, a scope will be needed (already on the benches).

2.1 Preliminary Testing

Use the procedure of Lab 2 to apply a voltage to a brush DC motor and to measure its position and velocity. Then, attach the stepper motor to the DC motor through a shaft coupler. Do not connect the stepper motor to the amplifier yet.

2.2 Electrical Measurements

Measure the resistance of both phases of the stepper motor using the multimeter (YELLOW to GREEN and RED to BLACK) – **with nothing else connected to the motor**. Then, apply 5V to the DC motor using the interface from Lab 2. The coupled motors will spin at approximately 450 rpm. Observe the back-emf voltages of the stepper motor on the scope (use two probes on 1X or two BNC-to-alligator cables to hook to the motor – one on YELLOW and one on RED). Notice that the voltages are sinusoidal (although not perfectly). Capture the oscilloscope screen showing the back-emf voltages of both phases, while measuring amplitude and frequency. Determine the speed using dSPACE data, and determine the voltages using the multimeter. Note that the multimeter gives an rms value, so that the voltage must be multiplied by $\sqrt{2}$ to give a peak value. From those measurements, deduce an estimate for K. By comparing the speed measured in dSPACE and the frequency of the voltages on the scope, determine the number of pole pairs of the motor.

2.3 Open-loop Stepping

Download the *Lab4.mdl*, *Lab4.lax, Lab4.xml* and *Lab 4 Trigger Rule.txt* files from the lab web page. Build the Simulink model and create a new project & experiment structure in dSPACE using the .sdf file. The layout should look something like the one shown in figure 1. You can use the recorder and trigger files for the recorder and start trigger. The lab 4 experiment is designed to run in one of two modes, selected by a radio button setting. The first mode is the open-loop stepping mode to be used in this first part of the lab. The stepping mode applies a sequence of voltages (+/- 25 V) to phases A and B, producing a stepping of the motor. The user may select the frequency of the voltages that are applied by specifying the speed in RPM. The program takes care of the conversion, using the number of pole pairs.

Analyze the Simulink model. Note that there is a block to guarantee that the zero angular position is such that the rotor is aligned with phase A, as assumed in the model. To obtain such an alignment (approximately), the maximum voltage is applied to phase A for 0.5 seconds when the "Zero Encoder" button is pressed. The encoder is reset to zero after a short period of time (0.1 sec).

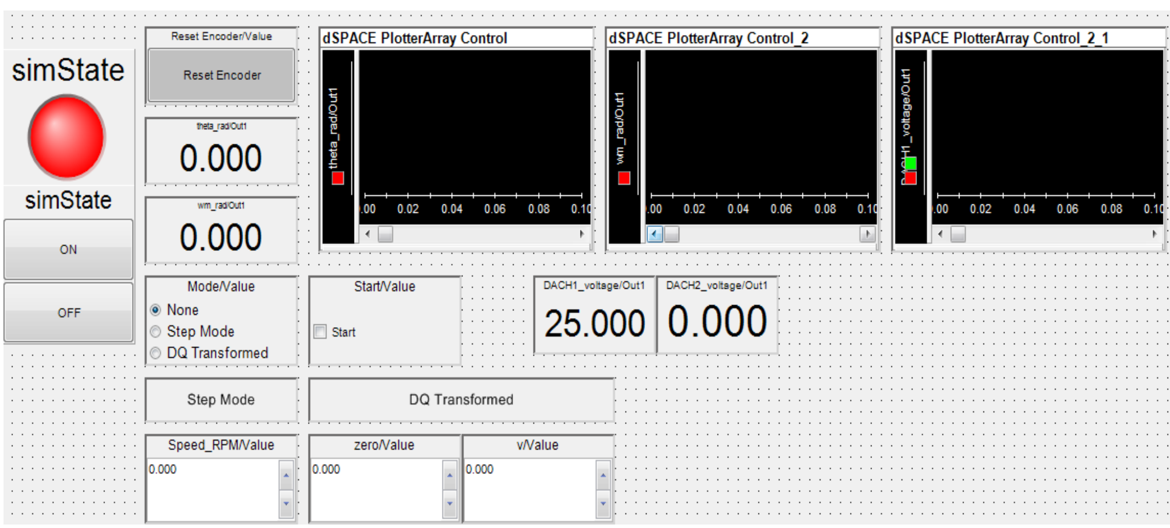

Figure 1: Layout

Connect the two phases of the stepper motor to the two outputs of the amplifier. Connect one channel to YELLOW (+) and GREEN (-), and connect the other channel to

RED (+) and BLACK (-). Remove the DC motor and replace it with a standalone encoder. Then, perform the following experiments:

- Zero the encoder using the “Zero Encoder” button in the layout which aligns the motor (note: the motor rotates slightly to align phase A). Be sure to wait for a second before starting the motor.

  Run the motor at low speed (10 rpm over 1 second) and capture the position and velocity response for a few steps in the plot window. *Note: If the motor rotates in the negative direction, swap the channel 1 and channel 2 sets of banana plugs to obtain a positive direction of rotation before proceeding with the next experiments.*
- Analyze your captured data in Matlab. Plot the position over time. Notice that the steps are not all identical. However, for a "typical" step, zoom in the response and estimate the frequency of oscillations and the percent overshoot of the response. Take the frequency of oscillations to be an estimate of *Im(p)* where *p* is the complex pole of a second-order approximation. Deduce an estimate of the complex pole p using

$$\%OS = 100\% * e^{-\pi\frac{Re(p)}{Im(p)}} \tag{1}$$

  Deduce an estimate of the inertia J from the imaginary part of the pole using (4). Report the percent overshoot, natural frequency, estimated pole location, and value for the inertia.
- Determine the maximum speed that can be reached if a large reference input of speed is applied at $t = 0$. Start with a reference value of 150 RPM, and apply larger and larger values, **stopping the motor in between**, until the motor no longer can run at the reference speed. Record the maximum reference input step. *Note: if the motor does not start, quickly turn off the input to avoid damage.*
- Determine the maximum speed that can be reached if the reference input is slowly increased from zero instead of being applied from the beginning. Record this maximum speed and compare to the speed determined in the previous step. **Uncheck the START box before continuing to the next section to stop applying voltage to the motor.**

2.4 Open-loop DQ Control

The second mode is a DQ mode that applies voltages to phases A and B through an inverse DQ transformation. It will be used for this part of the experiment. The program sets $v_d = 0$, which is a reasonable choice at low speeds. Study the operation of the program. Then, perform the following experiments:

Note: ensure that you zero the encoder before each start of the motor, or the motor may not be properly aligned – this will affect your results.

- Determine the maximum speed that can be reached if the maximum voltage up to $v_q = 25V$ is applied at $t = 0$. Compare the speed to the speed reached with open-loop stepping.
- Apply a 10V step, and observe that the response of the velocity is approximately that of a first-order system. Estimate the time constant, and compare it to RJ/K^2 , the value predicted by equation (9) using the values determined earlier.
- Apply steps of voltage equal to 5V, 10V, 15V, 20V, and 25V. Determine the velocities that are reached for each of these voltages. Then, plot $v_q - K\omega$ (with the value of K determined earlier) as a function of ω. According to equation (12), the function should be parabolic because the load torque is mostly the friction torque, and is pretty much constant.
- Using the method of your choice (for example, Matlab's *polyfit* function), determine the best values of a and b such that $v_q - K\omega \approx a + b\omega^2$. From the parameter a, determine the value of τ_{LF} using (12) and values of R and K determined in the first part of the lab.
- Compare the value of τ_{LF} to the value obtained for the Coulomb friction torque of the brush DC motor in Lab 2 (about 0.01 N.m). From the parameter b, obtain an estimate of the inductance L.
- Using the estimated values, calculate the eigenvalues of the 4x4 matrix in (2). Compare the values of the oscillatory poles to the values estimated with the step responses.
- Indicate what the maximum torque of the motor is for a supply voltage of 25V. Compare the value of the torque to that obtained for a 2A limit for the brush DC motor (about 0.2 N.m). Also compare the maximum value of the speed reached in the open-

loop experiments to the maximum speed for the brush DC motor under a 25V supply limit (about 2,370rpm).

Requirements for Full Credit: The list below is a reference for your benefit. Be sure to include comments and explanation for all work performed and results observed/produced.

- Introduction with stated objectives.
- Value of resistances (both phases) and back-emf constant.
- Scope capture of waveforms.
- Plot of typical step. Estimates of frequency of oscillation and percent overshoot.
- Max velocity values for large reference input, sudden step, and progressive stepping.
- Max velocity values for 25V, sudden step, and progressive stepping.
- Plot of speed *vs.* time, estimate of time constant and comparison to prediction using eqn (9).
- Plot of the 10 V speed vs. time response.
- Plot of $v_\mathrm{q} - K\omega$ as a function of ω. Estimates of a and b, τ_F and L.
- Comparison of Coulomb friction torque to the brush DC motor.
- Predicted eigenvalues, comparison of poles with results of step responses.
- Value of max torque and max speed of stepper motor **and** comparison to max torque and max speed of brush DC motor.

- Conclusion with reference to stated objectives. Describe what worked well and did not work well in this lab, and make suggestions for possible improvements.

***Be sure to LABEL the axes of all your plots and to include UNITS on all of your values. Comments should also always accompany any plot.**

Make certain you have: One oscilloscope screen capture, and a minimum of four plots.