1. For the MOS differential pair with a common-mode voltage V_{CM} applied, as shown below, let $V_{DD}=V_{SS}=1V$, $k_n'=0.4mA/V^2$, $(W/L)_{1,2}=12.5$, $V_t=0.5V$, $I=0.2mA$, and $R_D=10k\Omega$ (neglect channel-length modulation). Assume that the current source I requires a minimum voltage of 0.4V to operate properly. (worth 2 problems)

(a) Find V_{GS} for each transistor.

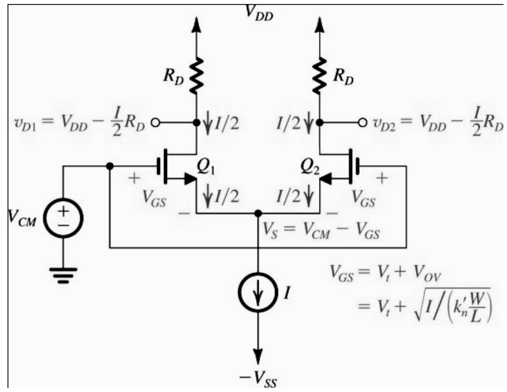
(b) For $V_{CM}=0$ find V_S, I_{D1}, I_{D2}, V_{D1}, and V_{D2}.

(c) Repeat (b) for $V_{CM}=0.3V$.

(d) Repeat (b) for $V_{CM}=-0.1V$.

(e) What is the highest permitted value of V_{CM} for which Q1 and Q2 remain in saturation?

(f) If the current source requires a minimum of 0.2V across it to operate correctly, what is the lowest value allowed for Vs and hence for V_{CM}?



note that $I_{D1} = I_{D2} = \dfrac{I}{2} = \dfrac{0.2\text{ mA}}{2}$

$= 0.1\text{ mA}$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$V_{ov} = \sqrt{\frac{2I_D}{k_n'(W/L)}} = \sqrt{\frac{2(0.1\text{ mA})}{0.4\text{ mA}/\text{V}^2(12.5)}}$$

$= 0.2\text{ V}$

$V_{Gs} = V_{nt} + V_{ov} = 0.5 + 0.2 = 0.7\text{ V}$

(b) If $V_{cm} = 0$,

$V_{s1} = V_{s2} = V_G - V_{GS} = 0 - 0.7 = -0.7\text{ V}$

$I_{D1} = I_{D2} = \dfrac{0.2\text{ mA}}{2} = 0.1\text{ mA}$

$V_{D1} = V_{D2} = V_{DD} - I_{D1}R_D$

$= 1\text{V} - (0.1\text{ mA})(10\text{ K}) = 0\text{ V}$

(c) Now, if $V_{ICM} = 0.1\text{ V}$,

$V_{S1} = V_{S2} = V_c - V_{GS} = 0.1\text{ V} - 0.7\text{ V}$

$= -0.6\text{V}$

Since I is a constant current source,
I_{D1} and I_{D2} remain at 0.1 mA
This means that
V_{D1} and V_{D2} are still 0 V

(d) $V_{ICM} = -0.1\text{ V}$,

$V_{S1} = V_{S2} = V_G - V_{GS} = -0.1\text{ V} - 0.7$

$= -0.8\text{ V}$

Still, $I_{D1} = I_{D2} = 0.1\text{ mA}$

$V_{D1} = V_{D2} = 0\text{ V}$

(e) $V_{Cm}(\text{max}) = V_{DD} - I_D R_D - V_{ov} + V_{GS}$

$= 1 - (0.1\text{ mA})(10\text{ K}) - 0.2 + 0.7 = +0.5\text{V}$

(f) $V_S(\text{min}) = -V_{SS} + V_{CS}(\text{min})$

$= -1 + 0.2 = -0.8\text{ V}$

$V_{CM}(\text{min}) = V_S(\text{min}) + V_{GS} = -0.8\text{ V} + 0.7\text{ V}$

$= -0.1\text{ V}$

2. For the differential amplifier of Problem 1, let V_{G2}=0 and V_{G2}= V_{id}. Find the value of V_{id} that corresponds to each of the following situations:

(a) i_{D1}=i_{D2}=0.1mA; (b) i_{D1}=0.15mA and i_{D2}=0.05mA; (c) i_{D1}=.2mA and i_{D2}=0(Q2 just cuts off); (d) i_{D1}=0.05mA and i_{D2}=0.15mA; (e) i_{D1}=0 (Q1 just cuts off) and i_{D2}=0.2mA. For each case, find v_s, v_{D1}, v_{D2}, and (v_{D2}-v_{D1}).

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'(W/L)}} = \sqrt{\frac{2(0.1\text{ mA})}{0.4\text{mA}/V^2(12.5)}} = 0.2\text{ V}$$

(a) $V_{GS} = V_{OV} + V_t = 0.2\text{ V} + 0.5\text{ V} = 0.7\text{ V}$

$V_S = V_G - V_{GS} = 0 - 0.7\text{ V} = -0.7\text{ V}$

$V_{D1} = V_{D2} = V_{DD} - i_{D1}R_D = 1.0\text{ V} - 0.1\text{ mA}$
$(10\text{ k}\Omega) = 0\text{ V}$

$V_{D2} - V_{D1} = 0\text{ V}$

(b) For $i_{D1} = 0.15$ mA, $i_{D2} = 0.05$ mA,

$$i_{D1} = \frac{I}{2} + \frac{I}{V_{OV}} \cdot \frac{v_{id}}{2} \rightarrow v_{id} = \left(\frac{2i_{D1}}{I} - 1\right) \cdot V_{OV}$$

$$v_{id} = \left[\frac{2(0.15\text{ mA})}{0.2\text{ mA}} - 1\right](0.2\text{ V}) = 0.1\text{ V}$$

$$V_{GS1} = \sqrt{\frac{2(0.15\text{ mA})}{0.4\text{ mA}/\text{V}^2(12.5)}} + 0.5\text{V} = 0.745\text{ V}$$

$V_S = V_G - V_{GS1} = 0.1\text{ V} - 0.745\text{ V} = -0.645\text{ V}$

$V_{D1} = V_{DD} - i_{D1}R_D = 1.0\text{ V} - 0.15\text{ mA}(10\text{ k}\Omega)$
$= -0.5\text{ V}$

$V_{D2} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = +0.5\text{ V}$

$V_{D2} - V_{D1} = 1.0\text{ V}$

(c) $i_{D1} = 0.2\text{ mA},\ i_{D2} = 0:$

$V_{G1} = v_{id} = \sqrt{2} \cdot V_{OV} = 1.414(0.2\text{ V}) = 0.283\text{ V}$

$$V_{GS} = \sqrt{\frac{2(0.2\text{ mA})}{0.4\text{ mA}/\text{V}^2(12.5)}} + 0.5\text{V} = 0.783$$

$V_S = V_G - V_{GS} = 0.283\text{V} - 0.783\text{V} = -0.5\text{V}$

$V_{D1} = 1.0\text{ V} - (0.2\text{ mA})(10\text{ k}\Omega) = -1.0\text{ V}$

$V_{D2} = +1.0\text{ V}$

$V_{D2} - V_{D1} = 2.0\text{ V}$

(d) $\left.\begin{matrix} i_{D1} = 0.05\text{ mA} \\ i_{D2} = 0.05\text{ mA} \end{matrix}\right\}$ opposite case of (b)

For example,

$$v_{id} = \left[\frac{2(0.05\text{ mA})}{0.2\text{ mA}} - 1\right](0.2\text{ V}) = -0.1\text{ V}$$

$$V_{GS} = \sqrt{\frac{2(0.05\text{ mA})}{0.4\text{ mA}/\text{V}^2(12.5)}} + 0.5\text{ V} = 0.641\text{ V}$$

$V_S = V_G - V_{GS} = -0.1\text{ V} - 0.641\text{V} = -0.741\text{ V}$

$V_{D1} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = +0.5\text{ V}$

$V_{D2} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = -0.5\text{ V}$

$V_{D2} - V_{D1} = -1.0\text{ V}$

(e) $i_{D1} = 0\text{ mA},\ i_{D2} = 0.2\text{ mA}$ is the opposite of (c):

$v_{id} = -\sqrt{2}(V_{OV}) = -\sqrt{2}(0.2\text{ V}) = -0.283\text{ V}$

For $i_{D2} = 0.2\text{ mA},\ V_{GS2} = 0.783\text{ V}$, So that

$V_S = -0.783\text{ V}$

$V_{D1} = 1.0\text{ V},$

$V_{D2} = -1.0\text{ V} \rightarrow V_{D2} - V_{D1} = -2\text{ V}$

The results are summarized in the following table:

Case	V_{id} (V)	i_{D1} (mA)	i_{D2} (mA)	V_s(V)	V_{D1}(V)	V_{D2} (V)	$V_{D2}-V_{D1}$(V)
(a)	0	0.1	0.1	−0.7	0	0	0
(b)	0.1	0.15	0.05	−0.645	−0.5	0.5	1.0
(c)	0.283	0.2	0	−0.5	−1.0	1.0	2.0
(d)	−0.1	0.05	0.15	−0.741	0.5	−0.5	−1.0
(e)	−0.283	0	0.2	−0.783	1.0	−1.0	−2.0

3. Consider the differential amplifier specified in Problem 1 with G2 grounded and $V_{G1}= V_{id}$. Let V_{id} be adjusted to the value that causes i_{D1}=0.11mA and i_{D2}=0.09mA. Find the corresponding values of V_{GS2}, V_S, V_{GS1}, and hence V_{id}. What is the difference output voltage (V_{D2}-V_{D1})? What is the voltage gain (V_{D2}-V_{D1})/V_{id}? What value of V_{id} results in i_{D1}=0.09mA and i_{D2}=0.11mA?

$V_{G1} = v_{id}$ $i_{D1} = 0.11$ mA

$V_{G2} = 0$ $i_{D2} = 0.09$ mA

$$I_D = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

For Q_1:

$$0.11 \text{ m} = \frac{1}{2} 5 \text{ m}(V_{GS1} - 0.5)^2$$

$\rightarrow V_{GS1} = 0.71$ V

For Q_2:

$$0.09 \text{ m} = \frac{1}{2} 5 \text{ m}(V_{GS2} - 0.5)^2$$

$\rightarrow V_{GS2} = 0.69$ V

$V_S = -V_{GS2} = -0.69$ V

$v_{id} = V_S + V_{GS1} = -0.69 + 0.71$
$= 0.02$ V

$V_{D2} - V_{D1} = 10 \text{ k}\Omega\ (i_{D1} - i_{D2})$
$= 10$ kV (0.11 2 0.09) m
$= 0.2$ V

thus

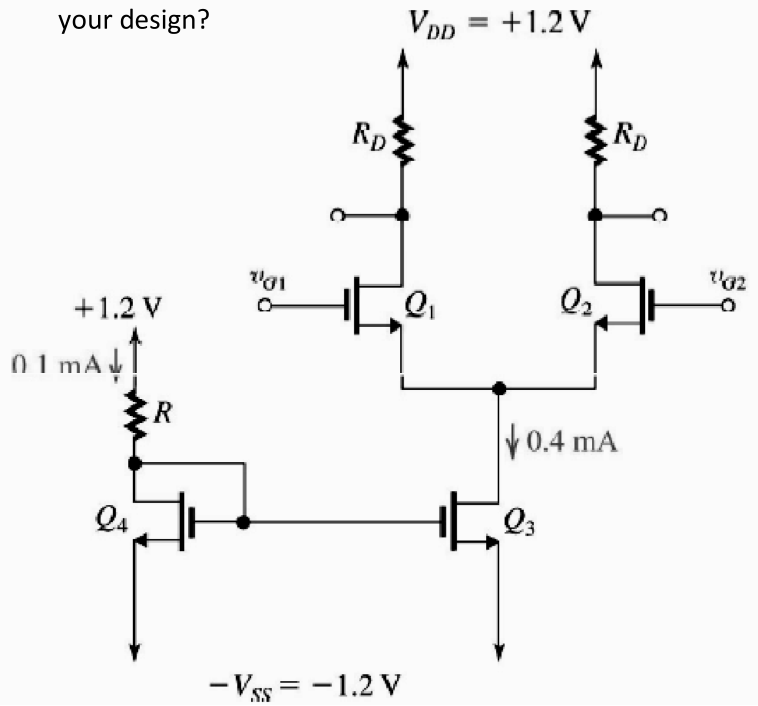
$$\frac{V_{D2} - V_{D1}}{v_{id}} = \frac{0.2}{0.02} = 10$$

when $i_{D1} = 0.09$ mA and

$i_{D2} = 0.11$ mA

is the reverse condition from the case we just studied, thus $v_{id} = -0.02$ V

4. Design the circuit shown below to obtain a dc voltage of +2V at each of the drains of Q1 and Q2 when $V_{G1}=V_{G2}=0V$. Operate all transistors at $V_{ov}=0.2V$ and assume that for the process technology in which the circuit is fabricated, $V_{tn}=0.5V$ and $k_n'=250\mu A/V^2$. Neglect channel-length modulation. Determine the values of R, R_D, and the W/L ratios of Q1, Q2, Q3, and Q4. What is the input common-mode range for your design?



$$V_{GS} = V_{tn} + V_{OV} = 0.5\ V + 0.2\ V = 0.7\ V$$

$$V_{D4} = V_{G4} = -V_{SS} + V_{GS} = -1.2\ V + 0.7\ V = -0.5\ V$$

$$R = \frac{V_{DD} - V_{D4}}{0.1\ mA} = \frac{1.2V - (-0.5\ V)}{0.1\ mA} = 17\ k\Omega$$

$$R_D = \frac{V_{DD} - V_{D1}}{0.4\ mA/2} = \frac{1.2V - 0.2\ V}{0.2\ mA} = 5\ k\Omega$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{0.4\ mA}{2}\left[k_n'\,V_{OV}^{\,2}\right]$$

$$= 0.2\ mA[(0.25\ mA/V^2)(0.2V)^2]^{-1} = 20$$

$$\left(\frac{W}{L}\right)_3 = 0.4\ mA[0.01\ mA]^{-1} = 40$$

$$\left(\frac{W}{L}\right)_4 = 0.1\ mA[0.01\ mA]^{-1} = 10$$

$$V_{Cm(max)} = V_{tn} + V_{DD} - (I/2)R_D$$

$$= 0.5\ V + 1.2\ V - (0.2\ mA)(5\ k\Omega) = 0.7\ V$$

$$V_{Cm(min)} = -V_{SS} + V_{OV3} + V_{tn} + V_{OV1}$$

$$= -1.2\ V + 0.2\ V + 0.5\ V + 0.2\ V = -0.3\ V$$

5. Design a MOS differential amplifier to operate from ±1V power supplies and dissipate no more than 2mW in the equilibrium state. The differential voltage gain A_d is to be 5 V/V and the output common-mode dc voltage is to be 0.5V. (Note: This is the dc voltage at the drains). Assume $k_n'=400\mu A/V^2$ and neglect the Early effect. Specify the required values of I, R_D, and W/L.

+1 V supplies not more than 2 mW $A_d = 5$ V/V

$V_D = 0.5$ V $K_n' = \mu_n C_{ox} = 0.4\ mA/V^2$

$$I = \frac{2\ mW}{1\ V - (-1\ V)} = 1\ mA$$

$$R_D = \frac{1\ V - 0.5\ V}{1/2\ I} = \frac{0.5\ V}{0.5\ mA} = 1\ k\Omega$$

$$g_m = \frac{A_d}{R_D} = \frac{5\ V/V}{1\ k\Omega} = 5\ mA/V$$

$$V_{OV} = \frac{I}{g_m} = \frac{1\ mA}{5\ mA/V} = 0.2\ V$$

$$\frac{W}{L} = 2(I/2)/(k_n'\,V_{OV}^2)$$

$$= 1\ mA/(0.4\ mA/V^2 \cdot (0.2\ V)^2) = 62.5$$

BECAUSE WE PICKED I = 1 mA THIS IS THE SOLUTION WITH THE HIGHEST ALLOWABLE POWER. THIS SOLUTION WILL ALSO THEREFORE HAVE THE WIDEST RANGE OF DIFFERENTIAL MODE OPERATION. AN INFINITE NUMBER OF OTHER SOLUTIONS EXIST.

6. An NMOS differential amplifier is operated at a bias current I of 0.4mA and has a W/L ratio of 32, $k_n' = \mu_n C_{ox} = 200\mu A/V^2$, V_A=10V, and R_D=5kΩ. Find $V_{ov}=(V_{GS}-V_t)$, g_m, r_o, and A_d.

$I = 0.4\text{ mA}\quad W/L = 32\quad k_n = \mu_n C_{ox}$

$= 200\ \mu\text{A}/\text{V}^2$

$V_A = 10\text{ V}\qquad R_D = 5\text{ k}\Omega$

$$V_{OV}' = \sqrt{I/k_n'\left(\frac{W}{L}\right)} = \sqrt{0.4/(0.2\cdot 32)}$$

$= 0.25\text{ V}$

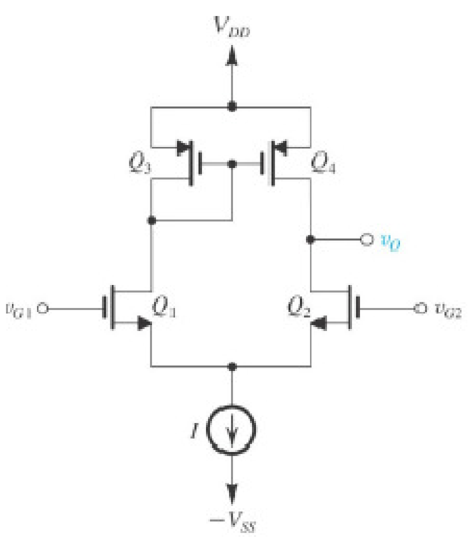
$$g_m = \frac{I}{V_{OV}} = \frac{0.4\text{ mA}}{0.25\text{ V}} = 1.6\text{ mA/V}$$

$$r_O = \frac{V_A}{I_D} = \frac{10\text{ V}}{0.2\text{ mA}} = 50\text{ k}\Omega$$

$$A_d = g_m(R_D \| r_O) = 1.6(5 \| 50)$$

$= 1.6(4.54) = 7.3\text{ V/V}$

7. An active-loaded NMOS differential amplifier operates with a bias current I of 100µA. The NMOS transistors are operated at V_{ov}=0.2V and the PMOS dives at $|V_{ov}|$=0.3V. The Early voltages are 20V for the NMOS and 12V for the PMOS transistors. Find G_m, R_o(output R), and A_d. For what value of load resistance is the gain reduced by a factor of 2?



$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

$$g_{m1} = g_{m2} = \frac{I_D}{V_{ov}/2} = \frac{50\ \mu\text{A}}{0.2\text{ V}/2} = 0.5\text{ mA/V}$$

$$G_m = g_{m1} = 0.5\text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I_D} = \frac{20\text{ V}}{0.05\text{ mA}} = 400\text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{12\text{ V}}{0.05\text{ mA}} = 240\text{ k}\Omega$$

$$R_o = r_{o2} \| r_{o4} = 400\text{ k} \| 240\text{ k} = 150\text{ k}\Omega$$

$$A_d = G_m R_o = (0.5\text{ mA/V})(150\text{ k}) = 75\text{ V/V}$$

Gain will be reduced by a factor of 2 if

$R_L = R_o = 150\text{ k}\Omega$