### ECE 5670/6670 - Lab 3

# PID Control of a Brush DC Motor

### **Objectives**

The objective of this lab is to experiment with a proportional-integralderivative (PID) control law for the positioning of a brush DC motor. Integral control is found useful to achieve zero-steady state errors in the presence of constant load disturbances. To avoid overshoot in the responses, simple modifications are applied to the control law. Controller parameters are selected by trial and error and using a procedure based on the principle of the symmetric optimum.

### 1. Introduction

Assuming voltage control of the motor, a PID control law is given by

$$v = k_p (\theta_{ref} - \theta) + k_I \int (\theta_{ref} - \theta) dt + k_D \frac{d}{dt} (\theta_{ref} - \theta)$$
(1)

where v is the voltage applied to the motor,  $\theta_{ref}$  is the desired position of the motor,  $\theta$  is the actual position of the motor, and  $k_p$ ,  $k_I$ , and  $k_D$  are the proportional, integral, and derivative gain parameters, respectively. Heuristically, one finds that, for increasing values of the parameters,  $k_p$  accelerates the speed of the response,  $k_D$  improves the damping, and  $k_I$  guarantees the return of the steady-state error to zero but slows down the system. However, the response of the closed-loop system depends on all three parameters in a complex manner, making manual tuning difficult.

Several modifications are brought to this control law in practice. The derivative action is usually only applied to  $\theta$ , so that large control values do not result from step changes in the reference input. Sometimes, the proportional term is also modified in order to avoid the presence of a low-frequency zero in the transfer function. To that effect, the contribution from the reference signal is reduced through a factor  $k_F < 1$ . The control law becomes

$$v = k_p \left( k_F \theta_{ref} - \theta \right) + k_I \int \left( \theta_{ref} - \theta \right) dt + k_D \frac{d}{dt}(\theta)$$
(2)

An important practical problem is also that of integrator wind-up. For large steps of the reference input, the response of the system exhibits an overshoot, even if no overshoot is observed for small steps. This problem occurs because the control input reaches a limit and the behavior of the system ceases to be linear. The integrator state then becomes large, leading to a peaking of the response. Several fixes exist for the problem, with two of them described in the course notes.

The performance of the PID control law can be evaluated using step inputs and various performance criteria. For example, the *rise time* is the time it takes for the output to rise from 10% to 90% of its final value. The *percent overshoot* is the amount by which the response overshoots its steady-state value. The *settling time* is the time it takes for the output to settle within 2% of its final value. The *steady-state error* is the difference between the steady-state value and the reference input.

## 2. Experiments

You will need:

- brush DC motor,
- dual power amplifier,
- small brush DC motor (to serve as a load),
- metal frame to mount the motors on, with a box of screws and a screwdriver.
- an encoder cable.

### 2.1 Preliminary Setup

Download the files *Lab3.mdl* and *Lab3.lax* from the lab web page and build the Simulink model. Create a new project & experiment structure in dSPACE using the .sdf file. The layout should look as Fig. 1 when you're done mapping the variables. Attach the smaller DC motor to the bigger one with a shaft coupler, and connect the encoder output to Inc 1 of the dSPACE breakout box. Connect banana-banana cables from both dc motors to the two sides of the power amplifier. Connect DACH 1 to the channel that powers the larger motor and DACH 2 to the channel that powers the smaller motor.

Important note: power is applied to the small DC motor through the red and black banana plugs. The yellow and green banana plugs are outputs from a tachometer and should not be used.



Figure 1: Layout

## 2.2 PID control - Manual Tuning

The experiment applies the PID control law (2) (without the derivative on the reference input) to the motor. Double-check that you have connected the DACH outputs to the proper motors through the amplifier. The program generates a step voltage of 6V on DACH 2 from time t = 1.5 to time t = 3 seconds. The torque generated by the smaller DC motor produces the effect of a change of load.

By trial and error, find the set of control parameters  $(k_p, k_I, k_D)$  that gives the best step response that you can find, considering the performance criteria listed above. Leave  $k_F$  at  $k_F = 1$ . Apply a reference input of 30 degrees. You will find that adjusting the three parameters is a tricky problem because of inter-dependencies. One approach consists in first setting  $k_I =$ 0 and finding the best possible values of  $k_P$  and  $k_D$ . Usually, the response improves when  $k_D$ increases, but there is a value beyond which vibrations arise because of discretization. For the parameter  $k_P$ , there is usually an optimum value beyond which the response becomes poorly damped. Once  $k_P$  and  $k_D$  are tuned,  $k_I$  may be raised in order to improve the steady-state error. Usually, the integral action leads to a deterioration of the transient response and a trade-off has to be reached. The proportional and derivative parameters may also have to be re-adjusted once  $k_I$  is raised.

#### Helpful hints:

PD manual tuning:  $k_p$  between 10 and 35,  $k_d$  between 0 and 1

PID manual tuning:  $k_p$  between 10 and 35,  $k_d$  between 0 and 1, and  $k_l$  between 900 and 1400 Recorder and Trigger files: You can use the recorder and trigger files provided. See the Lab1 handout for information on how to import these files. Use the time limit option for the stop trigger.

In your experiments, you are likely to observe the following phenomena:

- Without a load, fast responses can be achieved with a PD control law (k<sub>1</sub> = 0). The responses also remain good for large steps. However, a significant steady-state error appears under a load disturbance;
- With the integral action, the steady-state error goes to zero, but there is a tendency for the control law (1) to overshoot, sometimes with a sluggish return to the steady-state value.
- With sufficiently large steps of reference input (*e.g.*, 360 degrees), the response overshoots more than it does for small steps (even in percentage value).

Produce the following for your report:

- Plot of the response to 30° step reference using the best PD controller (position *vs.* time), with values of the PD gains. Use units of degrees and seconds for these plots.
- Plot of the response to 30° step reference using the best PID controller, with values of the PID gains.
- Plot of the response to a large step (*e.g.*, 360°) using the best PID controller, with values of the PID gains.

#### 2.3 Symmetric Optimum Design and Improvements

In the second part of the lab, the PID parameters are determined in two steps:

- First, the largest possible value of k<sub>D</sub> is determined experimentally. A value between 0.05 and 0.3 should be adequate.
- Second, the parameters  $k_p$  and  $k_l$  are determined using the principle of the symmetric optimum for the resulting transfer function.

The response of the motor from the voltage to the velocity can be approximately modeled by a first-order transfer function k/(s+a). For the motors in the lab,  $k \approx 1000$  and  $a \approx 100$ . When velocity feedback is applied, the transfer function from the position reference to the position becomes

$$P(s) = \frac{k}{s(s+a')} \text{ with } a' = a + kk_D$$
(3)

The parameters  $k_p$  and  $k_l$  determined by the principle of the symmetric optimum are related to the parameters k and a' through the relationships

$$k_{p} = \frac{(a')^{2}}{3k}$$

$$k_{I} = \frac{(a')^{3}}{27k}$$
(4)

Compute the values of  $k_p$  and  $k_l$  specified by the formulas and choose an appropriate value for  $k_F$  as suggested in the course notes. Then

 Fine-tune the values of k<sub>p</sub>, k<sub>I</sub>, k<sub>D</sub>, and k<sub>F</sub> experimentally to optimize the performance for a 30° step. Label your axes and use units of **DEGREES**.

- Modify the control law in Simulink so that performance remains acceptable for a large step (*e.g.*, 360 to 3600°). Specifically, implement an anti-windup modification by stopping the integration when the maximum voltage is reached (integrate zero instead of the error).
- Remember that you must stop the online calibration in dSPACE before building the Simulink model.
- Plot the response for the large step, showing that overshoot has been reduced. Label your axes and use units of **DEGREES**.
- Plot the response for a 30° step and for a period of time not exceeding 200ms. Calculate the values of the rise time, percent overshoot, settling time, and steady-state error with/without load.

#### **Requirements for Full Credit:**

The list below is a reference for your benefit. Be sure to include comments and explanation for all work performed and results observed/produced.

- Introduction with stated objectives.
- Plot of the response using the PD controller, with values of the PD gains.
- Plot of the response using the PID controller, with values of the PID gains.
- Plot of the response using the PID controller for a large step of reference input (overshoot should be observed and noted).
- Values of the PIDF gains given by the theory and eventually selected (2 sets of numbers).
- Screenshot of new Simulink model showing changes made to the control law.
- Plot of a large step response.
- Plot of the response showing the detail of the transient response (30° step and time period should not exceed 200 ms).
- Values of the rise time, settling time, percent overshoot, and steady-state error with and without load.
- Conclusion with reference to stated objectives. Describe what worked well and did not work well in this lab, and make suggestions for possible improvements.

#### \*Be sure to LABEL the axes of all your plots and to include UNITS on all of your values. Comments should also always accompany any plot.