

1. For a cascade MOS mirror utilizing devices with  $V_t=0.5V$ ,  $\mu_n C_{ox}=387\mu A/V^2$ ,  $W/L=3.6\mu m/0.36\mu m$ ,  $V'_A=10V/\mu m$ , and  $I_{REF}=100\mu A$ . Find the minimum voltage required at the output and the output resistance.



$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{REF} = 100 \ \mu\text{A}$$
  
Since  $I_D = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right) V^2 ov$   
 $V_{OV} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)}} = \sqrt{\frac{2(100 \ \mu\text{A})}{(387 \ \mu\text{A}/V^2) \left(\frac{3.6}{0.36}\right)}}$   
= 0.23 V  
The minimum output voltage is  
 $V_{\kappa n} + 2V_{ov} = 0.5 \ \text{V} + 2(0.23 \ \text{V}) = 0.96 \ \text{V}$   
To obtain the output resistance,  $R_O$ , we need  $g_{m3}$ .  
 $g_{m3} = \frac{I_{D3}}{V_{OV}/2} = \frac{2(0.1 \ \text{mA})}{0.23 \ \text{V}} = 0.87 \ \text{mA/V}$   
 $r_{O2} = r_{O3} = \frac{V_{A'}(L)}{I_D} = \frac{(5V/\mu\text{m})(0.36 \ \mu\text{m})}{0.1 \ \text{mA}}$   
= 18 k $\Omega$ . From eq. (7.77)  
 $R_0 \approx g_{m3}r_{o3}r_{o2} = (0.87 \ \text{mA/V})(18 \ \text{k}\Omega)^2$   
= 282 k $\Omega$ 

2. The BJT in the circuit below has  $v_{BE}$ =0.7V,  $\beta$ =100 and V<sub>A</sub>=50V. Find Ro.

$$I_{E} = \frac{V_{E} - V_{EE}}{R_{E}} = \frac{-0.7 - (-5)}{10 \text{ k}\Omega} = 0.43 \text{ mA}$$

$$g_{m} = \frac{I_{c}}{V_{r}} = \frac{0.43 \text{ mA}}{25 \text{ mV}} = 17.2 \text{ mA/V}$$

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$$R_{o} = r_{o}[1 + g_{m}(Re \parallel r_{\pi})]$$

$$R_{o} = (116.3 \text{ k}\Omega)$$

$$[1 + (17.2 \text{ mA/V})(10 \text{ k}\Omega \parallel 5.8 \text{ k}\Omega)] = 7.46 \text{ M}\Omega$$



## Examples #3

3. Design the basic BJT differential amplifier shown below to provide a differential input resistance of at least  $10k\Omega$  and a differential voltage gain of 100 V/V. The transistor  $\beta$  is specified to be at least 100. The available positive power supply is 5V.



$$g_{m} = \frac{I_{C}}{V_{T}}$$

$$R_{id} \ge 10 \text{ k}\Omega A_{d} = 100 \text{ V/V } V_{CC} = 5 \text{ V} \qquad I_{C} = g_{m}V_{T} = 20 \text{ mA/V} \cdot 25 \text{ mV} = 0.5 \text{ mA}$$

$$R_{id} = 10^{4} = 2r\pi = 2 \times \frac{\beta}{g_{m}} \qquad I = 2I_{C} = 1 \text{ mA}$$

$$Eqn 8.93 A_{d} = g_{m}R_{C}$$

$$g_{m} = \frac{2\beta}{R_{id}} = 20 \text{ mA/V} \qquad R_{C} = \frac{A_{d}}{g_{m}} = \frac{100}{20 \text{ mA/V}} = 5 \text{ k}\Omega$$



4. A BJT differential amplifier is biased from a 1mA constant-current source and includes a  $200\Omega$  resistor in each emitter. The collectors are connect to V<sub>cc</sub> via  $12k\Omega$  resistors. A differential input signal of 0.1V is applied between the two bases.

- (a) Find the signal current in the emitters ( $i_o$ ) and the signal voltage  $v_{be}$  for each BJT.
- (b) What is the total emitter current in each BJT?
- (c) What is the signal voltage at each collector? Assume  $\alpha$ =1.
- (d) What is the voltage gain realized when the output is taken between the two collectors?

$$I = 1\text{mA} \quad R_E = 200 \text{ V} \quad R_C = 12 \text{ kV}$$

$$V_{id} = 100 \text{ mV}$$

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}/2} = 50 \Omega$$
(a)  $i_e = \frac{V_{id}}{2(r_e + R_E)} = \frac{100 \text{ mV}}{2(250 \Omega)} = 0.2 \text{ mA}$ 

$$V_{be} = \frac{r_e}{r_e + R_E} \left(\frac{V_{id}}{2}\right) = 10 \text{ mV}$$
(b)
$$i_{E1} = \frac{I}{2} + i_e = 0.5 \text{ mA} + 0.2 \text{ mA} = 0.7 \text{ mA}$$

$$i_{E2} = \frac{I}{2} - i_e = 0.5 \text{ mA} - 0.2 \text{ mA} = 0.3 \text{ mA}$$
(c)
$$V_{C1} = -i_C R_C \approx -i_e \cdot R_C = -0.2 \text{ mA} \cdot 12 \text{ k\Omega}$$

$$= -2.4 \text{ V}$$

$$V_{C2} = +i_C R_C \approx i_e \cdot R_C = 0.2 \text{ mA} \cdot 12 \text{ k\Omega} = +2.4 \text{ V}$$
(d)  $A_d = V_{od} / V_{id} = \frac{4.8 \text{ V}}{100 \text{ mV}} = 48 \text{ V} / \text{V}$ 

5. Find the voltage gain and the input resistance of the amplifier shown below assuming  $\beta$ =100.



$$\frac{v_o}{v_i} = \frac{\alpha R_{C2}}{(2r_e + 2R_e)}$$

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{I/2} = \frac{25 \text{ mV}}{0.4 \text{ mA}/2} = 125 \Omega$$
Assuming  $\alpha \simeq 1$ ,  

$$\frac{v_o}{v_i} \simeq \frac{25 \text{ k}}{(2)(125) + 2(125)} = 50 \text{ V/V S}$$

$$R_i = (\beta + 1)(2r_e + 2R_e)$$

$$= (101)[2(125) + 2(125)] = 50.5 \text{ k}\Omega$$

6. A bipolar differential amplifier with I=0.5mA utilizes transistors for which V<sub>A</sub>=10V and  $\beta$ =100 and R<sub>c</sub>=10k $\Omega$ .

(a) the differential gain

(b) the common mode gain and the CMRR if the bias current I is generated using a simple current mirror.

(c) the common mode gain and the CMRR if the bias current I is generated using a Wilson mirror.



Equivalent

$$R_{EE} = r_{o3} = \frac{V_A}{I} = \frac{10 \text{ V}}{0.5 \text{ mA}} = 20 \text{ k}\Omega$$

$$r_{e2} = r_{e1} = r_e = \frac{V_T}{I/2} = \frac{25 \text{ mV}}{0.5 \text{ mA}/2} = 100 \Omega$$
Since  $\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 1$ ,
$$A_d \simeq \frac{R_C}{r_e} = \frac{10 \text{ k}}{0.1 \text{ k}} = 100 \text{ V/V}$$
(b)  $A_{cm} = \frac{\alpha \Delta R_C}{2R_{EE} + r_e} = \frac{(0.02)(10 \text{ k})}{2(20 \text{ k}) + 0.1 \text{ k}}$ 

$$= 0.00499 \text{ V/V}$$
CMRR(dB) = 20 log\_{10}  $\left| \frac{A_d}{A_{cm}} \right| = 20 \log_{10} \left| \frac{100}{0.00499} \right|$ 

$$= 86 \text{ dB}$$





7. Design a BJT differential amplifier that provides two single-ended outputs (at the collectors). The amplifier is to have a differential gain (to each of the two outputs) of at least 100 V/V, a differential input resistance  $\geq 10k\Omega$  and a common mode gain (to each of the two outputs) no greater than 0.1 V/V. Use a 2mA current source for biasing. Give the complete circuit with component values and suitable power supplies that allow for  $\pm 2V$  swing at each collector. Specify the minimum value that the output resistance of the bias current source must have. The BJTs available have  $\beta \geq 100$ . What is the value of the input common mode resistance when the bias source has the lowest acceptable resistance?

At  $I_c = 1$  mA,  $r_c = 25 \Omega$   $R_{id} = (\beta + 1) 2r_c = 5.05 \text{ k}\Omega < 10 \text{ K}$  $\Rightarrow$  need emitter resistors

In this case:



$$R_{id} = (\beta + 1) (2r_e + 2R_E) = 10 \text{ k}\Omega$$

$$A_d = 100 = \frac{R_C}{2(R_E + r_e)}$$

$$\Rightarrow R_C = 10 \text{ K}$$

$$A_{cm} = 0.1 \ge \frac{R_C}{2R_o + R_E + r_e}$$

$$\Rightarrow R_o \ge 50 \text{ k}\Omega$$
For  $\pm 2 \text{ swing } V_{c1} = V_{c2}$ 

$$= V_{CC} - \frac{I}{2}R_C = 2$$

$$\Rightarrow V_{CC} = 2 + 10^{-3} \times 10^4 = 12 \text{ V}$$
Choose  $V_{cc} = \pm 15 \text{ V}$  although 12 V is ok.  
 $2 R_{kon} = (\beta + 1) (2 R_e + R_E + r_e)$ 

$$\Rightarrow R_{icm} = 5 \text{ M}\Omega$$