1. For the MOS differential pair with a common-mode voltage V_{CM} applied, as shown below, let $V_{DD}=V_{SS}=1.5V$, $k_n'(W/L)=4mA/V^2$, $V_t=0.5V$, $I=0.4mA$, and $R_D=2.5k\Omega$ (neglect channel-length modulation). Assume that the current source I requires a minimum voltage of 0.4V to operate properly. (worth 2 problems)

(a) Find V_{GS} for each transistor.

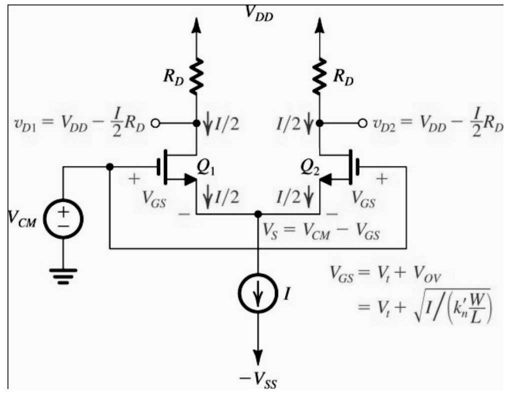
(b) For $V_{CM}=0$ find V_S, I_{D1}, I_{D2}, V_{D1}, and V_{D2}.

(c) Repeat (b) for $V_{CM}=1V$.

(d) Repeat (b) for $V_{CM}=-0.2V$.

(e) What is the highest permitted value of V_{CM}?

(f) What is the lowest value of V_{CM}?



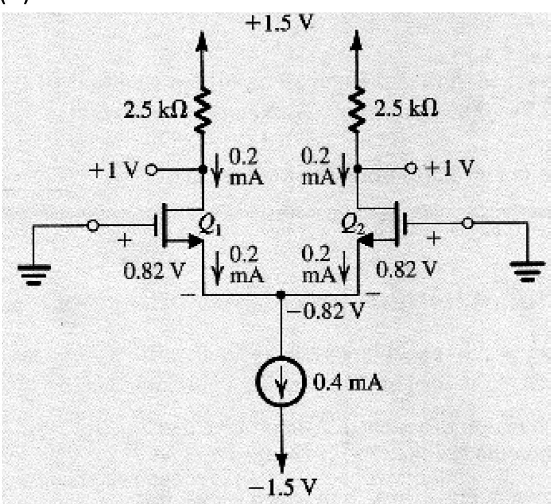
(a) $v_{G1} = v_{G2} = V_{CM}$

$I_{D1} = I_{D2} = \frac{I}{2}$ $= \frac{1}{2}k_n'(W/L)V_{OV}^2$ where Vov=(V_{GS}-V_t)

$\frac{0.4}{2} = \frac{1}{2} \times 4V_{OV}^2$ => $V_{OV} = 0.316 \text{ V}$

$V_{GS} = V_t + V_{OV} = 0.5 + 0.316 \simeq 0.82 \text{ V}$

(b)



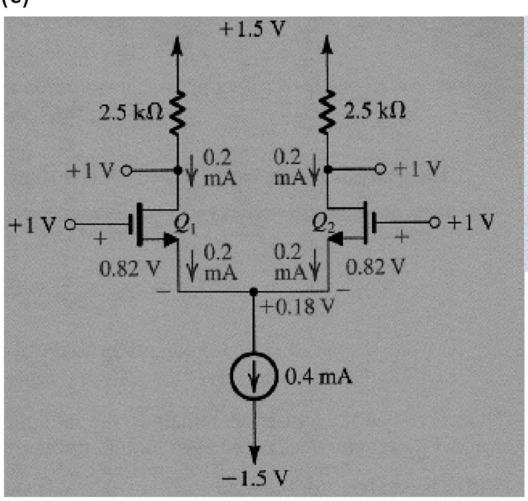
$V_S = V_G - V_{GS} = 0 - 0.82 = -0.82 \text{ V}$

$I_{D1} = I_{D2} = \frac{I}{2} = 0.2 \text{ mA}$

$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$

$= 1.5 - 0.2 \times 2.5 = 1 \text{ V}$

(c)



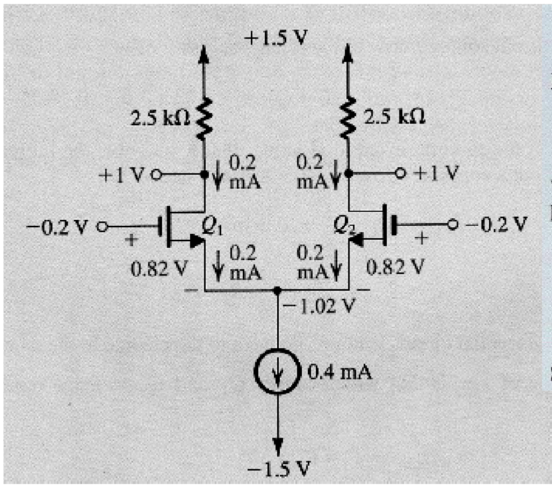
$V_S = V_G - V_{GS} = 1 - 0.82 = +0.18 \text{ V}$

$I_{D1} = I_{D2} = \frac{I}{2} = 0.2 \text{ mA}$

$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D = 1.5 - 0.2 \times 2.5 = +1 \text{ V}$

Observe that the transistors remain in the saturation region as assumed. Also observe that I_{D1}, I_{D2}, V_{D1}, and V_{D2} remain unchanged even though the common-mode voltage V_{CM} changed by 1 V.

(d)



$$V_S = V_G - V_{GS} = -0.2 - 0.82 = -1.02 \text{ V}$$

It follows that the current source I now has a voltage across it of

$$V_{CS} = -V_S - (-V_{SS}) = -1.02 + 1.5 = 0.48 \text{ V}$$

which is greater than the minimum required value of 0.4 V. Thus, the current source is still operating properly and delivering a constant current $I = 0.4$ mA and hence

$$I_{D1} = I_{D1} = \frac{I}{2} = 0.2 \text{ mA}$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D = +1 \text{ V}$$

So, here again the differential circuit is not responsive to the change in the common-mode voltage V_{CM}.

(e) The highest value of V_{CM} is that which causes Q_1 and Q_2 to leave saturation and enter the triode region. Thus,

$$V_{CM\max} = V_t + V_D$$

$$= 0.5 + 1 = +1.5 \text{ V}$$

(f) The lowest value allowed for V_{CM} is that which reduces the voltage across the current source I to the minimum required of $V_{CS} = 0.4$ V. Thus,

$$V_{CM\min} = -V_{SS} + V_{CS} + V_{GS}$$

$$= -1.5 + 0.4 + 0.82 = -0.28 \text{ V}$$

Thus, the input common-mode range is

$$-0.28 \text{ V} \le V_{CM} \le +1.5 \text{ V}$$

2. For the amplifier of Problem 1, find the input common-mode range for the case in which the two drain resistances R_D are increased by a factor of 2.

If R_D is doubled to 5 K,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2}(5 \text{ K}) = 0.5 \text{ V}$$

$$V_{CM_{\max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents I_{D1}, and I_{D2} are still 0.2 mA each.

$$V_{GS} = 0.82 \text{ V}$$

So, $V_{CM_{\min}} = V_{SS} + V_{CS} + V_{GS}$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

−0.28 V to 1.0 V

3. For the amplifier of Problem 1,

(a) Find the value of v_{id} that causes Q_1 to conduct the entire current I, and the corresponding values of V_{D1} *and* V_{D2}.

(b) Find the value of v_{id} that causes Q_2 to conduct the entire current I, and the corresponding values of V_{D1} *and* V_{D2}.

(c) Find the corresponding range of the differential output voltage (V_{D2}-V_{D1}).

(a) The value of v_{id} that causes Q_1 to conduct the entire current is $\sqrt{2}\,V_{OV}$

$\rightarrow \sqrt{2} \times 0.316 = 0.45\text{ V}$

then, $V_{D1} = V_{DD} - I \times R_D$

$= 1.5 - 0.4 \times 2.5 = 0.5\text{ V}$

$V_{D2} = V_{DD} = +1.5\text{ V}$

(b) For Q_2 to conduct the entire current:

$v_{id} = -\sqrt{2}\,V_{OV} = -0.45\text{ V}$

then,

$V_{D1} = V_{DD} = +1.5\text{ V}$

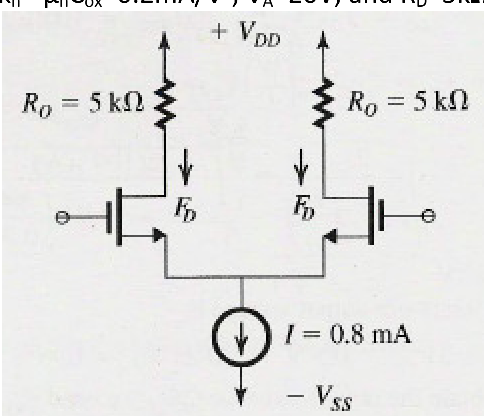
$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5\text{ V}$

(c) Thus the differential output range is:

$V_{D2} - V_{D1}$: from $1.5 - 0.5 = +1\text{ V}$

to $0.5 - 1.5 = -1\text{ V}$

4. A MOS differential amplifier is operated at a total current of 0.8mA, using transistors with a *W/L* ratio of 100, $k_n' = \mu_n C_{ox} = 0.2\text{mA/V}^2$, $V_A = 20\text{V}$, and $R_D = 5\text{k}\Omega$. Find $V_{ov} = (V_{GS} - V_t)$, g_m, r_o, and A_d.



$$I_D = \frac{I}{2} = \frac{0.8\text{ mA}}{2} = 0.4\text{ mA}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{OV})^2 \text{ So that}$$

$$V_{OV} = \sqrt{\frac{2\,I_D}{k_n'\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.4\text{ mA})}{0.2(\text{mA}/\text{V}^2)(100)}} = 0.2\text{ V}$$

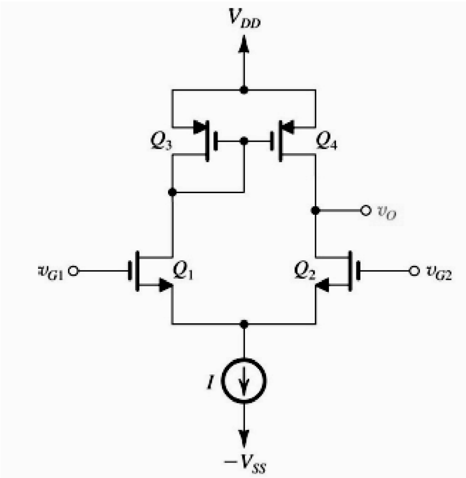
$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4\text{ mA}(2)}{0.2\text{ V}} = 4\text{ mA}/\text{V}$$

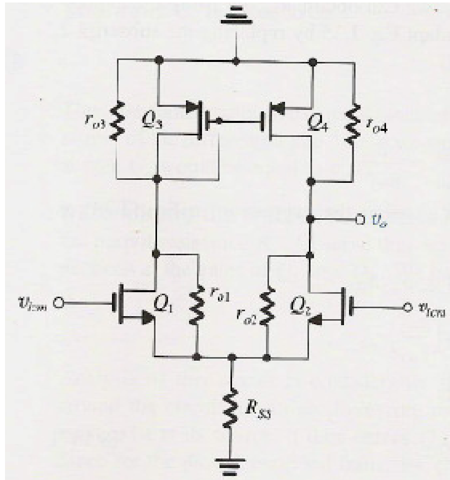
$$r_O = \frac{V_A}{I_D} = \frac{20\text{ V}}{0.4\text{ mA}} = 50\text{ k}\Omega$$

$$A_d = g_m(R_D \,\|\, r_O)$$

$$A_D = (4\text{ mA}/\text{V})(5\text{ K} \,\|\, 50\text{ K}) = 18.2\text{ V}/\text{V}$$

5. Prove that $A_{cm} \cong -\frac{r_{04}}{2R_{SS}}\frac{1}{(1+g_{m3}r_{03})} \cong -\frac{1}{2g_{m3}R_{SS}}$ for the active-loaded MOS differential amplifier below.





$$v_s = v_{icm}\frac{(2R_{SS} \| r_{o1})}{(2R_{SS} \| r_{o1}) + (1/g_{m1})}$$

$$\simeq v_{icm}$$

The short-circuit drain current i_o can be seen to be equal to the current through $2R_{SS}$; thus,

$$i_o = \frac{v_{icm}}{2R_{SS}}$$

which leads to

$$G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{1}{2R_{SS}}$$

$$R_{o1} = 2R_{SS} + r_{o1} + (g_{m1}r_{o1})(2R_{SS})$$

Similar results can be obtained for Q_2, namely, the same G_{mcm} and an output resistance R_{o2} given by

$$R_{o2} = 2R_{SS} + r_{o2} + (g_{m2}r_{o2})(2R_{SS})$$

the voltage v_{g3} can be obtained by multiplying $G_{mcm}v_{icm}$ by the total resistance between the d_1 node and ground,

$$v_{g3} = -G_{mcm}v_{icm}\left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)$$

This voltage in turn determines the current i_4 as

$$i_4 = g_{m4}v_{gs3} = g_{m4}v_{g3}$$

Thus,

$$i_4 = -g_{m4}G_{mcm}v_{icm}\left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)$$

Finally, we can obtain the output voltage v_o by writing for the output node,

$$G_{mcm}v_{icm} + i_4 + \frac{v_o}{R_{o2}} + \frac{v_o}{r_{o4}} = 0$$

$$v_o = -v_{icm}\frac{r_{o4} \| R_{o2}}{2R_{SS}}\left[1 - g_{m4}\left(R_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)\right]$$

Since $R_{o2} \gg r_{o4}$ and $R_{o1} \gg r_{o3}$, we can neglect both. Also, substituting $g_{m4} = g_{m3}$, we obtain the following expression for A_{cm},

$A_{cm} \equiv \frac{v_o}{v_{icm}} \simeq -\frac{r_{o4}}{2R_{SS}}\frac{1}{1+g_{m3}r_{o3}}$ if $g_{m3}r_{o3} \gg 1$ and $r_{o3} = r_{o4}$ then, $A_{cm} \simeq -\frac{1}{2g_{m3}R_{SS}}$

6. An active-loaded MOS differential amplifier as shown in Problem 5 is specified as follows: $(W/L)_n$=200, $(W/L)_p$=200, $k_n'=\mu_nC_{ox}=2k_p'= 2\mu_pC_{ox}=0.2mA/V^2$, $|V_A|$=20V, I=0.8mA, and R_{SS}=25kΩ. Calculate G_m, R_{out}, A_d, $|A_{cm}|$.

$$(W/L)_n \times \mu_nC_{ox} = 0.2\text{ m} \times 100 = 20\text{ m}\frac{A}{V}$$

$$(W/L)p \times \mu pC_{ox} = 0.1\text{ m} \times 200 = 20\text{ m}\frac{A}{V}$$

Since all transistors have the same drain current $(I/2)$ and the name product $W/L \times \mu C_{ox}$ then all transconductances g_m are identical.

$$|V_{OV}| = \sqrt{\frac{I_D}{20\text{ mA/V}}} = \sqrt{\frac{0.8\text{ mA}}{20\text{ mA/V}}} = 0.2\text{ V}$$

thus,

$$g_m = \frac{I_D}{V_{OV}} = \frac{(0.8\text{ mA}/2)}{0.2\text{ V}} = 4\text{ m}\frac{A}{V}$$

$$G_m = g_m = 4\text{ mA/V}$$

$$R_O = r_{O2} \| r_{O4}$$

$$r_{O2} = \frac{V_{An}}{I_{D2}} = \frac{20}{(0.8\text{ m}/2)} = 50\text{ k}\Omega$$

$$r_{O4} = \frac{V_{Ap}}{I_{D4}} = \frac{20}{(0.8\text{ m}/2)} = 50\text{ k}\Omega$$

thus,

$$R_O = 50 \| 50 = 25\text{ k}\Omega$$

$$A_d = G_mR_O = 4\frac{\text{mA}}{\text{V}} \times 25\text{ k}\Omega = 100\frac{\text{V}}{\text{V}}$$

$$A_{cm} \simeq \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005\text{ V/V}$$

$$CMRR = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005} = 20,000 \rightarrow 86\text{ dB}$$

7. Design a MOS differential amplifier to operate from ±1Vsupplies and dissipate no more than 2mW in its equilibrium state. Select the value of $V_{ov}=(V_{GS}-V_t)$ so that the value of v_{id} that steers the current from one side of the pair to the other is 0.4V. The differential voltage gain A_d is to be 5 V/V. Assume $k_n'=400\mu A/V^2$ and neglect the Early effect. Specify the required values of I, R_D, and W/L.

I=2mW/(1V-(-1V))=1mA

0.4V=$\sqrt{2}Vov$

Vov=0.2828V

R_D=Ad(Vov/ I)=5(.2828V/1mA)=1,414Ω

(W/L)= I /(kn'Vov^2)=1mA/(400e-6*.2828^2)≈32

8. In an active-loaded differential amplifier of the form shown in Problem 5, all transistors are characterized by $k_n'(W/L)=3.2mA/V^2$ and $|V_A|=20V$. Find the bias current I for which the gain v_o/v_{id}=100V/V.

Each transistor has $I_D = \frac{I}{2}$

$$r_{o2} = r_{o4} = r_o = \frac{|V_A|}{I_D}$$

$$A_d = \frac{1}{2} g_m r_o$$

Since $g_m = \frac{I_D}{V_{OV}/2}$,

$$A_d = \frac{1}{2}\left(\frac{2I_D}{V_{OV}}\right) \cdot \frac{|V_A|}{I_D} = \frac{|V_A|}{V_{OV}}$$ substituting, we find that

$$V_{OV} = \frac{|V_A|}{A_d} = \frac{20\ \text{V}}{100\ \text{V/V}} = 0.2\ \text{V}$$

$$I = 2I_D = (2)\frac{1}{2}k'(W/L)V_{ov}^2$$

$I = 3.2\ \text{mA/V}^2\ (0.2\ \text{V})^2 = 128\ \mu\text{A}$

9. It is required to design the active-loaded differential amplifier shown in Problem 5 to obtain a differential gain of 50 V/V. The technology available provides $\mu_n C_{ox}= 4\mu_p C_{ox}=400\mu A/V^2$, $|V_A|=10$, L=0.5µm, $|V_t|$=0.5V, and operates from ±1V supplies. Use a bias current I=200µA and operate all devices at $|V_{GS}-V_t|$=0.2V.

(a) Find the W/L ratios of the four transistors.

(b) If V_{CM}=0, what is the allowable range of v_o?

(c) If I is delivered by a simple NMOS current source operated at the same $V_{GS}-V_t$ and having the same channel length as the other four transistors, determine the CMRR obtained.

(a) $I_{D1}= I_{D2}= I_{D3}= I_{D4}= I/2$ $= \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$ $= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$$\left(\frac{W}{L}\right)_{1-2} = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{400\ \mu\text{A/V}\ (0.2\ \text{V})^2} = 12.5$$

For Q_3 and Q_4,

$$\left(\frac{W}{L}\right)_{3-4} = \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100\ \mu\text{A})}{100\ \mu\text{A/V}^2\ (0.2\ \text{V})^2} = 50$$

(b) Vomax=VG4+$|V_t|$=V_{DD}-$|V_{GS}|$+$|V_t|$=1-0.2=0.8V

Vomin=VG2-Vt=0-0.5=-0.5V

Range is -0.5 to 0.8V

(c) Q_5 delivers $I = 200\ \mu\text{A}$, and $L = 0.5\ \mu m$,

$V_{ov} = 0.2$ V. So,

$$r_{O5} = \frac{|V_A'|}{I} \cdot L = \frac{(20\ \text{V}/\mu\text{m})(0.5\ \mu\text{m})}{0.2\ \text{mA}} = 50\ \text{k}\Omega$$

$$r_{o3} = r_{o4} = r_o = 100\ \text{k}\Omega$$

$$A_{cm} = \frac{v_O}{v_{icm}} \approx -\frac{r_{O4}}{2R_{SS}} \cdot \frac{1}{1 + g_{m3} r_{O3}} = -\frac{100\ \text{k}}{2(50\ \text{k})} \cdot \frac{1}{1 + (1\ \text{mA/V})(100\ \text{k})} = -0.01$$

$$CMRR(\text{dB}) = 20 \log_{10} \frac{|A_d|}{|A_{cm}|} = 20 \log_{10} \left(\frac{50}{0.01}\right) = 74\ \text{dB}$$