1. The noninverting op-amp configuration shown to the right provides a direct implementation of a feedback loop.

(a) Assume that the op amp has infinite input resistance and zero output resistance. Find an expression for the feedback factor β.

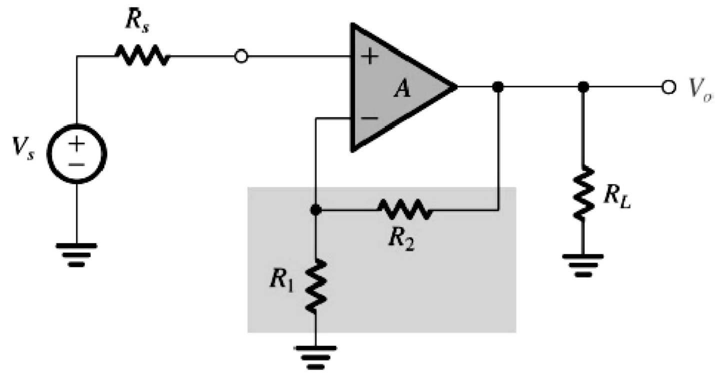
(b) Find the condition under which the closed-loop gain A_f is almost entirely determined by the feedback network.

(c) If the open-loop gain $A=10^4$ V/V, find R_2/R_1 to obtain a closed-loop gain A_f of 100 V/V.

(d) What is the amount of feedback in decibels?

(e) If V_s=1V, find V_o, V_f, and V_i.

(f) If A decreases by 20%, what is the corresponding decrease in A_f?



(a) β=R1/(R1+R2)

(b) A_f=A/(1+Aβ) if Aβ>>1 then A_f=A/(Aβ)= 1/(β)

(c) A_f= A/(1+Aβ)= $10^4/(1+10^4β)$=100

$10^4 =(1+10^4β)*100$

$\{(10^4/100)-1\}/10^4=β$ => β=9.9m=R1/(R1+R2) => R1=(R1+R2)*9.9m => R1-9.9mR1=9.9mR2

.990R1=9.9mR2 => (R2/R1)=(0.990/9.9m)=**100**

(d) (1+Aβ)= $(1+10^4*9.9\text{m})$=100 which is 20log(100)=40dB

(e) Vo=A_fV_s=100*1=**100V**; V_f=βVo=9.9m*100=**0.990V**; V_i=Vo/A=100/10^4=**10mV** **Note that in order to achieve Vo=100V, the power supply would need to be that large!**

(f) If A decreases by 20% => A=0.8×10^4V/V; Af=$0.8\times10^4/(1+0.8\times10^4\times9.9\text{m})$=**99.75V/V**

2. Consider the noninverting op-amp of Problem 1. Let the open-loop gain A have a low-frequency value of 10^4 and a uniform -6dB/octave rolloff at high frequencies with a 3dB frequency of 100 Hz. Find the low-frequency gain and the upper 3dB frequency of a closed-loop amplifier with R_1=1kΩ and R_2=90kΩ.

A=10^4 and f_H=100Hz

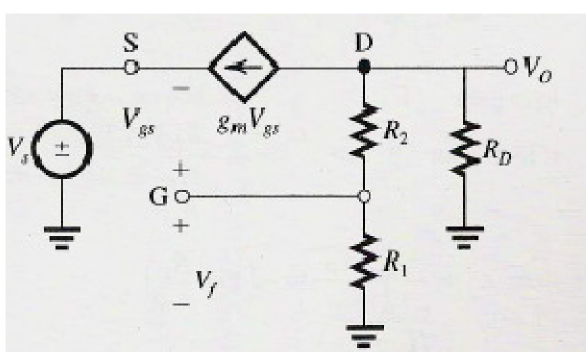
β=R1/(R1+R2)=1k/91k=11m

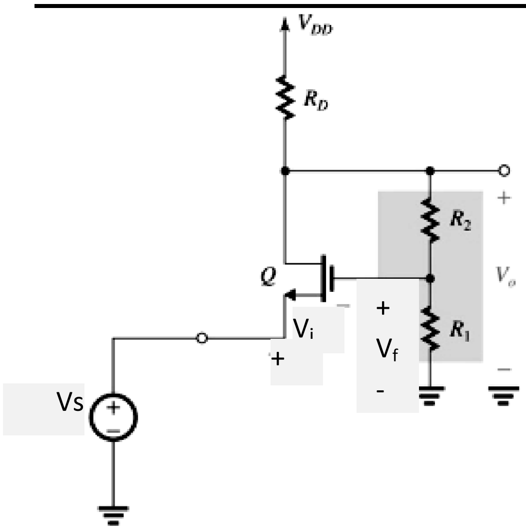
(1+Aβ)= $(1+10^4*11\text{m})$=111

A_f= A/(1+Aβ)= $10^4/(111)$=90V/V

$f_{Hf}=f_H(1+Aβ)$=100*(111)=11.1kHz

3. For the circuit below let $(R_1+R_2)\gg R_D$. Using small-signal analysis, find expressions for the open-loop gain $A\equiv V_o/V_i$; the feedback factor $β\equiv V_f/V_o$; and the closed loop gain $A_f\equiv V_o/V_s$. For $Aβ\gg1$, find an approximate expression for A_f. Neglect r_o.





Open-loop gain: without R_2 and R_1 ang the gate grounded:

$$V_O = -g_m V_{gs} \times R_D$$

$$V_{gs} = -V_S \Rightarrow A = g_m R_D$$

Feed-back factor:

$$\beta = \frac{V_f}{V_O} \Rightarrow V_f = \frac{R_1}{R_1 + R_2} \cdot V_O \rightarrow \beta = \frac{R_1}{R_1 + R_2}$$

Closed-loop gain A_f

$$A_f = \frac{V_O}{V_S}$$

$$V_O = -g_m V_{gs} \times \{(R_1 + R_2) \parallel R_D\}$$

$$= -g_m V_{gs} \cdot \frac{(R_2 + R_1)R_D}{R_2 + R_1 + R_D}$$

but $R_2 + R_1 >> R_D \quad R_2 + R_1 + R_D \simeq R_2 + R_1$

$$\rightarrow V_O = \left(-g_m V_{gs} \cdot \frac{(R_2 + R_1)R_D}{R_2 + R_1}\right)$$

$$= -g_m V_{gs}, R_D$$

$$-V_{gs} = V_S - V_f = V_S - \frac{R_1 V_O}{R_1 + R_2}$$

$$\Rightarrow V_O = g_m R_D \left\{ V_S - \frac{R_1 V_O}{R_1 + R_2} \right\}$$

$$\Rightarrow V_O \left\{ 1 + \frac{g_m R_D R_1}{R_1 + R_2} \right\} = g_m R_D V_S$$

Thus:

$$A_f = \frac{V_O}{V_S} = \frac{g_m R_D}{1 + g_m R_D R_1 / (R_1 + R_2)}$$

if $A \cdot \beta >> 1 \quad \Rightarrow (g_m R_D) \cdot R_1 / (R_1 + R_2) >> 1$

$$A_f = \frac{g_m R_D}{g_m R_D R_1 / (R_1 + R_2)} = \frac{R_1 + R_2}{R_1}$$

$$= 1 + \frac{R_2}{R_1}$$

4. A negative-feedback amplifier has a closed-loop gain $A_f = 100$ and an open-loop gain $A=10^4$. What is the feedback factor β? If a manufacturing error results in a reduction of A to 10^3, what closed-loop gain results? What is the percentage change in A_f corresponding to this factor of 10 reduction in A?

$$A_f = \frac{A}{1+A\beta} = 100$$

$$\Rightarrow A\beta = \frac{10^4}{100} - 1 = 99$$

$$\Rightarrow \beta = \frac{99}{10^4} = 9.9 \times 10^{-3}$$

If $A = 10^3$, then

$$A_f = \frac{10^3}{1 + 10^3 \times 9.9 \times 10^{-3}} = 91.74$$

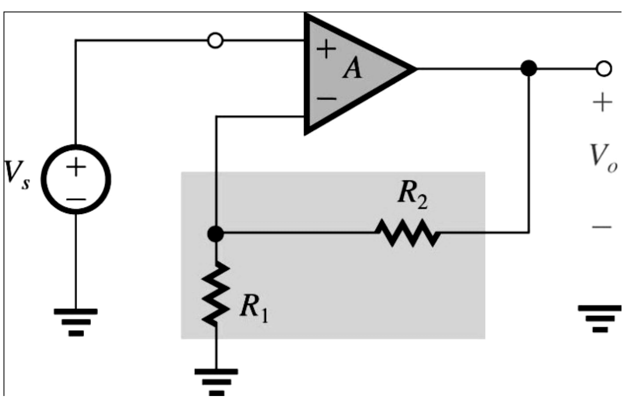
$$\frac{\Delta A_f}{A_f} = \frac{91.74 - 100}{100} = -8.26\%$$

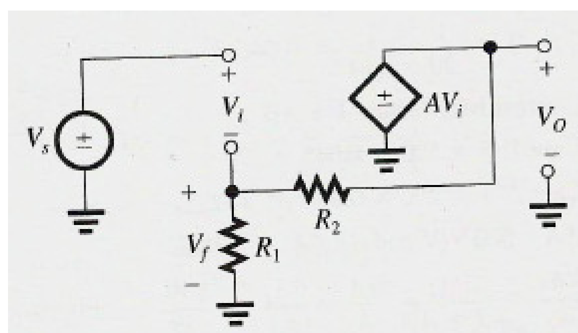
5. Consider the op-amp circuit shown below where the op amp has infinite input resistance and zero output resistance but finite open-loop gain A.

(a) Convince yourself that $\beta=R_1/(R_1+R_2)$

(b) If $R_1=10k\Omega$, find R_2 that results in $A_f = 10$ V/V for the following three cases: (i) A=1000V/V; (ii) A = 100 V/V; (iii) A=12 V/V.

(c) For each of the three cases in (b), find the percentage change in A_f that results when A decreases by 20%. Comment on the results.





$$V_f = \beta V_O = \frac{R_1}{R_1 + R_2} \cdot V_O$$

$$\Rightarrow \beta = \frac{R_1}{R_1 + R_2}$$

(b) $R_1 = 10\ \text{k}\Omega$, $A_f = 10$ V/V, what is R_2 if:

(i) $A = 1000$ V/V

$$A_f = \frac{A}{1 + \beta A} \Rightarrow \beta = \frac{1}{A_f} - \frac{1}{A}$$

$$= \frac{1}{10} - \frac{1}{10^3} = 0.099$$

$$\beta = \frac{R_1}{R_1 + R_2} \Rightarrow R_2 = R_1\left(\frac{1-\beta}{\beta}\right)$$

$$= 10\ \text{K}\frac{(1 - 0.099)}{0.099} = 91.01\ \text{k}\Omega$$

(ii) $A = 100$ V/V

$$\beta = \frac{1}{10} - \frac{1}{100} = 0.09;$$

$$R_2 = 10\ \text{K}\frac{(1 - 0.09)}{0.09} = 101.11\ \text{K}$$

(iii) $A = 12$

$$\beta = \frac{1}{10} - \frac{1}{12} = 0.0167;$$

$$R_2 = 10\ \text{K}\frac{(1 - 0.0167)}{0.0167} = 588.8\ \text{k}\Omega$$

(c) if A decreases by 20%

(i) $A = 0.8 \times 1000 = 800$ V / V

$$A_f = \frac{800}{1 + (0.099)(800)} = 9.975$$

$$\frac{\Delta A_f}{A} = \frac{9.975 - 10}{10} = -0.25\%$$

(ii) $A = 0.8 \times 100 = 80$ V / V

$$A_f = \frac{80}{1 + (0.09)(80)} = 9.756$$

$$\frac{\Delta A_f}{A} = \frac{9.756 - 10}{10} = -2.44\%$$

(iii) $A = 0.8 \times 12 = 9.6$ V / V

$$A_f = 9.6 / (1 + (0.0167)(9.6)) = 8.27$$

$$\frac{\Delta A_f}{A} = \frac{8.27 - 10}{10} = -17.26\%$$

6. Consider an amplifier having a midband gain A_M and a low-frequency response characterized by a pole at $s=-\omega_L$ and a zero at $s=0$. Let the amplifier be connected in a negative-feedback loop with a feedback factor β. Find an expression for the midband gain and the lower 3dB frequency of the closed-loop amplifier. By what factor have both changed?

$$A(S) = Am \frac{S}{S + W_L}$$

$$A_f(S) = \frac{Am\dfrac{S}{S+W_L}}{1+\dfrac{Am\,S}{S+W_L}\beta} = \frac{Am\,S}{S+W_L+Am\beta S}$$

$$= \frac{Am}{1+Am\beta}\cdot\frac{S}{S+\dfrac{W_L}{1+Am\beta}}$$

Thus

$$Am_f = \frac{Am}{1+Am\beta}$$

$$W_{Lf} = \frac{W_L}{1+Am\beta}$$

Both decreased by same amount

7. A capacitively coupled amplifier has a midband gain of 1000 V/V, a single high-frequency pole at 10kHz, and a single low frequency pole at 100 Hz. Negative feedback is employed so that the midband gain is reduced to 10. What are the upper and lower 3dB frequencies of the closed loop gain?

$$A_f = \frac{A}{1+A\beta} = 10$$

$$\rightarrow 10 = \frac{1000}{1+A\beta} \Rightarrow (1+A\beta) = 100$$

$$f_L' = f_L/(1+A\beta) = 100/100 = 1\text{ Hz}$$

$$f_H' = f_H \times (1+A\beta) = 10\text{K} \times 100 = 1\text{ MHz}$$

9. A series-shunt feedback amplifier employs a basic amplifier with input and output resistances each of 2kΩ and gain A=1000 V/V. The feedback factor β=0.1V/V. Find the gain A_f, the input resistance R_{if}, and the output resistance R_{of} of the closed loop amplifier.

$$A_F = \frac{A}{1+A\beta} = \frac{1\times10^3}{1+1\times10^3\cdot0.1}$$
$$= 9.9\text{ V/V}$$

$$R_{if} = R_i(1+A\beta) = 2(101) = 202\text{ k}\Omega$$

$$R_{of} = \frac{R_O}{(1+A\beta)} = \frac{2}{101} = 19.8\ \Omega$$

10. (worth 2 problems) The circuit below shows a series-shunt amplifier in which the three MOSFETs are sized to operate at $|V_{ov}|$=0.2V. Let $|V_t|$=0.5V and $|V_A|$=10V. The current sources utilize single transistors and thus have output resistances equal to r_o.

(a) Show that the feedback is negative.

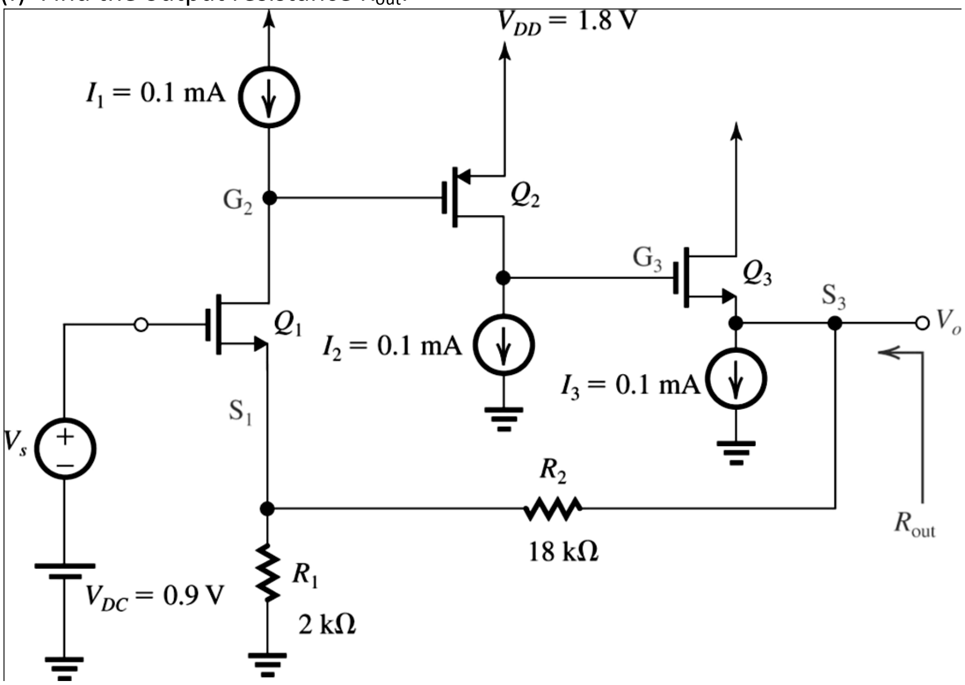
(b) Assuming the loop gain to be large, what do you expect the closed loop voltage gain Vo/Vs to be approximately?

(c) If Vs has a zero dc component, find the dc voltages at nodes S1, G2, S3, and G3. Verify that each of the current sources has the minimum required dc voltage across it for proper operation.

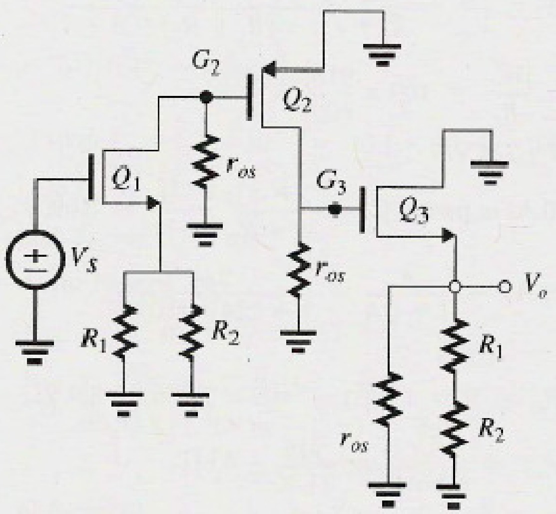
(d) Find the *A* circuit. Calculate the gain of each of the three stages and the overall voltage gain, *A*.

(e) Find β.

(f) Find the output resistance R_{out}.



a) Transistors Q_1 and Q_2 are used in CS configuration. Therefore an increase in V_s causes the small-signal drain voltage of Q_1 to increase, followed by a voltage increase at the drain of Q_2. Transistor Q_3 is used in CD configuration. An increase in gate voltage at Q_3 results in an increase at the output V_O (source of Q_3) which through the voltage dividing feed-back causes V_f to increase. The feed-back is indeed negative.



b) If the loop gain $1 + A\beta$ is large then $A\beta >> 1$

$$A_f = \frac{A}{1 + A\beta} \simeq \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 10$$

c) Find DC voltages : $V_{GS} - V_T = V_{OV} \rightarrow V_{GS}$ $= V_{OV} + V_T$ for all transistors $|V_{GS}| = 0.2 + 0.5$ $= 0.7$ V.

Then:

$V_{S1} = V_{DC} - V_{GS1} = 0.9 - 0.7 \rightarrow V_{S1}$ $= 0.2$ V

$V_{GS} = 0.2$ V

$V_{G3} = V_{GS3} + V_{S3} = 0.7 + 0.2 = 0.9$V

$V_{G2} = V_{DD} - V_{SG2} = V_{DD} - 0.7$

For all current sources to operate in saturation

$|V_{DS}| \geq |V_{OV}| \qquad |V_{OV}| = 0.2$ V

For source $I_1 : |V_{DS}| = V_{DD} - V_{G2} = 0.7$ V

I_2: $V_{DS} = V_{G3} = 0.9$ V

I_3: $V_{DS} = V_{S3} = 0.2$ V.

d) obtain the A-circuit

Load of feed-back network at the input: $R_1 \parallel R_2$

Load of feed-back network at the output: $R_1 + R_2$

The A-circuit is:

Where $r_{os} = r_O$ is the output resistance of the current sources

Gain of each stage:

$$\begin{cases} \text{All } g_m\text{'s} = 2I_D / V_{OV} = 2 \times 0.1 \text{ m}/0.2 \\ \qquad = 1 \text{ mA/V} \\ \text{all } r_o\text{'s} = V_A / I_D = 10/0.1 \text{ m} = 100 \text{ k}\Omega \end{cases}$$

For Q_1: $A_{V1} = \dfrac{V_{G2}}{V_S} = g_{m\,eff}\,(r_{Oeff} \parallel r_{OS})$

$$\frac{V_{G2}}{V_S} = \frac{g_{m1}}{1 + g_{m1}R_S} \cdot (r_{O1}(1 + g_{m1}R_S) \parallel r_{OS});$$

$R_S = R_1 \parallel R_2$

$R_S = 2\text{ K} \parallel 18\text{ K} = 1.8\text{ K}$ and $1 + g_m R_s$ $= 1 + 1.8 = 2.8$

$$\Rightarrow \frac{V_{G2}}{V_S} = \frac{1\text{ m}}{2.8}[(100\text{ K} \times 2.8) \parallel 100\text{ K}]$$

$= 26.3$ V/V

For Q_2:

$$A_{V2} = \frac{V_{G3}}{V_{G2}} = g_{m2}(r_{O2} \parallel r_{OS})$$

$r_{o2} = r_{os}$

$g_{m2} = g_m$

$$\frac{V_{G3}}{V_{G2}} = g_m \frac{r_O}{2} = 1\text{ m} \times \frac{100\text{ K}}{2} = 50\text{ V/V}$$

For Q_3:

$r_{OS} \parallel (R_1 + R_2) = 100\text{ K} \parallel (18\text{ K} + 2\text{ K})$ $= 16.7$ kΩ

For a common-drain amplifier:

$$A_V = \frac{r_O \parallel R_L}{(r_O \parallel R_L) + \dfrac{1}{gm}}$$

where $R_L = r_{OS} \parallel (R_1 + R_2)$

$$\Rightarrow A_{V3} = \frac{V_O}{V_{G3}} = \frac{100\text{ K} \parallel 16.7\text{ K}}{(100\text{ K} \parallel 16.7\text{ K}) + 1/1\text{ m}}$$

$= 0.93$

Find the overall voltage-gain:

$A = A_{Vi} \cdot A_{V2} \cdot A_{V3} = 26.3 \times 50 \times 0.93$

$= 1223$ V/V

(e) Find β: $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{2}{2 + 18} = 0.1$

(f) $A_f = \dfrac{V_O}{V_S} = \dfrac{A}{1 + A\beta} = \dfrac{1223}{1 + 0.1 \times 1223}$

$= 9.92$ V/V

which is $\approx \dfrac{1}{\beta} = 10$ as found in (b)

(g) For the common-drain stage:

$R_o \simeq \dfrac{1}{g_m} \parallel (r_{OS} \parallel R_1 + R_2) = 1\text{ K} \parallel 16.7\text{ K}$

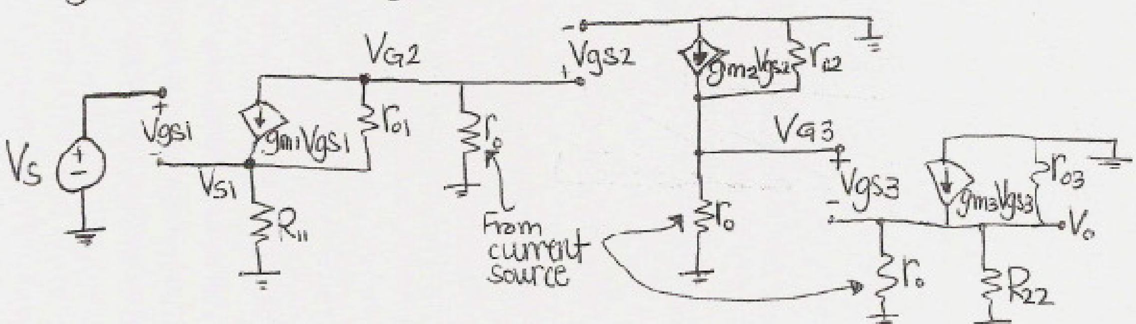
$= 944\ \Omega$

$R_{Of} = \dfrac{R_O}{1 + A\beta} = \dfrac{944}{1 + 0.1 \times 1223} = 7.66\ \Omega$

Since $R_L = \infty \Rightarrow R_{\text{out}} = R_{\text{of}}$

(d) Alternate Solution

Hybrid-π small signal model



$R_{11} = R_1 \| R_2 = 2k \| 18k = \dfrac{(2k)(18k)}{20k} = 1.8k$

$r_o = \dfrac{V_A}{I_D} = \dfrac{10}{100\mu} = 100k\Omega$

$g_m = \dfrac{2I_D}{V_{ov}} = \dfrac{2(100\mu)}{0.2} = 1m\ A/V$

$r_o = r_{o1} = r_{o2} = r_{o3}$

① $\dfrac{V_{S1}}{R_{11}} - g_{m1}(V_S - V_{S1}) + \dfrac{(V_{S1} - V_{G2})}{r_{o1}} = 0 \Rightarrow V_{S1}\left(\dfrac{1}{R_{11}} + g_{m1} + \dfrac{1}{r_{o1}}\right) - \dfrac{V_{G2}}{r_{o1}} = g_{m1} V_S$

② $+g_{m1}(V_S - V_{S1}) + \dfrac{(V_{G2} - V_{S1})}{r_{o1}} + \dfrac{V_{G2}}{r_o} = 0$

③ $\dfrac{V_{G3}}{r_o} - g_{m2} V_{G2} + \dfrac{V_{G3}}{r_{o2}} = 0$

④ $\dfrac{V_o}{r_o} - g_{m3}(V_{G3} - V_o) + \dfrac{V_o}{R_{22}} + \dfrac{V_o}{r_{o3}} = 0$

Solve ② ⇒ $V_{G2}\left(\frac{1}{r_{o1}}+\frac{1}{r_o}\right)-V_{S1}\left(g_{m1}+\frac{1}{r_{o1}}\right)+g_{m1}V_S=0$

$$V_{G2}\left(\frac{1}{r_{o1}}+\frac{1}{r_o}\right)+g_{m1}V_S=V_{S1}\left(g_{m1}+\frac{1}{r_{o1}}\right)$$

① $V_{S1}\left(\frac{1}{1.8k}+1m+\frac{1}{100k}\right)-\frac{V_{G2}}{100k}=1mV_S$

$$V_{S1}(1.566m)-10\mu V_{G2}=1mV_S$$

② $20\mu V_{G2}+1mV_S=V_{S1}(1m+10\mu)$

$$V_{S1}=\frac{20\mu V_{G2}}{(1m+10\mu)}+\frac{1mV_S}{(1m+10\mu)}=19.8mV_{G2}+.990V_S$$

plug into ① ⇒

$$19.8mV_{G2}(1.566m)+.990V_S(1.566m)-10\mu V_{G2}=1mV_S$$

$$31\mu V_{G2}+1.55mV_S-10\mu V_{G2}=1mV_S$$

$$21\mu V_{G2}=(1m-1.55m)V_S$$

$$\frac{V_{G2}}{V_S}=-26.2\ V/V$$

From ③ ⇒ $\frac{V_{G3}}{V_{G2}}\left(\cancel{\frac{1}{r_o}}+\frac{1}{r_o}\right)=\frac{g_mV_{G2}}{\frac{1}{r_o}+\frac{1}{r_o}}=50\ V/V$

From ④ ⇒ $\frac{V_o}{V_{G3}}\left(\cancel{\frac{1}{r_o}+g_{m3}+\frac{1}{R_{22}}}+\frac{1}{r_o}\right)=\frac{g_{m3}V_{G3}}{\frac{1}{r_o}+g_{m3}+\frac{1}{\underset{R_1+R_2}{R_{22}}}+\frac{1}{r_o}}$

$$\frac{V_o}{V_{G3}}=\frac{1m}{10\mu+1m+\underbrace{\frac{1}{20k}}_{50\mu}+10\mu}=0.93\ V/V$$

$$A=\frac{V_o}{V_{G3}}\times\frac{V_{G3}}{V_{G2}}\times\frac{V_{G2}}{V_S}=-0.93\times50\times26.2=\boxed{-1218\ V/V}$$