

Homework #3 solutions:

**PROBLEM No 1:**

- (a) Explain the difference between a direct and indirect bandgap material.
- (b) Which type of material (direct or indirect) result in applications in Light emitting Diodes?
- (c) A piece of GaAs emits red light at room temperature due to recombination of electrons and holes. If the bandgap of GaAs is 1.43 eV, calculate the wavelength of emitted light.

**Solutions:**

(a) A direct bandgap material conserves the momentum during transition, i.e. the k-value of the electron does not change as it makes the transition. Usually this happens when the conduction band minimum and the valence band maximum are at the same k-value.

For indirect bandgap material, the transition takes place between conduction band minimum and valence band maximum that are at different k-values, and hence they change their momentum with the help of lattice oscillations (phonons). This makes the transition probability really low. So these materials are not good for optoelectronic devices that require efficient transitions.

(b) Direct bandgap materials are suitable for optoelectronic applications for reasons mentioned above.

(c) The energy of the emitted light will have the same energy as the bandgap i.e. 1.43 eV. The wavelength is then given as

$$\lambda = \frac{1240}{1.43} \text{ nm} = 867.13 \text{ nm}$$

**Problem No. 2**

- (a) Explain the formation of band when N atoms are together and their orbitals interact. How many orbitals will there be in each band?
- (b) What basic principle is being followed when equal energy orbitals of two interacting atoms split into bonding and antibonding orbitals?
- (c) If the lattice constant of a semiconductor A is bigger than another semiconductor B, then which one has the larger bandgap?
- (d) Fill the gaps: A bandstructure is basically a map of electron energy vs. electron .....
- (e) You are given two direct bandgap material and a flashlight which can have continuously variable photon coming out at your will. How can you find out which one of the semiconductors has a lower bandgap?

- (f) How do you explain that in an intrinsic material the electron and hole concentrations have to be the same, but in an extrinsic material they are always different?

**Solution:**

- (a) When  $N$  atoms are close enough for their orbitals to interact, the degenerate (equal energy) energy states of the orbitals are forced to change slightly due to Pauli's exclusion principle, which says that no two electrons can have the exact same state (defined by 4 quantum numbers). Thus the split energy levels form a band of allowable energies for the electrons to occupy.

There will  $N$  orbitals in each band.

- (b) The basic principle followed is Pauli's exclusion principle  
 (c) The semiconductor B with smaller lattice constant will have the larger bandgap  
 (d) Momentum  
 (e) By applying a voltage across the semiconductors, and slowly increasing the energy of light, we can find out which one starts conducting at lower photon energies. That will have the lower bandgap.  
 (f) In intrinsic material, electrons and holes are created in pairs (called electron-hole pair) whether they are created by light or thermal energy. However, in extrinsic material, they originate from the dopant atoms, and hence the concentrations are different.

**Homework #4 solutions:**

1. The forbidden energy band of GaAs is 1.42 eV. (a) Determine the minimum frequency of an incident photon that can interact with a valence electron and elevate the electron to the conduction band. (b) What is the corresponding wavelength?

(a)  $E = h\nu$

Then

$$\nu = \frac{E}{h} = \frac{(1.42)(1.6 \times 10^{-19})}{(6.625 \times 10^{-34})} \Rightarrow$$

$$\underline{\nu = 3.43 \times 10^{14} \text{ Hz}}$$

(b)

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.43 \times 10^{14}} = 8.75 \times 10^{-7} \text{ m}$$

or

$$\underline{\lambda = 0.875 \text{ } \mu\text{m}}$$

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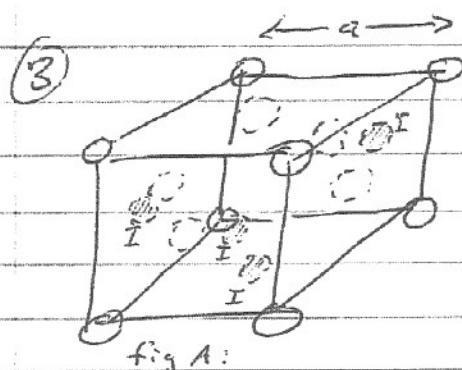
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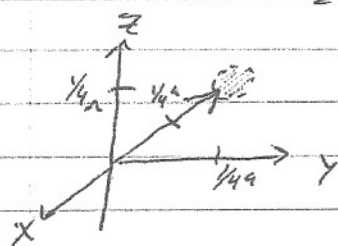
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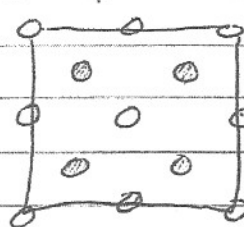
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HW #3

The location of the Si atoms are described as the following:

- consider the cubic structure above. There is an atom located on each corner of the cubic structure. This gives 8 atoms
- There is also an atom located on each side or face of the structure. Since there are six sides or faces there are an additional 6 atoms to those located on the corners.
- finally the location of the last atoms are the most difficult to visualize. There are 4 atoms located inside the cube. They are located  $\frac{1}{4}a$  in the  $x$  &  $y$  &  $z$  directions (see fig A: they are labeled w/ an I)



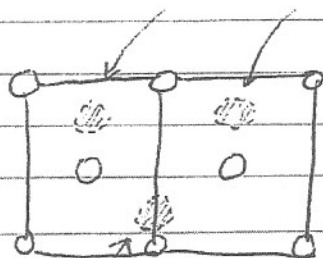
Top view:



Side view

Fig B.

Two of these atoms are located in the top half region & two are located on the bottom half region.



(2 there, other one is located directly behind this one)

2. Plot the Fermi-Dirac probability function, given by Equation (2.29), over the range  $-0.2 \leq (E - E_F) \leq 0.2$  eV for (a)  $T = 200$  K, (b)  $T = 300$  K, and (c)  $T = 400$  K.

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$T(K)$	$kT(eV)$
200	0.01727
300	0.0259
400	0.03453

$(E - E_F) (eV)$	$f_F(\text{for } 200K)$
0.2	$9.34 \times 10^{-6}$
0.1	$3.05 \times 10^{-3}$
0	0.5
-0.1	0.9970
-0.2	0.999991

$f_F(\text{for } 300K)$	$f_F(\text{for } 400K)$
$4.43 \times 10^{-4}$	$3.04 \times 10^{-3}$
$2.06 \times 10^{-2}$	$5.24 \times 10^{-2}$
0.5	0.5
0.9794	0.9476
0.99956	0.9970

3.

Determine the probability that an allowed energy state is occupied by an electron if the state is above the Fermi level by (a)  $kT$ , (b)  $3kT$ , and (c)  $6kT$ .

$$\begin{aligned} \text{(a)} \quad f_F &= \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \\ &= \frac{1}{1 + \exp(1)} = 0.269 \end{aligned}$$

(b)

$$f_F = \frac{1}{1 + \exp(3)} = 0.0474$$

(c)

$$f_F = \frac{1}{1 + \exp(6)} = 2.47 \times 10^{-3}$$

4.

Assume that the electrons in a material follow the Fermi–Dirac distribution function and assume that the Fermi level is 5.50 eV. Determine the temperature at which there is a 0.5 percent probability that a state 0.20 eV above the Fermi level is occupied by an electron.

$$f_F = 0.005 = \exp\left[\frac{-0.20}{0.0259}\right]$$

or

$$\exp\left(\frac{+0.20}{kT}\right) = \frac{1}{0.005} = 200$$

Then

$$\frac{+0.20}{kT} = \ln(200)$$

or

$$kT = \frac{0.20}{\ln(200)} = 0.03775 \text{ eV}$$

We can write

$$0.03775 = (0.0259) \left( \frac{T}{300} \right)$$

which yields

$$T = 437 \text{ K}$$