3.2. Laser Diodes

A semiconductor laser diode is basically an LED structure with mirrors for optical feedback. This feedback causes photons to retrace their path back through the gain region. These photons induce the stimulated emission of other photons, which are identical to the photons that initiated the process. Amplification by stimulated emission continues until the total gain in the cavity equals the total losses, which is known as the lasing condition.

Any optical cavity supports spatial and longitudinal modes. We will largely ignore spatial mode structure by assuming that the laser operates on a single TEM_{00} spatial mode, which is almost universally true for communications lasers (the exception is for the high power 980 nm and 1480 nm diodes used for pumping optical amplifiers). The longitudinal modes of the cavity are separated by the frequency spacing

$$\Delta \nu = \frac{c}{2L},$$

where \( L \) is the cavity length. Many longitudinal modes may lie under the gain curve of the forward-biased diode, and, since semiconductor gain is an inhomogeneously-broadened process, many of these modes can achieve the lasing condition. Therefore, without additional mode reduction mechanisms, semiconductor lasers emit over a broad spectral range (but still much narrower than LEDs).

Semiconductor diodes are typically built from heterostructures, meaning that different materials are sandwiched together to form the p-n junction. By using an intrinsic region of higher refractive index as the depletion region, two major benefits are obtained: gain confinement and optical mode confinement. Both of these effects serve to greatly increase the performance of laser diodes.
Frequency-selective mechanisms can be used to force a laser diode to operate on a single longitudinal mode, thus dramatically reducing the lasing spectral width. The major mechanisms used today rely on the use of refractive index gratings within the laser cavity. The first type of structure is the distributed feedback (DFB) structure, in which the grating lies within the gain region and produces large loss for all but one longitudinal mode. The second structure is the distributed bragg reflector (DBR) in which two gratings serve as high-reflectance mirrors for one wavelength, or mode.

3.2.1. Light-current characteristics

Lasing in semiconductor diodes can be described by the following coupled differential equations:

\[
\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - GP
\]

\[
\frac{dP}{dt} = GP + R_{sp} \frac{P}{\tau_p}.
\]

In these equations, \( N \) is the number of electrons in the conduction band, \( I \) is the injection current, \( \tau_c \) is the carrier lifetime, \( G \) is the net rate of stimulated emission, \( P \) is the number of photons emitted into the lasing mode, \( R_{sp} \) is the rate of spontaneous emission into the lasing mode, and
\[ \tau_p \text{ is the photon lifetime in the cavity. The stimulated emission rate } G \text{ can be approximated as} \]

\[ G = G_N (N - N_o), \]

where \( G_N \) = \( \Gamma v_g \sigma_g / V \), \( \Gamma \) is the mode confinement factor, \( V \) is the active volume, \( v_g \) is the photon group velocity, \( \sigma_g \) is the differential gain coefficient, and \( N_o = N_T V \) is the transparency carrier number (where \( G = 0 \)) with \( N_T \) the threshold carrier density. Finally,

\[ \tau_p = \frac{1}{v_g (\alpha_{\text{mirrors}} + \alpha_{\text{cavity}})} \]

is the photon lifetime and is related to the total loss within the cavity arising from absorption within the cavity and transmission through the mirrors. The loss due to the mirrors, with intensity reflectance coefficients \( R_1 \) and \( R_2 \), is given by

\[ \alpha_{\text{mirrors}} = -\frac{1}{2L} \ln (R_1 R_2). \]

At steady state, we can find the conditions for lasing and the output power. Setting the time derivatives to zero, we get

\[ 0 = GP + R_{sp} - \frac{P}{\tau_p} = G_N (N - N_o) P + R_{sp} - \frac{P}{\tau_p} \]

\[ 0 = \frac{I}{q} \frac{N}{\tau_c} - GP = \frac{I}{q} \frac{N}{\tau_c} - G_N (N - N_o) P. \]

Using the first equation, the carrier number at steady-state can be written

\[ N = N_o + \frac{(P/\tau_p - R_{sp})}{G_N P} \text{ ignore } R_{sp} \]

\[ = N_o + \frac{1}{G_N \tau_p} \]

\[ \equiv N_{th}, \]

which means that the number of carriers is clamped such that gain equals loss. We can see this more clearly by considering the rate of spontaneous emission at steady-state,

\[ G = G_N (N_{th} - N_o) = \Gamma v_g g, \]

where \( g \) is the gain. Substituting for \( N_{th} \), we get

\[ g = \frac{G_N (1/G_N \tau_p)}{\Gamma v_g} = \frac{1}{\tau_p \Gamma v_g} = \frac{\alpha_{\text{mirror}} + \alpha_{\text{cavity}}}{\Gamma} = \text{loss}. \]

Now, we can use the carrier density in the second steady-state equation to obtain an expression for the photon number

\[ P = \tau_p \left( \frac{I}{q} - \frac{N_o + 1/G_N \tau_p}{\tau_c} \right) \]

\[ = \frac{\tau_p I}{q} - \left( N_o + \frac{1}{G_N \tau_p} \right) \frac{\tau_p}{\tau_c} \]

\[ = \frac{\tau_p}{q} \left[ I - q \left( N_o + \frac{1}{G_N \tau_p} \right) \frac{1}{\tau_c} \right]. \]
Here, $P$ represents the number of photons internal to the cavity,

$$P_{\text{int}} = \frac{\tau_p}{q} (I - I_{\text{th}}),$$

where $I_{\text{th}}$ is the threshold current defined as

$$I_{\text{th}} = \frac{q}{\tau_c} \left( N_0 + \frac{1}{G_N \tau_p} \right) = \frac{q}{\tau_c} N_{\text{th}},$$

which is the current at which lasing begins. Note that applied current greater than $I_{\text{th}}$ increases the number of photons in the cavity, while the carrier number remains constant.

The power emitted from the laser cavity is related to the photon number within the cavity through the group velocity and transmission at the mirror (with a factor of 1/2 since the output is taken only from one end)

$$P_{\text{ext}} = \frac{1}{2} \left( v_g \alpha_{\text{mirror}} \right) \hbar \omega P_{\text{int}} = \frac{\hbar \omega}{2q} \frac{\alpha_{\text{mirror}}}{\alpha_{\text{mirror}} + \alpha_{\text{cavity}}} (I - I_{\text{th}}).$$

The slope efficiency of a laser is defined as the change in output power with respect to applied current

$$\frac{dP_{\text{ext}}}{dI} = \frac{\hbar \omega}{2q} \eta_d,$$

where the differential quantum efficiency is defined

$$\eta_d = \frac{\alpha_{\text{mirror}}}{\alpha_{\text{mirror}} + \alpha_{\text{cavity}}}. $$

The external quantum efficiency is defined

$$\eta_{\text{ext}} = \frac{\text{photon-emission rate}}{\text{electron injection rate}} = \frac{2P_{\text{ext}}/\hbar \omega}{I/q} = \frac{2q P_{\text{ext}}}{\hbar \omega I} = \eta_d \left( 1 - \frac{I_{\text{th}}}{I} \right).$$

The wall-plug efficiency

$$\eta_{\text{tot}} = \frac{\hbar \omega}{q V_0} \eta_{\text{ext}} \approx \frac{E_g}{q V_0} \eta_{\text{ext}} \frac{\eta_{\text{ext}}}{\text{applied voltage}}$$

can approach 50% for a diode laser. This is significantly greater than for any other type of laser.

The operation of a diode laser is dependent on temperature. This is sometimes expressed as the dependence of the threshold current

$$I_{\text{th}}(T) = I_{\text{th}} e^{T/T_0}.$$ 

For InGaAsP lasers (1.3-1.6 \( \mu \text{m} \)), $T_0 \sim 50 - 70$ K, while for GaAs (800 nm), $T_0 \sim 120$ K. Therefore, telecom lasers (such as InGaAsP) must be temperature stabilized, which is accomplished with a thermo-electric (TE) cooler integrated on the package.
3.2.2. Modulation response

Following a procedure similar to that used for the LED, we assume time-dependent current injection of the form

\[ I(t) = I_b + I_m f_p(t), \]

where \( I_b \) is the bias current level and \( f_p \) represents the current modulation waveform. In digital systems, \( f_p \) is a square pulse, while in analog systems, \( f_p \) is a sinusoid.

For high-speed modulation, we have to include some additional effects in the rate equations:

\[
\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - GP
\]

\[
\frac{dP}{dt} = GP + R_{sp} - \frac{P}{\tau_p}
\]

\[
\frac{d\phi}{dt} = \frac{1}{2} \beta_c \left[ G_N (N - N_0) - \frac{1}{\tau_p} \right].
\]

The last rate equation describes phase modulation of the optical field due to the change in bandgap with carrier density. This effect is called bandgap renormalization, and \( \beta_c \) is called the linewidth enhancement factor and has values \( \beta_c \sim 4 - 8 \). We also must take into account the fact that the stimulated emission rate coefficient depends on the number of photons in the cavity due to spatial and spectral hole-burning, carrier heating, and two-photon absorption:

\[ G = G_N (N - N_0) (1 - G_{NL} P) \]

which causes a slight reduction in \( G \) as \( P \) increases. The coefficient \( G_{NL} \sim 10^{-7} \).

In general, these coupled equations must be solved numerically because they are nonlinear. However, we can obtain a frequency-response estimation by assuming that the transient operation of the laser lies well within the linear regime of the P-I relationship. This is ensured by the conditions \( I_b > I_{th} \), so that lasing occurs at the bias current, and small-signal modulation \( I_m << I_b - I_{th} \), so we can linearize about the bias levels. Therefore, our current modulation waveform is

\[ f_p(t) = \sin(\omega_m t) \text{ or } e^{+j\omega_m t} \]

and the resulting photon number and carrier density expressions

\[
P(t) = P_b + |p_m| \sin(\omega_m t + \theta_m)
\]

\[
N(t) = N_b + |n_m| \sin(\omega_m t + \psi_m)
\]

Instead of using complex amplitudes, we have explicitly separated the magnitude and phase of the photon and electron density responses. Because of cavity lifetime, the phases of \( P \) and \( N \) will be different, so that \( \theta_m \neq \psi_m \). Bandgap renormalization will be a small effect in the small-signal regime since the change in carrier number is small.

The small-signal modulation of the photon number is

\[
p_m(\omega_m) = \frac{P_b G_N I_m / q}{(\Omega_R + \omega_m - i\Gamma_R)(\Omega_R - \omega_m + i\Gamma_R)},
\]

where

\[
\Omega_R = \sqrt{GG_N P_b - (\Gamma_p - \Gamma_N)^2 / 4}
\]

\[
\Gamma_p = \frac{R_{sp}}{P_b} + G_{NL} GP_b
\]

\[
\Gamma_N = \frac{1}{\tau_c} + G_N P_b
\]
Ω_R is the relaxation oscillation frequency (which is due to the difference in carrier lifetime to photon lifetime), and Γ_R is the damping rate of the relaxation oscillations. The modulation transfer function is written

\[ H(\omega_m) = \frac{p_m(\omega_m)}{p_m(0)} = \frac{\Omega_R^2 + \Gamma_R^2}{(\Omega_R + \omega_m - i\Gamma_R)(\Omega_R - \omega_m + i\Gamma_R)} \]

For limiting values of modulation frequency \( \omega_m \), we have

\[ \begin{align*}
\omega_m &< < \Omega_R & |H(\omega_m)| = 1 \\
\omega_m &= \Omega_R & |H(\omega_m)| = \text{max} \\
\omega_m &> > \Omega_R & |H(\omega_m)| \rightarrow 0
\end{align*} \]

The 3 dB down point in modulation amplitude occurs for the frequency

\[ f_{3dB} = \frac{1}{2\pi} \left[ \Omega_R^2 + \Gamma_R^2 + 2(\Omega_R^4 + \Omega_R^2\Gamma_R^2 + \Gamma_R^4)^{1/2} \right]^{1/2} \approx \sqrt{\frac{3G_N}{4\pi^2q}} (I_b - I_{th}) \quad \text{for} \quad \Gamma_R << \Omega_R \]

Electrical parasitics usually limit bandwidth to less than 10 GHz, but 25 GHz and greater has been obtained.

For digital current modulation, the laser is biased near threshold and modulated well above threshold to reproduce a digital output with minimal low value and maximum high value to maximize threshold discrimination. This type of modulation is known as large-signal modulation, and must be solved numerically. The major effect in large-signal modulation is chirp caused by large changes in carrier density, and therefore, large changes in bandgap. Chirp can be detrimental in optical communications.
In the figure, the rise time is 100 ps and the fall time is 300 ps. The chirp on the optical pulse can be described by

$$\delta \omega(t) = - \frac{d\phi}{dt} = - \frac{1}{2} \beta_c \left[ G_N (N - N_0) - \frac{1}{\tau_p} \right].$$

Because $\beta_c > 0$, current modulation of a laser diode always produces a down chirp ($C < 0$). Down chirp is very bad in the anomalous dispersion regime $\beta_2 < 0$ (i.e. $D > 0$). In the normal dispersion regime, the pulse will initially compress to the transform limit, and then expand rapidly due to the extra bandwidth.
3.2.3. Laser noise

The two fundamental noise sources in lasers are due to spontaneous emission and electron-hole recombination. Photons emitted through spontaneous emission are uncorrelated with those emitted due to stimulated emission from the lasing mode and lead to intensity and phase fluctuations of the emitted light on a time scale of about 100 ps. Since electron-hole recombination is a series of discrete events, lasers have an additional source of intensity noise called shot noise. Intensity fluctuations degrade the SNR, while phase fluctuations increase the spectral linewidth \( \sigma_\omega \). Shot noise will be discussed in the next chapter.

Again, in order to describe laser noise, we return to the rate equations

\[
\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_c} - GP + F_N(t)
\]
\[
\frac{dP}{dt} = \left( G - \frac{1}{\tau_p} \right) P + R_{sp} + F_P(t)
\]
\[
\frac{d\phi}{dt} = \frac{1}{2} \beta_c \left[ G_N (N - N_0) - \frac{1}{\tau_p} \right] + F_\phi(t).
\]

The noise terms \( F_N, F_P, F_\phi \) are called Langevin forces. They are assumed to be Gaussian random processes with zero mean, with correlation functions

\[
\langle F_i(t) F_j(t') \rangle = 2D_{ij} \delta(t - t'),
\]

where \( i, j \in \{ N, P, \phi \} \). \( D_{ij} \) is known as the diffusion coefficient, which is the degree of correlation between noise terms. The dominant sources of noise arise from the diffusion coefficients \( D_{pp} = R_{sp}P \) and \( D_{\phi\phi} = R_{sp}/4P \), which are amplitude and phase fluctuations due to spontaneous emission.

For intensity noise, we use the intensity autocorrelation function

\[
C_{pp}(\tau) = \frac{\langle \delta P(t) \delta P(t + \tau) \rangle}{\bar{P}^2},
\]

where \( \bar{P} \) is the average photon number and \( \delta P = P - \bar{P} \) represents photon number fluctuation. The Fourier transform of \( C_{pp} \) is called the relative intensity noise, or

\[
\text{RIN}(\omega) = \int_{-\infty}^{\infty} C_{pp}(\tau) e^{-j\omega \tau} d\tau,
\]

which is also the product of \( \mathcal{F} \{ \delta P \} \) with itself, or otherwise known as the power spectral density of amplitude noise. An approximate expression for the laser RIN is given by

\[
\text{RIN}(\omega) \sim \frac{2R_{sp}}{\bar{P}} \left\{ \left( \frac{\Omega_R^2 - \omega^2}{\Gamma_R^2} \right) + G_N \bar{P} \left[ G_N \bar{P} \left( 1 + \frac{N}{\tau_c R_{sp} \bar{P}} \right) - 2\Gamma_N \right] \right\}.
\]
Laser RIN is enhanced near the relaxation oscillation frequency $\Omega_R$, and decreases as $\omega >> \Omega_R$ since the laser is not able to respond to those frequencies.

The signal to noise ratio is defined

$$SNR = \frac{\overline{P}}{\sigma_p}$$

where $\sigma_p = \text{RMS noise}$

$$\sigma_p = C_{pp}(0)^{-1/2}$$

Above a few mW output power, $SNR > 20$ dB and is well approximated by the expression

$$SNR = \sqrt{\frac{G_{NL} R_{sp} \tau_p}{P}}$$

The SNR saturates at about 30 dB.

Mode partition noise results from the interference of multiple longitudinal modes of the laser, and can occur even for a single mode (SLM) laser. Typically, SLM lasers suppress side modes by 20 dB or more. Even though the total output power is constant, the distribution among modes can change. There is an anticorrelation between the main and side lobes which enhances RIN for main mode by 20 dB in the 0-1 GHz range. MPN will be discussed further later on.

The spectral linewidth $S(\omega)$ is the Fourier transform of the field autocorrelation function

$$S(\omega) = \int_{-\infty}^{\infty} \Gamma_{EE}(t) e^{-j(\omega-\omega_c)\tau} d\tau,$$

where

$$\Gamma_{EE}(t) = \langle E^*(t)E(t+\tau) \rangle,$$
and $E(t) = \sqrt{P} e^{j\phi}$. The frequency $\omega_c$ is the carrier frequency. If we neglect intensity fluctuations (i.e. assume that $P = \bar{P}$) then

$$\Gamma_{EE}(t) = \langle e^{j\Delta \phi(t)} \rangle = e^{-\langle \Delta \phi^2(\tau) \rangle / 2},$$

where

$$\Delta \phi(\tau) = \phi(t + \tau) - \phi(t)$$

is assumed to be a Gaussian random process. We can then evaluate

$$\langle \Delta \phi^2(\tau) \rangle = \frac{R_{sp}}{2P} \left\{ (1 + \beta^2_c b) \tau + \frac{\beta^2_c b}{2 \Gamma_R \cos \delta} \left[ \cos(3\delta) - e^{-\Gamma_R \tau \cos(\Omega_R \tau - 3\delta)} \right] \right\}$$

with the definitions

$$b = \frac{\Omega_R}{\sqrt{\Omega^2_R + \Gamma^2_R}}, \quad \delta = \tan^{-1} \left( \frac{\Gamma_R}{\Omega_R} \right).$$

Because $\Omega_R \gg \Gamma_R$, we can assume $b = 1$, and obtain a linewidth

$$\Delta \nu = R_{sp} \frac{(1 + \beta^2_c)}{4\pi P},$$

which is valid for a Lorentzian lineshape. The linewidth is enhanced by the factor $(1 + \beta^2_c)$, which is the origin of the term linewidth enhancement factor. The linewidth $\Delta \nu$ typically saturates at 1-10 MHz for $\bar{P} \geq 10$ mW.